

# Combinatorics and toric topology of Stasheff polytopes

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## Abstract

The Stasheff polytopes  $K_n$ ,  $n > 2$ , appeared in the Stasheff paper “Homotopy associativity of H-spaces” (1963) as the spaces of homotopy parameters for maps determining associativity conditions for the products  $a_1 \dots a_n$ ,  $n > 2$ .

Stasheff polytopes are the focus of attention of various research areas. Nowadays they have become well-known due to applications of operad theory in physics. There is a growing number of different approaches, such that bracketing, polygon dissection, plane trees, intervals and so on, which result in Stasheff polytopes.

We will describe combinatorics and toric topology of Stasheff polytopes using several constructions of these polytopes.

We will show that the two-parameter generating function  $U(t, x)$ , enumerating the number of  $k$ -dimensional faces of the  $n$ -th Stasheff polytope, satisfies the famous Burgers-Hopf equation  $U_t = UU_x$ .

We will discuss some applications of this result including an interpretation of the Dehn–Sommerville relations in terms of the Cauchy problem, and the Cayley formula in terms of conservation laws.

## References

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