The Search for Baryonic Popcorn

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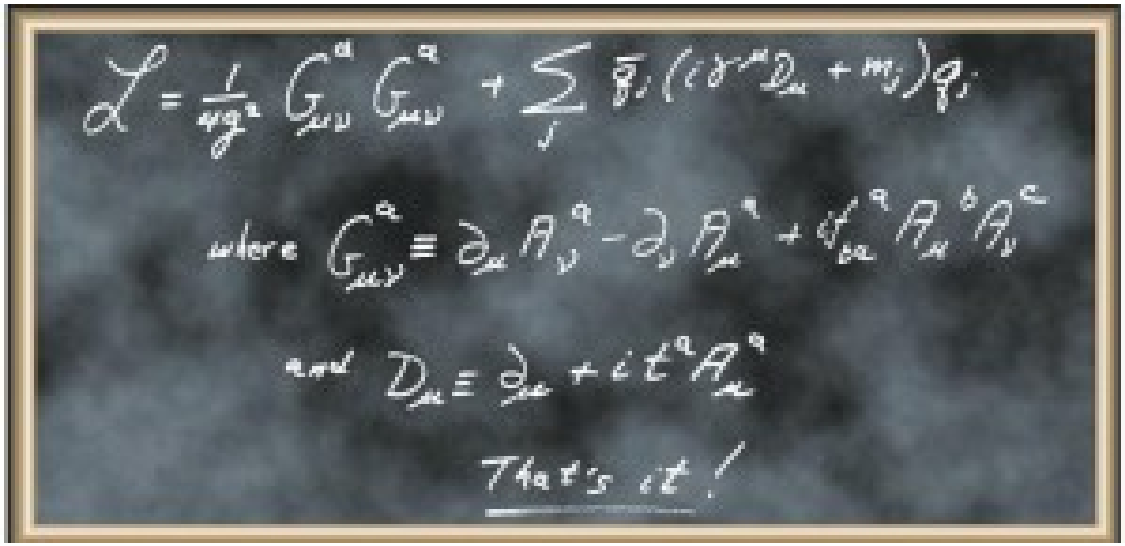
Outline

- Intro: QCD and Skyrmions
- The Sakai-Sugimoto Model
- A Low Dimensional Analogue of Sakai-Sugimoto
- Homogeneous Ansatze in Sakai-Sugimoto

Introduction: QCD is Hard

- Quantum Chromodynamics (QCD) is the quantum theory of the strong nuclear interaction involving quarks and gluons



- QCD is difficult to study: perturbation theory is complicated and limited in scope, numerical computations are resource-intensive
- We can study a low energy effective field theory instead: Skyrmions!

Introduction: Skyrmions (I)

- The Skyrme model is a nonlinear theory of pions given by

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(R_\mu R^\mu\right) + \frac{1}{16}\mathrm{Tr}\left([R_\mu, R_\nu][R^\mu, R^\nu]\right)$$

  where $R_\mu = (\partial_\mu U)U^{-1}$, $U \in SU(2)$.

- Obtained as a low-energy EFT of QCD in the large N_c limit
- Model admits topological soliton solutions, classified by a winding number

$$B = -\frac{1}{24\pi^2}\int \varepsilon_{ijk}\mathrm{Tr}\left(R_i R_j R_k\right)\, d^3x$$

- Solitons in this model (called Skyrmions) are identified with baryons with nucleon number B

Introduction: Skyrmions (II)

- Binding energies in the Skyrme model are too high!
- Many candidate solutions to reduce binding energies: sextic models, lightly bound models
- Here we will discuss a holographic approach to reducing binding energies: the Sakai-Sugimoto model

