XKNOTJOB DOCUMENTATION

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1. Introduction

XKnotJob is a Java programme designed to calculate Steenrod Squares for the stable Khovanov homotopy type of Lipshitz-Sarkar and the $\mathfrak{sl}_n$-stable homotopy type for bipartite diagrams of Jones-Lobb-Schütz. It can also perform other calculations such as Khovanov cohomology and the Jones polynomial.

The program is started with the command

```
java -jar XKnotJob.jar
```

or with

```
java -jar -Xms16g -Xmx16g XKnotJob.jar
```

if you want to allow the programme to use more RAM (which may be a good idea for certain calculations).

In case of problems you may want to recompile the source code which is contained in the file `XKnotJob.java`.

Other files that come with the programme are

```
rolfsen.gld  2-11_links.gld  2-10_matched.gld  torfour.gld
```

The first one gives the Rolfsen knot table, which was created from Knotscape. The second file lists all prime links with at most eleven crossings and was created using the data from the Knot Atlas. The third one is a list of matched diagrams for some of the knots in the Rolfsen table. Some of these knots are known to have no matched diagram, see [1], such as 9_{35}. But there are some knots with nine crossings missing for which it is not known whether they are not bipartite. For all prime knots with up to eight crossings a matched diagram is given. The last file is a list of those prime knots with up to 16 crossings which have 4-torsion in their Khovanov cohomology.

2. Diagrams

XKnotJob describes knots and links in form of *glued diagrams* which are similar to the planar diagram presentations commonly used. We allow certain rational tangles as in Figure 2 which we then treat as a box with four edges. Each edge in the diagram is numbered starting from 1, and each box has five numbers associated to it, the first giving the number of crossings in the tangle and the remaining four identifying the number of the edges, with the first number referring to the lower right corner, and then proceeding counter clockwise. A negative number for the crossing means the mirror of its absolute value. We allow 0 for the crossing, meaning that the right lower corner and right upper corner are connected, and also the left lower corner with the left upper corner.

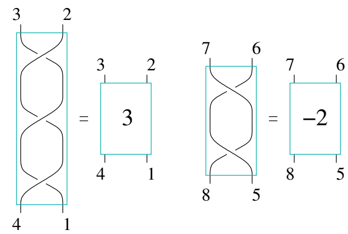

FIGURE 1. Simple tangles used in glued diagrams.

Upon starting XKnotJob has five knots and links, which are all torus links but with somewhat unusual diagrams. These diagrams were designed to simplify Steenrod square calculations by combining crossings. The pretzel link $P(-2,2,2)$ agrees with the $3,3$-torus link if we ignore orientations.

2.1. **Add Links.** Additional links can be entered by choosing the 'Add Link' button. This opens a window with various choices.

2.1.1. *Glued diagram.* This allows you to enter a diagram as described above.

2.1.2. *DT-Code.* This can be used to enter a Dowker–Thistlethwaite code such as `4 6 2` for the trefoil. If the option 'Combine crossings' is selected, the program will put together crossings that can be combined to rational tangles as in Figure 2.

The option 'Improve via R-III' can be selected to combine further crossings. It basically checks which Reidemeister-III moves are possible, and whether they can be used to minimize multi-crossings in a glued diagram. One should not use this for larger knots, as it can take a very long time to go through all possibilities and there is currently no option to cancel the process. It is usually ok for knots with up to 16 crossings, however the standard diagram for the $5,4$-torus knot is already taking forever (or longer than I have been willing to wait).

2.1.3. *Alphabetical DT-Code.* This accepts the Dowker–Thistlethwaite code in alphabetical form for links, such as `hcbcccdFgHEab`. Don't ask me what this means, it does seem to work though.

2.1.4. *Braid Code.* This accepts a braid code such as `cababAbab`. The first letter gives the number of strands, while the following letters describe the crossings, with `a` standing for the first going over the second. To mirror this crossing, use the capital letter `A`. The code `e` is accepted, and leads to the unlink with five components.

2.1.5. *Concatenate Links.* Choose two existing links and combine them to a new one. It is possible to mirror one ot both diagrams, and if a link has several components, it can be chosen which components are combined.

2.1.6. *Disjoint Union.* Just like concatenation, only simpler.

2.1.7. *Torus Links.* This creates the p,q-torus link, where q is the number of strands. The maximal number for p is 60, and the maximal number for q is 10. This is somewhat random, and XKnotJob can't do very much with the $60,10$-torus link.

