Higgs decays in the dimension-6 Standard Model Effective Field Theory at one-loop

Student Seminar

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Work done in collaboration with Rhorry Gauld and Ben Pecjak; full details in 1512.02508.

Outline

Standard Model Effective Field Theory

Standard Model

$$\begin{aligned}\mathcal{L}_{\rm SM} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &+ i\bar{\psi}\not{D}\psi \\ &+ \bar{\psi}_i Y_{ij} H \psi_j + \text{h.c.} \\ &+ |D_\mu H|^2 - V(H)\end{aligned}$$

Standard Model Effective Field Theory

Standard Model

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}\not{D}\psi + \bar{\psi}_i Y_{ij} H \psi_j + \mathrm{h.c.} + |D_\mu H|^2 - V(H)$$

SMEFT

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{D5}} + \mathcal{L}_{\mathrm{D6}} + \mathcal{L}_{\mathrm{D7}} + \ldots$$

where,

$$\mathcal{L}_{\mathrm{D}k} = \sum_i C_{ki} Q_{ki}$$

- Q_{ki} are simply operators built from SM d.o.f of dimension k, while i runs over all operators available at that dimension which satisfy Lorentz and $SU(3) \times SU(2) \times U(1)$ gauge symmetry

Standard Model Effective Field Theory

For this talk we'll restrict ourselves to

$$\begin{aligned}\mathcal{L}_{\mathrm{SMEFT}} &= \mathcal{L}_{\mathrm{SM}} \ + \ \mathcal{L}_{\mathrm{D6}} \\ &= \mathcal{L}_{\mathrm{SM}} \ + \ \sum_{i=1}^{59} C_i Q_i\end{aligned}$$

- Baryon number conserving operators only
- 80 Generated from Buchmuller & Wyler, but over complete basis
- Minimal basis of 59 operators in 1008.4884
- The Wilson coefficients are dimensionful, $C_i = \frac{\tilde{C}_i}{\Lambda^2_{\mathsf{NP}}}$
- Choice of basis

"It is really amazing that no author of almost 600 papers that quoted Ref. [3] over 24 years has ever decided to rederive the operator basis from the outset to check its correctness. As the current work shows, the exercise has been straightforward enough for an M. Sc. thesis..."

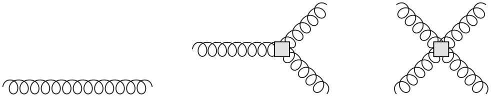
Motivation for SMEFT

- The rationale behind extending the standard model in this manner stems from the idea that the SM is simply a low energy effective field theory
- If new physics exists *at high energy*, then the effects of integrated out new particles should manifest itself as non-renormalisable operators
- Same idea as four-quark operators, Fermi-theory, Higgs EFT (large m_t limit) ...
- In the abscence of direct hint of a particular model, this is a general way to proceed

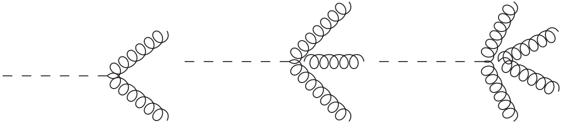
Example Operators

$$Q_{\mathsf{H}G} = H^\dagger H \; G^A_{\mu\nu} G^{A\mu\nu}$$

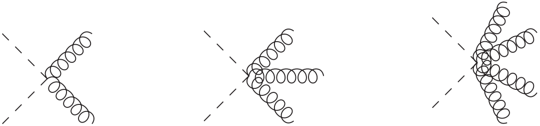
$v^2 G \cdot G$



$vh G \cdot G$



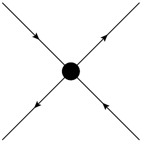
$h^2 G \cdot G$



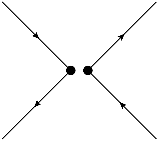
Four Fermion operators

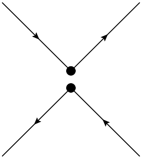
- One needs to be careful when dealing with four-fermion operators
- Different possible Dirac structures for the same vertex

- Consider the operators
  $Q_{ll} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$,
  $Q_{le} = (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$



- $Q_{ll} = (\bar{e} \gamma_\mu \nu_e)(\bar{\nu}_\mu \gamma^\mu \mu)$ gives the same vertex as
  $Q_{le} = (\bar{\nu}_\mu \gamma_\mu \nu_e)(\bar{e} \gamma^\mu \mu)$, modulo handedness of massive particles.