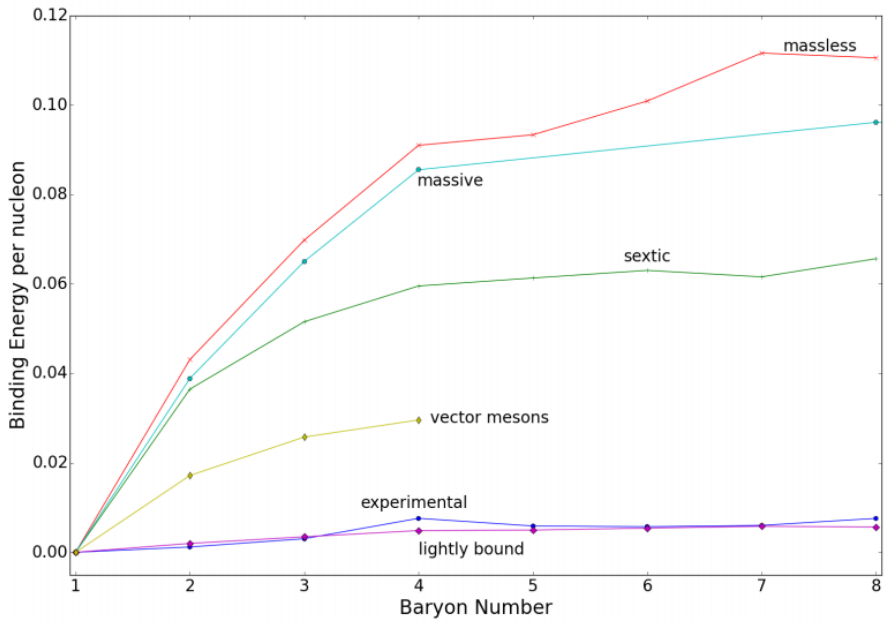


Figure: From M. Gillard, D. Harland, M. Speight 2015, arXiv:1501.05455

The Sakai-Sugimoto Model (I)

- Gravity description: N_f probe D8 branes in background of N_c D4 branes wrapped on a spatial circle.
- Leads to cigar shaped geometry

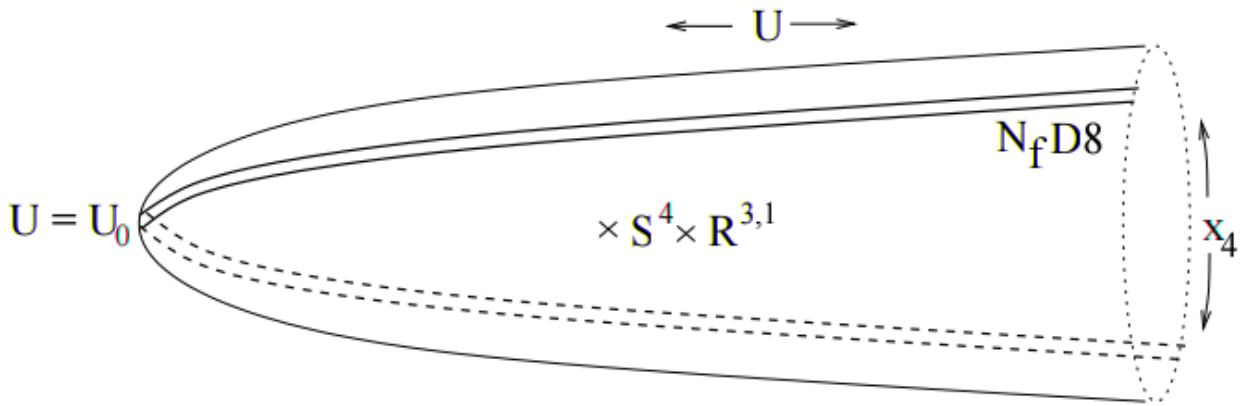


Figure: From M. Rozali et al 2008, arXiv:0708.1322

- Bulk solitons in this model correspond to extended Skyrmions (i.e. baryons) on the boundary

The Sakai-Sugimoto Model (II)