2.2. **Remove Links.** You can choose several links from the list to be removed.

2.3. **Load Links.** The standard format in XKnotJob for links is `.gld`, but there are two other formats which are supported.

The `.dtc` files allow you to load files which have been created using 'Knotscape'. Simply save a list of knots in 'Knotscape', but call the file `something.dtc` in order for XKnotJob to recognize it.

Also, `.adc` files are allowed, which are basically just a list of alphabetical DT-Codes. Such files can be obtained for example from 'Knotkit'. Again files need to come with the `.adc` extension for XKnotJob to recognize them.

2.4. **Save Links.** You can choose several links from the list to be saved as a `.gld` file.

3. Link invariants

3.1. **Khovanov Cohomology.** You can choose between reduced and unreduced Khovanov cohomology. The integral version is the standard version, rational and mod k versions are available through universal coefficients. The cohomology groups can be saved as a LaTeX-file.

3.2. **$\mathfrak{sl}_n$-Cohomology, $n > 2$.** This only works if the diagram is a matched diagram, as the programme follows the Krasner algorithm for matched diagrams [3]. Only the unreduced version is available, and again it is done with integer coefficients.

3.3. **Stable Homotopy Info.** This calculates the action of Sq^2 and Bockstein homomorphisms on the Khovanov Cohomology with $\mathbb{Z}/2\mathbb{Z}$-coefficients. For matched diagrams there is also the option for $\mathfrak{sl}_n$-cohomology. The programme first calculates the cohomology to see which quantum degrees have a chance for non-trivial actions. After that one has the option between 'Stable Type' and 'Steenrod Action'. The algorithm is explained in [5]. There is an option to save information as a LaTeX-file.

3.3.1. *Stable Type.* This gives an integral calculation of a graph which symbolizes the stable Khovanov homotopy type of Lipshitz-Sarkar [4] or the $\mathfrak{sl}_n$-stable homotopy type for matched diagrams of Jones-Lobb-Schütz [2]. In many cases this determines the stable homotopy type.

3.3.2. *Steenrod Action.* The resulting graph has as vertices generators of the $\mathbb{Z}/2\mathbb{Z}$-cohomology with black edges referring to a non-trivial action of Sq^1, and blue edges referring to a non-trivial action of Sq^2. This can be derived from the 'Stable Type', but the calculation will be faster for more complicated links. If the integral cohomology does not have 2^p-torsion for $p \geq 2$, we can derive the same information from this as from 'Stable Type'.

3.3.3. *Jones Polynomial.* Both the reduced and unreduced Jones Polynomial are calculated. The 'Copy'-buttons can be used to copy them into the clipboard with the LaTeX-option making it compatible to a LaTeX-file.

3.3.4. *HOMFLYPT-Polynomial.* This is only available for matched diagrams, as the calculation simplifies considerably. The current version cannot handle disjoint union or crossings which have less than four different edges going into them. Maybe I'll finish that at some point.

3.3.5. *DT/Gauss-Code.* The DT-code is only calculated for knots, but the Gauss-Code also works for links. The 'Copy'-buttons can be used to copy them into the clipboard. Using the DT-Code to create a new diagram leads to the mirror diagram, but I'm not too bothered about that.

4. Other options

4.1. **Edit diagram.** You can change the diagram here, give the link a different name or add a comment.

4.2. **Mirror diagram.** Basically just changes the number of each crossing by a factor of -1. The speed of the stable homotopy calculations can greatly differ whether one uses the original diagram or its mirror. Experimentation is needed to figure out which one is faster though.

4.3. **Components.** Lists the components of the link, thus indicating their orientation. If a component consists of more than one edge, its orientation can be reversed.

References

[1] S. Duzhin, M. Shkolnikov, *Bipartite knots*, Fund. Math. 225 (2014), 95–102.
[2] D. Jones, A. Lobb, D. Schütz, *An $\mathfrak{sl}_n$-stable homotopy type for matched diagrams*, preprint.
[3] D. Krasner, *A computation in Khovanov-Rozansky homology*, Fund. Math. 203 (2009), 75–95.
[4] R. Lipshitz, S. Sarkar, *A Steenrod square on Khovanov homology*, J. Topol. 7 (2014), 817–848.
[5] A. Lobb, P. Orson, D. Schütz, *Calculating Steenrod squares in flow categories using handle cancellation*, preprint.

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