$1 : X^3$

Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2 : H^6$

Q_H	$(H^\dagger H)^3$

$3 : H^4 D^2$

$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5 : \psi^2 H^3 + \text{h.c.}$

Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4 : X^2 H^2$

Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6 : \psi^2 X H + \text{h.c.}$

Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7 : \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

$8 : (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$

$8 : (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$

....

Effect of Dimension-6 operators

Adding such operators alters the definition of the SM parameters at tree level.

Effect of Dimension-6 operators

- Consider $Q_H = (H^\dagger H)^3$ which alters the shape of the Higgs doublet potential (at order v^2/Λ^2).

$$V(H) = \lambda(H^\dagger H - \frac{1}{2}v^2)^2 - C_H(H^\dagger H)^3$$

which gives a new minimum,

$$\langle H^\dagger H\rangle = \frac{v^2}{2}\left(1 + \frac{3C_H v^2}{4\lambda}\right) \equiv \frac{1}{2}v_T^2$$

- We should thus use v_T as our new vev: filters into masses, Higgs interactions, ...

Effect of Dimension-6 operators

- Similarly we get effective mass and Yukawa matrices

$$
\begin{aligned}
[M_f]_{rs} &= \frac{v_T}{\sqrt{2}}\left([Y_f]_{rs} - \frac{1}{2}v_T^2 C^*_{\substack{fH\\sr}}\right) \\
[\mathcal{Y}_f]_{rs} &= \frac{1}{\sqrt{2}}\left([Y_f]_{rs}\,[1 + C_{H,\mathrm{kin}}] - \frac{3}{2}v_T^2 C^*_{\substack{fH\\sr}}\right) \\
&= \frac{1}{v_T}\,[M_f]_{rs}\,[1 + C_{H,\mathrm{kin}}] - \frac{v_T^2}{\sqrt{2}} C^*_{\substack{fH\\sr}}
\end{aligned}
$$

- Can lead to flavour violating effects
- Impose MFV: essentially require that the mass and Yukawa matrices are simultaneously diagonalisable

Effect of Dimension-6 operators

- We also need to redefine the gauge fields and couplings..
  - E.g. $H^\dagger H G^A_{\mu\nu} G^{A\mu\nu}$
- The actual expressions for the new terms are not relevant to the talk however
- We'll denote objects with a bar as those that appear in the covariant derivative in the broken phase of the theory

$$D_\mu = \partial_\mu + i\frac{\bar{g}_2}{\sqrt{2}}\left[W^+_\mu T^+ + W^-_\mu T^-\right] + i\bar{g}_Z\left[T_3 - \bar{s}^2 Q\right] Z_\mu + i\bar{e}QA_\mu$$

$$\bar{e} = \bar{g}_2 \sin\bar{\theta} - \frac{1}{2}\cos\bar{\theta}\bar{g}_2 v_T^2 C_{\mathsf{HWB}}$$

Input Parameters

- Before proceeding, it's necessary to specify the input parameters. How do we want the answer expressed?
- Choose to work with the following independent, physical parameters
  - $\bar{e}, m_H, M_W, M_Z, m_f, C_i$
- In practise, we'll choose to eliminate M_W in terms of the Fermi-constant G_F. We'll come to this later.