- Yang-Mills Chern-Simons theory in (4+1)-dimensions:

$$S = -\int \sqrt{-g}\left\{\frac{1}{2}\mathrm{Tr}\left(\mathcal{F}_{\Gamma\Delta}\mathcal{F}^{\Gamma\Delta}\right)\right\} dx^4 dz - \frac{\pi}{9\lambda}\int \omega_5(\mathcal{A}_\Gamma)\, d^4x dz$$

with $\mathcal{A}_\Gamma \in U(2)$ and $\mathcal{F}_{\Gamma\Delta} = \partial_\Gamma \mathcal{A}_\Delta - \partial_\Delta \mathcal{A}_\Gamma + i[\mathcal{A}_\Gamma, \mathcal{A}_\Delta]$

- Spacetime of this model is "AdS-Like", described by warped metric

$$ds^2 = H(z) dx_\mu dx^\mu + \frac{1}{H(z)} dz^2\,, \quad H(z) = \left(1+z^2\right)^{2/3}$$

- Static soliton solutions satisfy the Bogomolny bound

$$E \geq 8\pi^2 |B|\,, \quad B = \frac{1}{8\pi^2}\int \varepsilon_{ijk}\mathrm{Tr}\left(F_{iz}F_{jk}\right)\, d^3\boldsymbol{x} dz$$

The Sakai-Sugimoto Model (III)

- Advantage: Only one free parameter, λ. After expanding as Skyrmions coupled to a tower of vector mesons, all interaction coefficients are fixed.
- Advantage: Model without Chern-Simons term is BPS - has no binding energy. Large λ should then give small binding energies
- Disadvantage: The Sakai-Sugimoto model is still hard!
- Single $B = 1$ soliton computed numerically by exploiting symmetries (S. Bolognesi, P. Sutcliffe 2013, arXiv:1309.1396), but that's about it!
- Some analytical and numerical work for up to 2 vector mesons (P. Sutcliffe 2011, arXiv:1101.2402) assuming solutions share Skyrmion symmetries

Baryonic Popcorn?

- Can ask what happens in high density limit of Sakai-Sugimoto model
- Two main proposals: Dyon Salt vs. Baryonic Popcorn
- Dyon Salt: solitons sit at tip of the cigar and split into a pair of oppositely charged dyons in a salt-like configuration. Dual to Skyrme crystal at high densities
- Baryonic Popcorn: a series of *popcorn transitions* occurs where 3-d soliton crystal develops additional layers in the holographic direction
- Can investigate this phenomenon qualitatively by looking at various approximations

The Baby Skyrme Model (I)

- The baby Skyrme model is a (2+1)-dimensional $O(3)$-sigma model given by

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{\kappa^2}{4}(\partial_\mu \boldsymbol{\phi} \times \partial_\nu \boldsymbol{\phi}) \cdot (\partial^\mu \boldsymbol{\phi} \times \partial^\nu \boldsymbol{\phi})$$

  where $\boldsymbol{\phi}$ is a unit vector corresponding to the pion field

- First term is the $O(3)$-sigma term: scale invariant so low-dimensional analogue of Yang-Mills instanton term
- Second term is the baby Skyrme term: low-dimensional analogue of Chern-Simons term
- Low dimensionality of the model means full numerical solutions are easy to obtain

The Baby Skyrme Model (II)

- Can use symmetries to set up configurations with different numbers of layers in holographic direction
- Energetically favourable to split into two/three layers at critical densities - popcorn!

