Radiation dosimetry through statistical analysis of biomarkers

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Radiation

 Radiation is energy in the form of waves or particles that travels through space or some material (includes heat, radio waves, light,...)

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- When we talk about radiation, we often mean ionizing radiation (α and β particles, γ-rays, X-rays, neutrons...), which carries enough energy to ionize atoms or molecules. lonizing radiation can cause serious damage to cells, tissues, and DNA.



Biodosimetry

 Dosimetry is the measurement of the absorbed dose delivered by ionizing radiation. The absorbed dose is measured in Gy (Joules per kg).

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- A biomarker (short for: "biological marker") is "any substance, structure, or process that can be measured in the body or its products and influence or predict the incidence of outcome or disease" (WHO).
- Biodosimetry is a dosimetry technique which exploits the information provided by radiation-sensitive biomarkers (usually, radiation-induced damage inside the cell nucleus) to infer the radiation dose.

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- Statistics comes into play when establishing the link between the biomarker and the absorbed dose.

Radiation biodosimetry

- Radiation accident or incident leading to irradiated blood lymphocytes.
- Need rapid and reliable procedures to determine the radiation dose contracted by individuals.
- Members of the public do not usually wear radiation dosimeters...
- Hence, triage and clinical decision making needs to rely on biomarkers to estimate the contracted radiation dose.

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- Most commonly used biomarker: Chromosome aberrations, such as dicentric chromosomes:



Cytogenetic biomarkers

Example: Frequencies of dicentrics (= aberrant chromosome having two centromeres) in n = 4400 lymphocytes after in vitro 'whole body' exposure with 200 kV X-rays.

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Xi	0	1	2	3	4	5	6	7	n _i
1	1715	268	15	2	0	0	0	0	2000
2	638	298	56	8	0	0	0	0	1000
3	247	225	85	37	6	0	0	0	600
4	99	129	92	52	21	5	2	0	400
5	48	88	97	99	36	25	5	2	400

▶ x_i : dose (in Gy) used to irradiate blood sample i, i = 1, ..., 5.

y_{ij}: counts of dicentric aberrations in *j*-th cell of blood sample *i*, *j* = 1,... n_i.

Dose-response modelling

These are count data, so a natural choice for the response distribution is Poisson, that is

$$f(y_{ij}|x_i) = e^{-\lambda_i} \frac{\lambda_i^{y_{ij}}}{y_{ij}!}$$

There is agreement in the biodosimetry literature that the Poisson means λ_i can be described by a parametric model

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$$\lambda_i = E(y_{ij}|x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$$

For parameter estimation, firstly set up the likelihood function:

$$L = \prod_{i,j} f(y_{ij}|x_i) = \prod_{i,j} e^{-\lambda_i} \frac{\lambda_i^{y_{ij}}}{y_{ij}!} \propto \prod_i e^{-n_i\lambda_i} \lambda_i^{\sum_j y_{ij}}$$

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• One can conveniently work at the aggregated data level, with data $(x_i, y_i) = (x_i, \sum_{j=1}^{n_i} y_{ij})$.

Aggregated data

Let y_i = ∑_j y_{ij}. Then the aggregated data are

Xi	ni	Уi
1.0	2000	304
2.0	1000	434
3.0	600	530
4.0	400	590
5.0	400	892

• Graphically, with circle size $\propto n_i$.



 'Empirical dose-response curve'

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Poisson regression

Model in terms of aggregated data:

$$\lambda_i \equiv E(y_i|x_i)/n_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$$

where $y_i \sim \text{Pois}(n_i \lambda_i)$.

Poisson regression model fitted via ML

special case of generalized linear models

Fitted dose-response curve, $\hat{\lambda}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{\beta}_2 x_i^2$:



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Inverse regression

Dose estimation is an inverse regression problem:

- We have a model for the dicentric count, y_i , given dose x_i .
- ▶ In practice, we want to estimate x_i given y_i!
- For instance, assume a patient has been admitted to hospital due to potential radiation exposure. A sample of $n_0 = 200$ lymphocytes was analyzed, yielding $y_0 = 150$ dicentrics.



Dose estimation from calibration curve

Mathematically, this is not a big problem. Assume the observed ratio of dicentrics is R = ^{y0}/_{n0}. Then we have

$$R = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2$$

which can be solved wrt x as

$$\hat{x} = \frac{-\hat{\beta}_1 + \sqrt{\hat{\beta}_1^2 - 4\hat{\beta}_2(\hat{\beta}_0 - R)}}{2\hat{\beta}_2}$$

• With $R = \frac{150}{200} = 0.75$, this gives

$$\hat{x} = 2.745.$$

Uncertainty

- Of course, this estimation is not *exact*.
- Can one specify the uncertainty in this process?