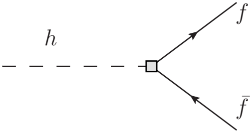
Tree-level Higgs decay: SMEFT style

With the SMEFT framework now in place, it is possible to study the decay of the Higgs in this context. The tree-level decay amplitude for the Higgs to fermions is straight forward. Simply the effective Yukawa coupling from earlier dressed with external spinors.

$$i\mathcal{M}^{(0)}(h \to f\bar{f}) = -i\bar{u}(p_f)\left(\mathcal{M}^{(0)}_{f,L}P_L + \mathcal{M}^{(0)*}_{f,L}P_R\right)v(p_{\bar{f}})$$

where

$$\mathcal{M}^{(0)}_{f,L} = \frac{m_f}{v_T}\left[1 + C_{H,\mathrm{kin}}\right] - \frac{v_T^2}{\sqrt{2}}C^*_{fH}$$



Renormalisation Proceedure

- One-loop calculation proceeds in two parts:
  - Bare one-loop matrix elements
  - UV counter-terms
- Renormalise masses and electric charge in the on-shell scheme
- Renormalise Wilson coefficients in the $\overline{\mathrm{MS}}$ scheme
  - Standard for EFT calculations

Renormalisation

- Wavefunction, mass, and electric charge renormalisation
- Defining the renormalised fields in terms of bare ones, indicated with the superscript (0)

$$
\begin{aligned}
h^{(0)} &= \sqrt{Z_h} h = \left(1 + \frac{1}{2}\delta Z_h\right) h \\
f_L^{(0)} &= \sqrt{Z_f^L} f_L = \left(1 + \frac{1}{2}\delta Z_f^L\right) f_L \\
f_R^{(0)} &= \sqrt{Z_f^R} f_R = \left(1 + \frac{1}{2}\delta Z_f^R\right) f_R
\end{aligned} \tag{1}
$$

$$
M^{(0)} = M + \delta M \qquad \bar{e}_0 = \bar{e} + \delta \bar{e} \tag{2}
$$

Renormalisation

- The on-shell scheme gives us our renormalisation conditions

$$\begin{aligned}\delta Z_f^L =& -\widetilde{\mathrm{Re}}\,\Sigma_f^L(m_f^2) + \Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2) \\ & - m_f^2 \frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\underbrace{\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2)}_{\text{Two-point functions}}\right]\Bigg|_{p^2=m_f^2}\end{aligned}$$

$$\begin{aligned}\delta Z_f^R =& -\widetilde{\mathrm{Re}}\,\Sigma^{f,R}(m_f^2) \\ & - m_f^2 \frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2)\right]\Bigg|_{p^2=m_f^2}\end{aligned}$$

$$\delta Z_h = -\mathrm{Re}\frac{\partial \Sigma^H(k^2)}{\partial k^2}\Bigg|_{k^2=m_H^2}$$

Renormalisation

- The mass counterterms are computed as

$$\delta m_f = \frac{m_f}{2}\widetilde{\mathrm{Re}}\left(\Sigma_f^L(m_f^2) + \Sigma_f^R(m_f^2) + \Sigma_f^S(m_f^2) + \Sigma_f^{S*}(m_f^2)\right)$$

$$\frac{\delta M_W}{M_W} = \widetilde{\mathrm{Re}}\,\frac{\Sigma_T^W(M_W^2)}{2M_W^2}$$

- The electric charge renormalisation can also be computed from two-point functions

$$\frac{\delta \bar{e}}{\bar{e}} = \frac{1}{2}\frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}\bigg|_{k^2=0} + \frac{(v_f - a_f)}{Q_f}\underbrace{\frac{\Sigma_T^{AZ}(0)}{M_Z^2}}_{\text{Subleading in a limit we'll consider}}$$

Wilson Coefficient renormalisation

We use the $\overline{MS}$ scheme for the renormalisation of the Wilson coefficients. To one-loop order, we can write

$$C_i^{(0)} = C_i(\mu) + \frac{\delta C_i(\mu)}{16\pi^2} = C_i(\mu) + \frac{1}{2\hat{\epsilon}}\frac{1}{16\pi^2}\dot{C}_i(\mu)$$

$$\dot{C}_i(\mu) \equiv 16\pi^2 \left(\mu \frac{d}{d\mu} C_i(\mu)\right)$$

But the anomalous dimension mixes the operators $\mu \frac{d}{d\mu} C_i(\mu) = \Gamma_{ij} C_j(\mu)$.