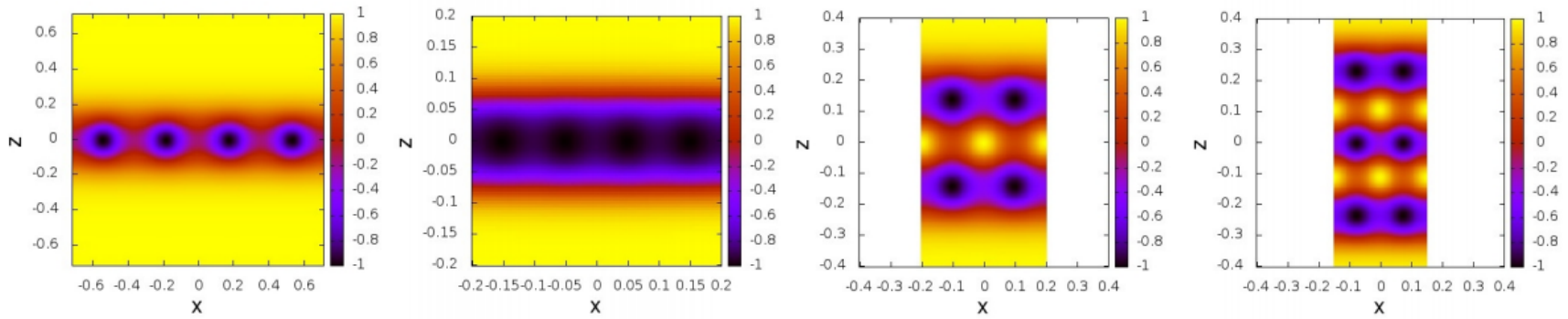


Figure: From S. Bolognesi, P. Sutcliffe 2013, arXiv:1311.2685

- Could also couple pion field to a vector meson field instead of baby Skyrme term (M. Elliot-Ripley 2015, arXiv:1503.08755) - same results qualitatively

Back to Sakai-Sugimoto: Homoegenous Ansatze

- Sakai-Sugimoto model is hard - high dimensionality, coupled nonlinear PDEs...
- At high densities, might expect soliton solutions to become approximately homogeneous i.e. only depending on holographic direction
- Can find an ansatz in the baby Skyrme model which gives spatially homogeneous energy density and baryon number density - reproduces popcorn phenomenon
- Can we find something similar in the full Sakai-Sugimoto model?

First Homogeneous Ansatz (I)

- First split gauge field into abelian and $SU(2)$ parts: $\mathcal{A}_\Gamma = \frac{1}{2}\hat{A}_\Gamma + A_\Gamma$
- Static homogeneous ansatz has non-zero components

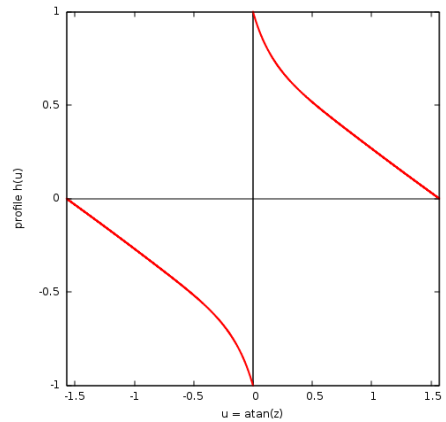
$$\hat{A}_0 = \omega(z)\,, \quad A_i = -\frac{\alpha}{2} h(z)\sigma_i$$

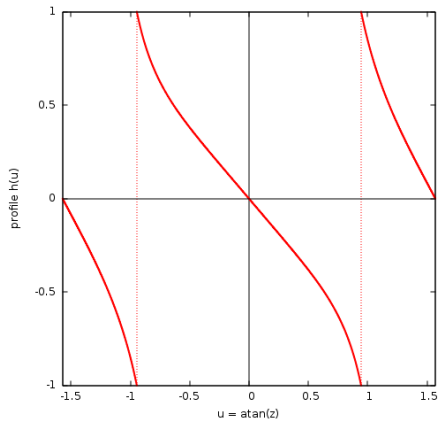
with $h(z)$ odd, $h, \omega \to 0$ as $|z| \to \infty$ and $h(0^+) = 1$

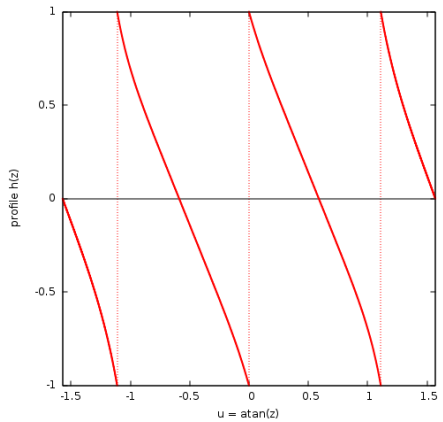
- Discontinuity at $z = 0$ required to ensure non-zero topological charge
- α related to topological charge per 3-volume ρ by $\alpha^3 = 4\pi^2\rho$
- Can generalise this construction to two- and three-layer configurations by placing discontinuities at the locations of each layer.
- Discontinuity inspired by solution to the self-dual Yang Mills equations within this ansatz: $h(z) = 1/\alpha z$

