AQT Summary

Below is a brief summary of the topics you should know from the lectures. You should also be able to do all the questions from the problems sheets (including homework) and the relevant questions from last year's exam paper.

1 Classical Theory

It is important to have a good understanding of relativity, particularly index notation and the concepts of time-like, light-like (null) and space-like vectors.

You should also understand how to derive the Euler-Lagrange equations for a field theory, and certainly know what the Euler-Lagrange equations and definition of conjugate momenta are.

You should be familiar with the two standard forms of point particle and string actions (the forms with and without square roots, called Nambu-Goto and Polyakov actions in the string case.)

2 Scalar Quantum Field Theory

First you must be familiar with canonical quantisation and the (bosonic) simple harmonic oscillator in quantum mechanics.

You should know what the canonical quantisation rules are for a field theory and be able to apply these rules to quantise free scalar field theory. In particular you should be able to write down the general classical solutions (as a sum of plane waves) and use the quantisation procedure to quantise the arbitrary coefficients – specifically you should understand the procedure for 3+1 dimensions and should be able to derive the results in the simpler case of 1+1 dimensions, with the standard boundary conditions (Neumann, Dirichlet or periodic.)

You should understand what normal ordering is and why it is necessary to get sensible results. You should understand how the above procedure leads to a particle interpretation of quantum field theory.

3 Quantising the String

After fixing reparametrisation invariance, the bosonic Polyakov string action has a simple form as a sum of D free scalar fields, X^μ, with the general classical solutions being an arbitrary sum of left- and right-moving waves on the worldsheet for closed strings, while open string boundary conditions relate the left- and right-moving modes.

However, there are constraints which arise from the Polyakov action before fixing the reparametrisation invariance. One method of dealing with these constraints is to solve them explicitly using "light-cone quantisation". Then $X^\pm$ are essentially fixed and X^I are arbitrary solutions of the two-dimensional wave equation, which can be expressed as Fourier series. After quantisation these Fourier coefficients become (bosonic) SHO creation/annihilation operators. The corresponding coefficients for X^- are quadratic in these operators and give rise to the Virasoro algebra.

You should be able to derive the spectrum of open and closed bosonic strings (at level $0, 1, 2, \ldots$) and understand how self-consistency leads to restrictions on the allowed number of spacetime dimensions D, and normal-ordering constant 'a' associated with L_0.

4 Symmetries and Conserved Currents

You should know what Noether's theorem is, and what is meant by a conserved current and charge, and the relation to symmetry generators in the quantum theory. You should be able to use this to calculate conserved currents for actions with specific symmetries including translations and rotations.

You should know what gauge symmetries are (Abelian and non-Abelian with unitary gauge groups) and how the gauge potential A_μ and gauge field strength $F_{\mu\nu}$ transform under gauge transformations. You should also know what the gauge invariant Yang-Mills action is, as well as the gauge covariant derivative D_μ.

5 Fermions

Fermions refer to objects with an anti-commuting nature, as opposed to bosons which naturally commute. Typically this means replacing commutators with anti-commutators in quantum mechanics. In particular you should be very familiar with the fermionic simple harmonic oscillator in quantum mechanics. In 3+1 dimensions fermions have spin $\frac{1}{2}, \frac{3}{2}, \ldots$ whereas bosons have spin $0, 1, 2, \ldots$.

You should know what the Dirac equation is and be aware that in 3+1 dimensions it describes particles with spin $\frac{1}{2}$. You should know how to manipulate the Γ-matrices which obey the Clifford algebra and know that the solutions of the Dirac equation are spinors which are acted upon by these Γ-matrices. You should be aware of the interpretation of solutions of the Dirac equation in terms of the Dirac sea (which relies on the Pauli exclusion principle) and how this leads to the concept of anti-particles.

You should know what the action for a free spinor field is and that its Euler-Lagrange equations give the Dirac equation. You should know what the canonical quantisation procedure is for fermions. You should also know what the generalisation of the spinor action to spinors transforming in the fundamental representation of a gauge group is.

6 Superstring Theory

You should know that the superstring worldsheet action (after the worldsheet reparametrisations have been fixed) is a sum of free scalars and spinors. You should be familiar with manipulations of two-dimensional anti-commuting Majorana spinors – in particular you should be able to derive various identities (involving products of two spinors with Γ-matrices including the Fierz identity) and use them, e.g. to show that the superstring action is invariant under the worldsheet supersymmetry transformations.

You should be generally familiar with the concepts involved in quantising the superstring action, particularly how the method generalises that of the bosonic string. You should know how to manipulate the super-Virasoro generators. You should know how to find the superstring spectrum and know what the GSO projection is.