MATH4171:
Riemannian Geometry

Topology prerequisites.

In this course I shall often want to say or write "topology" or "open set". Those of you who have taken a topology course will have familiarity with these concepts, but those of you who have taken just the Differential Geometry prerequisite may not have seen these notions outside of low dimensions.

There is a natural distance function d on n-dimensional real space $\mathbf{R}^n$ which just measures the Euclidean distance between two points,

$$d : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}, d(x, y) = |x - y|.$$

We define an *open ball* $B(x; r)$ of radius $r > 0$ around a point $x \in \mathbf{R}^n$ to be the set of all points strictly less than distance r from x:

$$B(x; r) = \{y \in \mathbf{R}^n \ : \ d(x, y) < r\}.$$

Then an *open subset* of $\mathbf{R}^n$ is defined to be a subset $U \subseteq \mathbf{R}^n$ such that every point in U is contained in an open ball that is *also* wholly in U:

$$U \subseteq \mathbf{R}^n \text{ open} \iff (x \in U \implies \exists r > 0 \text{ such that } B(x; r) \subseteq U).$$

Speaking informally, we say U is open if every point in U has some "elbow room" in U.

Now, starting with a set X, a *topology* on X is just a collection of subsets of X that are called, again, *open* sets. These open sets have to satisfy a few properties (I can't just take a random collection of subsets of X and call them open) and you can look these up online if you like. If U is an open set containing a point x, we call U an *open neighbourhood* of x.

The open sets defined earlier for $\mathbf{R}^n$ satisfy the required properties and so define a topology on $\mathbf{R}^n$. When we talk about *the* topology of $\mathbf{R}^n$, this is the topology that we mean.

If we need to know more about topology later in the course I shall make another handout.