First Homogeneous Ansatz (II)

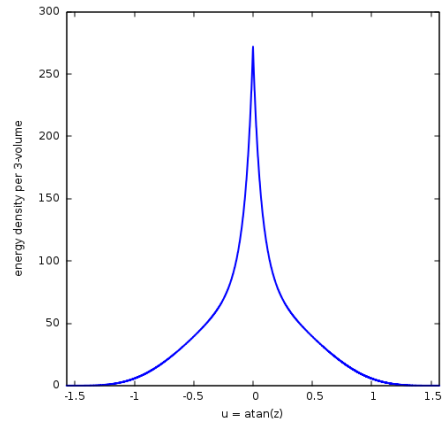
- Can find solutions numerically by solving ODEs

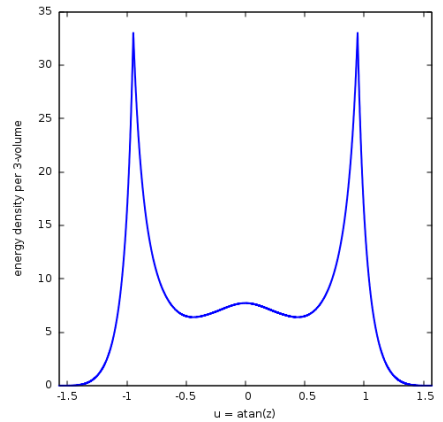


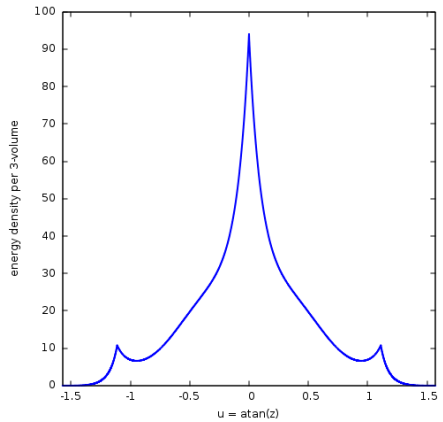




- Although gauge fields are discontinuous, physical quantities are continuous

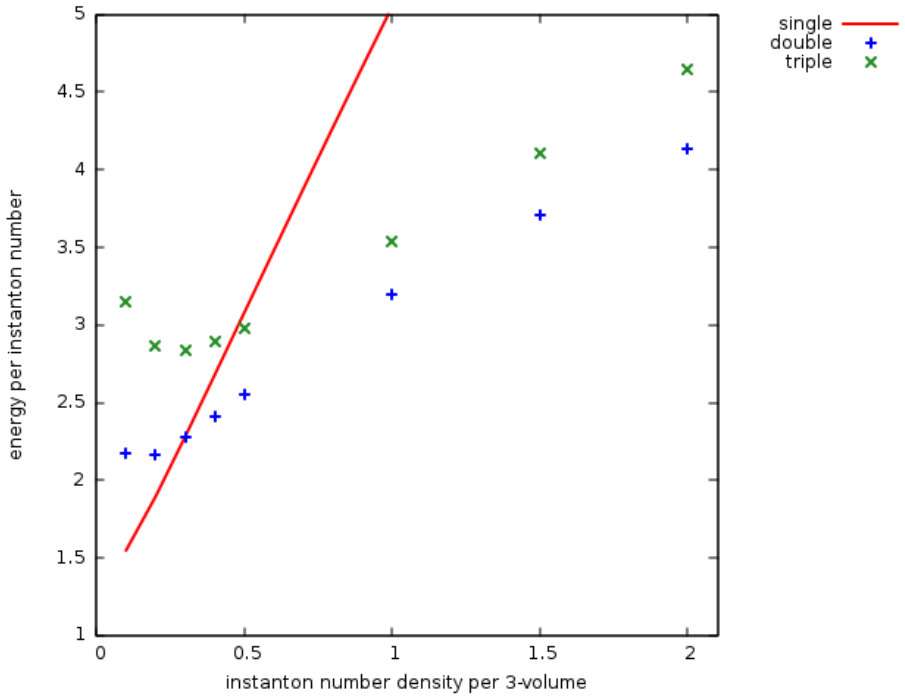






First Homogeneous Ansatz (III)

- Transition from one- to two-layers can be observed, although three-layer configurations seem unfavourable



Second Homogeneous Ansatz (I)