These were recently fully worked out in a set of three papers by Alonso, Manohar, Jenkins & Trott.

Counter-term Construction

The counterterm for the $h \to f\bar{f}$ decay amplitude can now be written as

$$i\mathcal{M}^{\text{C.T.}}(h \to f\bar{f}) = -i\bar{u}(p_f)\left(\delta\mathcal{M}_L P_L + \delta\mathcal{M}_L^* P_R\right)v(p_{\bar{f}})$$

where we distinguish SM and dimension-6 contributions through the notation

$$\delta\mathcal{M}_L = \frac{1}{16\pi^2}\left(\delta\mathcal{M}_L^{(4)} + \delta\mathcal{M}_L^{(6)}\right) + \ldots$$

Counter-term Construction

Form of the counter-terms

$$\delta\mathcal{M}_L^{(4)} = \frac{m_f}{v_T}\left(\frac{\delta m_f^{(4)}}{m_f} - \frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right)$$

$$\begin{aligned}\delta\mathcal{M}_L^{(6)} =& \left(\frac{m_f}{v_T}C_{H,\mathrm{kin}}\right)\left(\frac{\delta m_f^{(4)}}{m_f} - \frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right)\\ &- \frac{v_T^2}{\sqrt{2}}C_{bH}^*\left(2\frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right)\\ &+ \frac{m_f}{v_T}\left(\frac{\delta m_f^{(6)}}{m_f} - \frac{\delta v_T^{(6)}}{v_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_f^{(6),L} + \frac{1}{2}\delta Z_f^{(6),R*}\right)\\ &+ \frac{m_f}{v_T}\delta C_{H,\mathrm{kin}} - \frac{v_T^2}{\sqrt{2}}\delta C_{fH}^*\end{aligned}$$

Recap

Right, where do we stand...

- Chosen input parameters
  - $\bar{e}, m_H, M_W, M_Z, m_f, C_i$
- Calculated tree-level decay $i\mathcal{M}^{(0)}(h \to f\bar{f})$
- Chosen renormalisation proceedure
  - Masses & electric charge in on-shell scheme
  - Wilson coefficients in $\overline{MS}$ scheme
  - These gave prescriptions for how to construct the counter-terms
- We have expressions for the counter-terms $i\mathcal{M}^{\text{C.T.}}(h \to f\bar{f})$

We now have the ingredients to calculate the one-loop corrections...

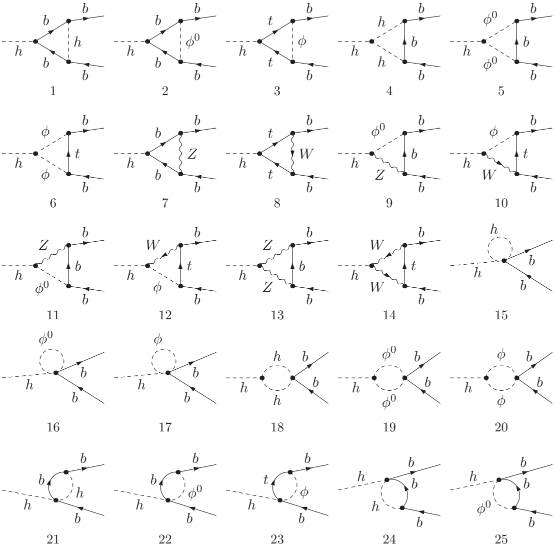
$$\mathcal{M}^{(1)}(h \to f\bar{f}) = \mathcal{M}^{(1),\text{bare}} + \mathcal{M}^{\text{C.T.}}$$

We will do this calculation in the limit of vanishing gauge couplings and further, only keep the log dependence or pieces proportional to m_t in the finite part.

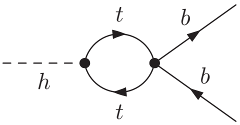
Sample Diagrams

The dimension-6 contributions are inserted onto the relevant vertices, and contributions to $\mathcal{O}(1/\Lambda_{NP}^2)$ are kept.

Note the presence of Diagrams 15-17 which are generated solely by Class 5 operators.



