MATH4171:

Riemannian Geometry

Homework 7

Assigned on 29th November 2011
Starred problems due on Tuesday December 6th

Please submit solutions on or before the due date to Andrew Lobb's pigeonhole in the Mathematics Coffee Room on the 1st floor of the Mathematics Dept.

Problems:

1. (*) Let $S^2 = \{(x, y, z) \in \mathbf{R}^3 \ : \ x^2 + y^2 + z^2 = 1\}$ be the unit sphere inside 3-space, with the induced metric from the standard Euclidean metric on $\mathbf{R}^3$.

   Let c be the curve on S^2 given by

   $$c(t) = \left(\frac{1}{\sqrt{2}}, \frac{\sin t}{\sqrt{2}}, \frac{\cos t}{\sqrt{2}} \right),$$

   and let $v \in T_{c(0)}S^2$ be given by

   $$v = (0, 1, 0) \in T_{c(0)}S^2 \subset T_{c(0)}\mathbf{R}^3.$$

   Find $X(t) = (a_1(t), a_2(t), a_3(t))$ where $X(t) \in T_{c(t)}S^2 \subset T_{c(t)}\mathbf{R}^3$ is the unique (parallel) vector field along c determined by the parallel condition

   $$\frac{D}{dt}X(t) = 0,$$

   and by

   $$X(0) = v \in T_{c(0)}S^2.$$

2. Let $\mathbf{H}^2 = \{z \in \mathbf{C} \ : \ \mathrm{Im}(z) > 0\}$ be the upper-half plane with its usual hyperbolic metric. Let c be the curve in $\mathbf{H}^2$ given by $c(t) = i + t$ for $t \in \mathbf{R}$. Identifying the tangent space to each point of $\mathbf{H}^2$ in the usual way

with $\mathbf{C}$, find the parallel vector field $X(t) \in \mathbf{C} = T_{c(t)}\mathbf{H}^2$ along c, which is determined by its value at $t = 0$:

$$X(0) = 1 \in \mathbf{C} = T_i\mathbf{H}^2.$$