- Instead, introduce homogeneity at the level of the field strength F_{IJ}
- Motivated by holographic expansion of the gauge fields:

$$A_i(\boldsymbol{x}, z) = iR_i(\boldsymbol{x})\psi_+(z) + \dots$$

where R_i satisfy $\partial_i R_j - \partial_j R_i = [R_i, R_j]$

- Static ansatz has non-zero components

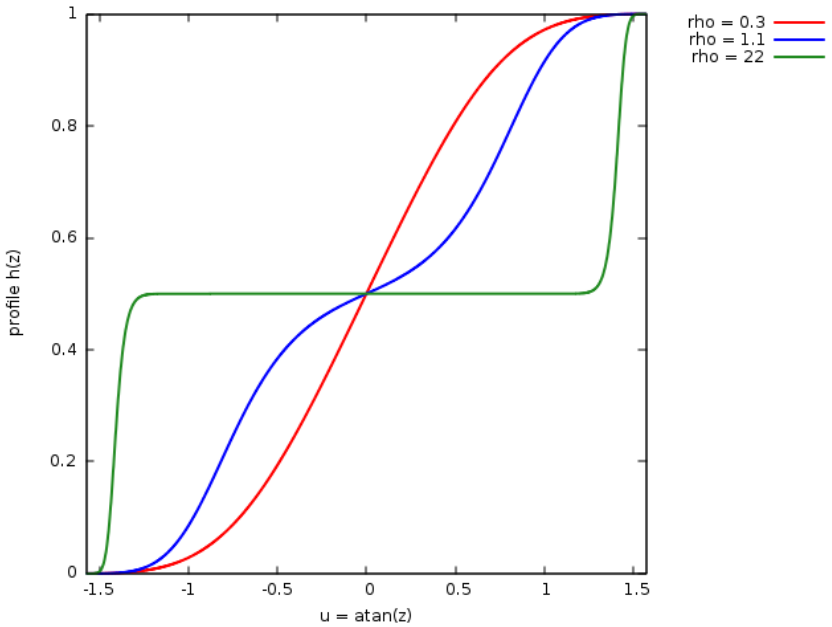
$$\hat{A}_0 = \omega(z)\,, \quad F_{ij} = -\frac{\alpha^2}{2}h(h-1)\varepsilon_{ijk}\sigma_k\,, \quad F_{iz} = \frac{\alpha}{2}h'\sigma_i$$