Bare one-loop Matrix Elements



We'll discuss the contribution from four-fermion operators.

Four-Fermion operators

We denote the non-vanishing contribution for the sum of all four-fermion diagrams to the bare matrix element by

$$i\mathcal{M}_8^{(1),\text{bare}}(h \to f\bar{f}) = -i\frac{1}{16\pi^2}\bar{u}(p_f)\left(C_{8,f}^{L,(1),\text{bare}}P_L + C_{8,f}^{R,(1),\text{bare}}P_R\right)v(p_{\bar{f}})$$

It is found that

$$\begin{aligned} C_{8,b}^{L,(1),\text{bare}} &= \frac{1}{v_T}\frac{1}{\epsilon}\Bigg[4m_b\left(3m_b^2 - \frac{m_H^2}{2}\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + 2m_\tau\left(3m_\tau^2 - \frac{m_H^2}{2}\right)C_{l\tau bq}^* \\ &\quad - m_t\left(3m_t^2 - \frac{m_H^2}{2}\right)\left((1+2N_c)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right)\Bigg] + C_{8,b}^{L,(1),\text{fin}} \end{aligned}$$

$$\begin{aligned} C_{8,b}^{L,(1),\text{fin}} &= \frac{1}{v_T}\Bigg[m_b\left(4\hat{I}_8^b - 6m_b^2 + m_H^2\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + 2m_\tau\hat{I}_8^\tau C_{l\tau bq}^* \\ &\quad - m_t\hat{I}_8^t\left((2N_c+1)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right)\Bigg] \end{aligned}$$

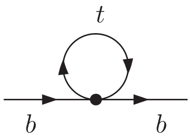
Four-Fermion operators

To renormalise we need to find all the four-fermion contributions from the expression for the counter term. Mass renormalisation as an example.

Recall: $\mathcal{M}_L^{(6)} = \left(\frac{m_f}{v_T} C_{H,\text{kin}}\right)\left(\frac{\delta m_f^{(4)}}{m_f} - \frac{\delta v_T^{(4)}}{v_T} + \ldots\right) + \frac{m_f}{v_T}\delta C_{H,\text{kin}} - \frac{v_T^2}{\sqrt{2}}\delta C_{fH}^*$

$$\delta m_b^{(6)} = \frac{1}{\epsilon}\left[\frac{m_t^3}{2}\left((2N_c+1)\left(C_{qtqb}^{(1)} + C_{qtqb}^{(1)*}\right) + c_{F,3}\left(C_{qtqb}^{(8)} + C_{qtqb}^{(8)*}\right)\right)\right.$$
$$\left. - 4m_b^3\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + m_\tau^3\left(C_{l\tau bq} + C_{l\tau bq}^*\right)\right] + \delta m_b^{\text{fin}}(\mu)\,,$$

$$\delta m_b^{\text{fin}}(\mu) = \frac{m_t}{2}\hat{A}_0(m_t^2)\left((2N_c+1)\left(C_{qtqb}^{(1)} + C_{qtqb}^{(1)*}\right) + c_{F,3}\left(C_{qtqb}^{(8)} + C_{qtqb}^{(8)*}\right)\right)$$



Four-fermion Operators

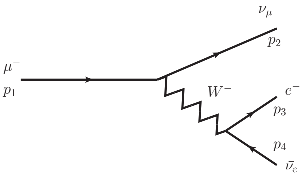
After cancelling the divergences, we find

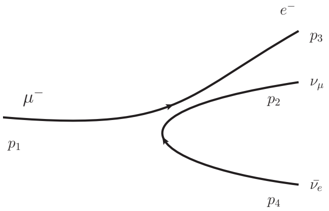
$$
\begin{aligned}
v_T C_{8,b}^{L,(1)} &= m_b(m_H^2 - 4m_b^2)\left(1 - 2\hat{b}_0(m_H^2, m_b^2, m_b^2)\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) \\
&+ m_\tau(m_H^2 - 4m_\tau^2)\hat{b}_0(m_H^2, m_\tau^2, m_\tau^2)C_{l\tau bq} \\
&+ \frac{m_t}{2}(m_H^2 - 4m_t^2)\hat{b}_0(m_H^2, m_t^2, m_t^2)\left((2N_c + 1)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right) \\
&- \frac{1}{2}\frac{v_T^2}{\sqrt{2}}\dot{C}_{bH}^{(4f)*}\ln\left(\frac{m_H^2}{\mu^2}\right)
\end{aligned}
$$

- In principle, this is it
- Substitute for each counter-term as needed
- But we want to express our answer in terms of G_F, not M_W...

