MATH4171:

Riemannian Geometry, Michaelmas 2011.

Homework 1

Assigned on 11th October.

Starred problems due on Tuesday November 8th

Please submit solutions on or before the due date to Andrew Lobb's pigeonhole in the Mathematics Coffee Room on the 1st floor of the Mathematics Dept.

Problems:

1. What is a smooth manifold?

2. (*) Let M^n be an n-dimensional smooth manifold. Show that there exists an atlas

$$\{(U_i \subseteq M^n, V_i \subseteq \mathbf{R}^n, \phi_i : U_i \to V_i), i \in I\}$$

inducing the same topology on M^n, such that V_i is the open unit ball in $\mathbf{R}^n$ for all $i \in I$.

3. Consider the *Lemniscate of Gerono* Γ, which is given as a subset of $\mathbf{R}^2$ by

$$\Gamma = \{(x, y) \in \mathbf{R}^2 \mid x^4 - x^2 + y^2 = 0\}.$$

You should go ogle a picture of this.

As is usual, we give Γ a topology induced by its inclusion in $\mathbf{R}^2$. We do this by setting the open subsets of Γ to be exactly those sets $\Gamma \cap U$ where U is an open subset of $\mathbf{R}^2$. Possibly making use of the previous question, show that Γ with this topology does not admit the structure of a smooth 1-manifold.

4. For $a \in \mathbf{R}$ define the subset Γ_a of $\mathbf{R}^3$ by

$$(x, y, z) \in \Gamma_a \iff xyz = a,$$

(and give Γ_a a topology induced by inclusion in $\mathbf{R}^3$). For which values of a does Γ_a have the structure of a smooth 2-manifold?