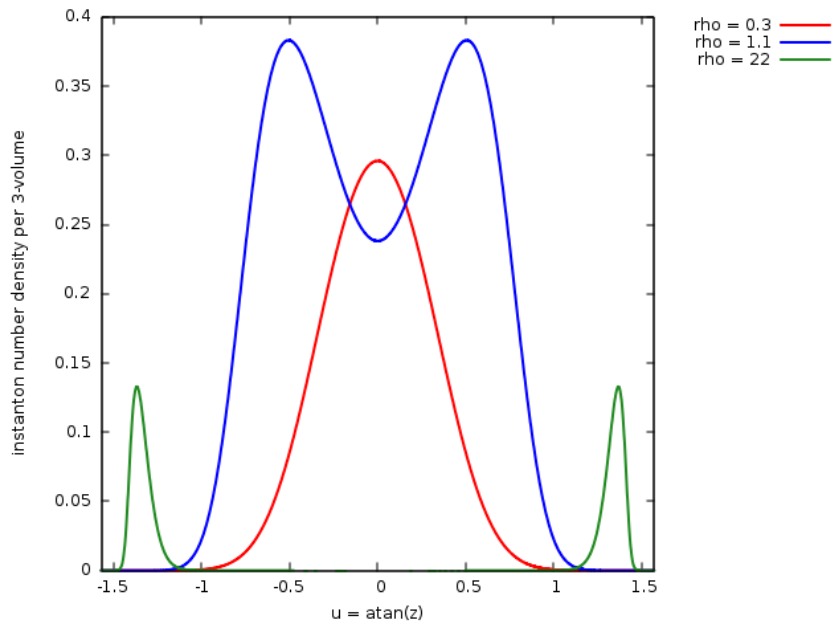
with $h(-\infty) = 0$, $h(\infty) = 1$

- α related to topological charge per 3-volume ρ by $\alpha^3 = 16\pi^2\rho$
- Disadvantage: cannot find gauge fields to reproduce this ansatz

Second Homogeneous Ansatz (II)

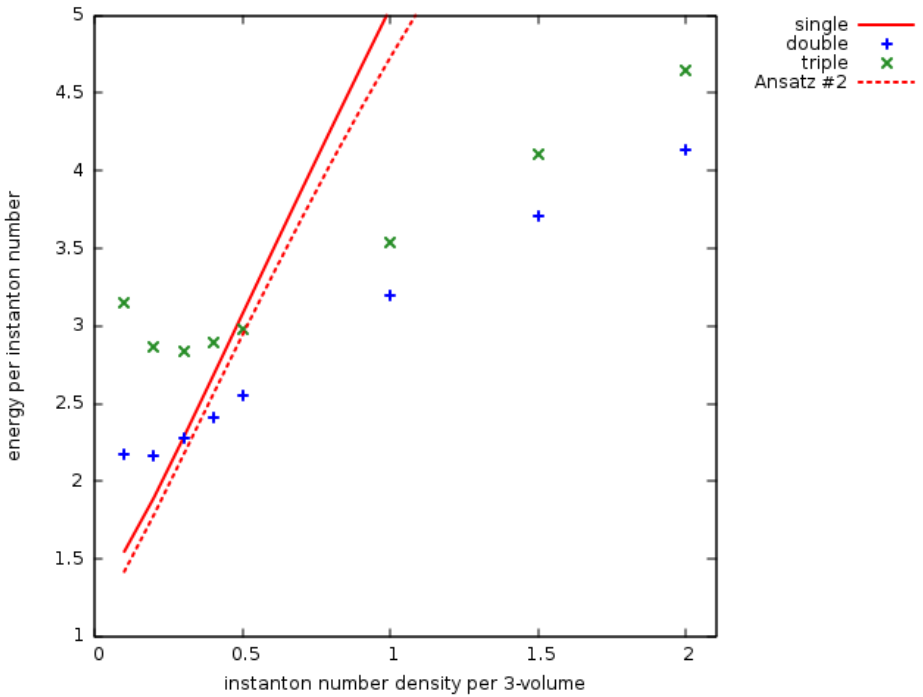
- Advantage: everything in this ansatz is smooth!
- Solutions are seen to split into two constituents and then separate





Second Homogeneous Ansatz (III)

- No split into three constituents: dyon salt? Or something else?



- Possibly haven't captured multi-chain behaviour (yet) in this ansatz?

Summary

- Introduced and motivated the Sakai-Sugimoto model as a holographic description of QCD
- Investigated different approaches to make calculations more tractable: low-dimensional analogues, homogeneous ansatze
- Evidence from low-dimensional models suggests popcorn transitions at critical densities
- Homogeneous ansatze in the full model may (tentatively) suggest a dyon salt arrangement/single popcorn transition - work in progress!

Thank you for listening.