Fermi Constant: G_F

- We said before we'd like to replace M_W with the Fermi constant G_F as one of our input parameters
- Defined and extracted from muon decay





$$\frac{1}{\sqrt{2}}\frac{1}{v_T^2} = G_F - \frac{1}{\sqrt{2}}\left(C^{(3)}_{\underset{ee}{Hl}} + C^{(3)}_{\underset{\mu\mu}{Hl}}\right) + \frac{1}{2\sqrt{2}}\left(C_{\underset{\mu e e \mu}{ll}} + C_{\underset{e \mu\mu e}{ll}}\right) ,$$

Fermi constant at one-loop

- Necessary to work out G_F to one-loop also

$$\frac{1}{\sqrt{2}}\frac{1}{v_T^2}(1+\underbrace{\Delta r}_{\substack{\text{1-loop}\\\text{non-QED}\\\text{corrections}}}) = G_F + \overbrace{\underbrace{\Delta R^{(6,0)}}_{\substack{\text{Obtained by}\\\text{matching with}\\\text{tree-level}\\\text{from}\\\text{before}}} + \Delta R^{(6,1)}}^{\substack{\text{Finite}\\\text{SMEFT}\\\text{contribution}}}$$

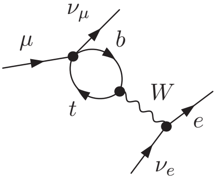
- In the limit of vanishing gauge couplings

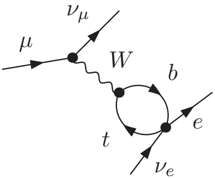
$$\Delta r = 2\left(\frac{\delta M_W}{M_W} - \frac{\delta v_T}{v_T}\right)$$

- This will have SM and dimension-6 contributions

Fermi constant at one-loop

- $\Delta R^{(6,1)}$ Obtained from the following diagrams





$$\Delta R^{(6,1)} = \frac{N_c m_t^2}{\sqrt{2} v_T^2} \left(C^{(3)}_{\substack{lq \\ \mu\mu 33}} + C^{(3)}_{\substack{lq \\ ee33}} \right) - \frac{1}{2\sqrt{2}} \left(\dot{C}^{(3)}_{\substack{Hl \\ ee}} + \dot{C}^{(3)}_{\substack{Hl \\ \mu\mu}} - \frac{1}{2} \left(\dot{C}_{\substack{ll \\ \mu e e \mu}} + \dot{C}_{\substack{ll \\ e\mu\mu e}} \right) \right) \ln \left(\frac{m_t^2}{\mu^2} \right) .$$

- It is noteworthy that the counter-terms take on a much simpler form when v_T is written in terms of G_F.

Noteworthy Points: Electric Charge renormalisation

Recall:

$$\frac{\delta \bar{e}}{\bar{e}} = \frac{1}{2} \frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}\bigg|_{k^2=0} + \text{Subleading...}$$

- Even in the limit of vanishing gauge couplings, we find it necessary to renormalise the charge.

$$\frac{\delta \bar{e}^{(4)}}{\bar{e}} = 0$$
$$\frac{\delta \bar{e}^{(6)}}{\bar{e}} = -\frac{C_\epsilon}{\epsilon} m_H^2 \hat{c}_w \hat{s}_w C_{HWB}$$

$C_\epsilon = \left(\frac{\mu}{m_t^2}\right)^\epsilon$

Noteworthy Points: Large cancelations

- Potentially dominant non-logarithmic contributions $\sim m_t^3$ in the final answers are cancelled by those in the mass renormalisation counter-terms $\delta m^{(6)}$
- Scheme dependent
- This would not happen if we'd renormalised the masses in $\overline{\mathrm{MS}}$

Implications for Phenomenology

It is possible to make some naïve estimates for the impact on SM phenomenology when also considering dimension-6 operators in fixed order.

The decay rate can be written as:

$$\Gamma(h \to f\bar{f}) = \underbrace{B_f}_{\text{Phase-space factor}} \Bigg[\overbrace{\underbrace{\Gamma_f^{(4,0)}}_{\mathcal{O}(1/\Lambda^0)} + \underbrace{\Gamma_f^{(6,0)}}_{\mathcal{O}(1/\Lambda^2)}}^{\text{Tree-level}} + \overbrace{\underbrace{\Gamma_f^{(4,1)}}_{\mathcal{O}(1/\Lambda^0)} + \underbrace{\Gamma_f^{(6,1)}}_{\mathcal{O}(1/\Lambda^2)}}^{\text{One-loop}} \Bigg]$$

$$\Gamma_f^{(4,0)} = \left[A_f^{(4,0)} \cdot A_f^{(4,0)}\right], \qquad \Gamma_f^{(4,1)} = \frac{1}{16\pi^2}\left[2A_f^{(4,0)} \cdot A_f^{(4,1)}\right],$$

$$\Gamma_f^{(6,0)} = \left[2A_f^{(4,0)} \cdot A_f^{(6,0)}\right], \quad \Gamma_f^{(6,1)} = \frac{1}{16\pi^2}\left[2\left(A_f^{(6,0)} \cdot A_f^{(4,1)} + A_f^{(4,0)} \cdot A_f^{(6,1)}\right)\right]$$

Implications for Phenomenology: Tree-level

Consider a tree-level comparison of dimension-6 and SM contributions. Numerically, at a scale of $\Lambda_{\rm NP} = 1$ TeV, for $h \to b\bar{b}$ this amounts to

$$\frac{\Gamma_b^{(6,0)}}{\Gamma_b^{(4,0)}} = -4.44\tilde{C}_{bH} + 0.03\left(4\tilde{C}_{H\Box} - \tilde{C}_{HD} - 2\left(\tilde{C}^{(3)}_{\substack{Hl\\ ee}} + \tilde{C}^{(3)}_{\substack{Hl\\ \mu\mu}}\right) + \left(\tilde{C}_{\substack{ll\\ \mu ee\mu}} + \tilde{C}_{\substack{ll\\ e\mu\mu e}}\right)\right)$$

For $\tilde{C}_{bH} \sim y_b$:

$$\frac{\Gamma_b^{(6,0)}}{\Gamma_b^{(4,0)}} = -0.12\frac{\tilde{C}_{bH}}{y_b} + \ldots$$

Implications for Phenomenology: One-loop

$$\frac{\Gamma_b^{(4,1)}}{\Gamma_b^{(4,0)}} = \frac{G_F m_t^2}{8\pi^2}\left(\frac{-18+7N_c}{3\sqrt{2}}\right) = 0.003\,,$$

$$\frac{\Gamma_b^{(6,1)}}{\Gamma_{C_{bH}}^{(6,0)}} \simeq -0.12 + 0.03\frac{\tilde{C}_{Htb}}{\tilde{C}_{bH}} + 0.13\frac{\tilde{C}_{qtqb}^{(1)}}{\tilde{C}_{bH}} + 0.03\frac{\tilde{C}_{qtqb}^{(8)}}{\tilde{C}_{bH}} + \ldots\,.$$

Conclusion and Summary

- SMEFT is a model independent way to account for possible decoupled BSM effects
- Calculated Higgs decays to b quarks at one-loop:
  - Select renormalisation scheme
  - Calculate Feynman diagrams
  - Cancelation of divergences
  - Rough Pheno implications
- Next step is to complete the calculation without vanishing gauge couplings...
- Can use renormalisation group running to resum higher order logs....
- *Work in progress..*