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▶ Uncertainty due to random variation of y₀, given x₀.



Uncertainty bounds

Combine the two sources of uncertainty ('Merkle's method', 1983):



► Here, a 95% confidence interval for the 'true' dose, x₀, is given as [2.04, 3.33].

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- Here, a 95% confidence interval for the 'true' dose, x₀, is given as [2.04, 3.33].
- Official uncertainty assessment routine suggested by the International Atomic Energy Agency [IAEA].

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- ► For the (potentially) exposed patient, count dicentrics, y₀, in a sample of n₀ cells, yielding 'test data likelihood'

 $L(y_0|\lambda, n_0) \propto e^{-n_0\lambda}\lambda^{y_0}$

where $\lambda = \beta_0 + \beta_1 x + \beta_2 x^2$, and x the (true, unknown) dose.

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$$p(x|y_0) \propto p(x) \int L(y_0|\lambda, n_0) \phi(\lambda|x) \, d\lambda$$

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 Integral has explicit solution via Hermite distribution (Higueras et al, 2015)

- Consider again the example before: Patient sample with $n_0 = 200$, $y_0 = 150$.
- ► Use the same estimated dose-response curve, $\hat{\lambda}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{\beta}_2 x_i^2$, as before:



Calibrative density for 'true' dose x, using R package radir:



Dose, x, Gy

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Dose estimate: Mode of calibrative density:



Dose, x, Gy

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▶ $\hat{x} = 2.75$.

Uncertainty assessment: 95% Credible intervals



Dose, x, Gy

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► *CI* = [2.48, 3.01]

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It's not as easy...

- NIHR-funded project (2014): Identification of appropriate response models for chromosomal aberration counts.
- Initially, carried out an extensive analysis of 11 in vitro calibration data sets.
- In turned out that the data set shown initially in this talk was the only one for which the Poisson assumption is (approximately) adequate.
- In most occasions, things are not quite as nice...
 - There is overdispersion (variance >> mean)
 - There is zero-inflation (more zero counts than one would expect under the Poisson model)

Example: Cobalt–60 γ – rays



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- Alternative models include
 - Zero-inflated regression model (ZIP)

$$P(Y_{ij} = y_{ij}) = \begin{cases} p + (1-p)\exp(-\lambda_i), & y_{ij} = 0, \\ (1-p)\exp(-\lambda_i)\lambda_i^{y_i}/y_i!, & y_{ij} > 0, \end{cases}$$

- Negative Binomial model (NB)
- Zero-inflated Negative Binomial model (ZINB)
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- However, the presence/type of violation of the Poisson model will often not be obvious (from the fitted calibration curve).
- Tools which can be used to assist in this question:
 - Model selection criteria, e.g AIC, BIC
 - Statistical tests, e.g. LR tests, Score tests
 - Diagnostic plot

Model selection

- Assume we we need to decide between one of the response distributions Poisson, ZIP, NB, ZINB.
- ► Use Akaike Information Criterion: Find model which minimizes AIC = -2 log L + 2p (Goodness-of-fit/ complexity trade-off).

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- ► For instance, for the two data sets displayed so far,

AIC for	X–rays	$\gamma extsf{-}rays$
Poisson	7622.55	7504.73
Zero-inflated Poisson (ZIP)	7623.57	7490.36
Negative Binomial (NB)	7624.56	7489.10
ZINB	7626.39	7491.44

► For the X-rays, the Poisson model well supported.

▶ For the *γ*-rays, evidence for NB or perhaps ZIP model.

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- ► Use Akaike Information Criterion: Find model which minimizes AIC = -2 log L + 2p (Goodness-of-fit/ complexity trade-off).
- ► For instance, for the two data sets displayed so far,

AIC for	X–rays	$\gamma extsf{-}rays$
Poisson	7622.55	7504.73
Zero-inflated Poisson (ZIP)	7623.57	7490.36
Negative Binomial (NB)	7624.56	7489.10
ZINB	7626.39	7491.44

- ► For the X-rays, the Poisson model well supported.
- ▶ For the γ -rays, evidence for NB or perhaps ZIP model.
- Model selection methods...
 - give a useful indication of a suitable model;
 - are easily implemented;
 - do not tell whether one model is significantly better than another.

Score test

Say
$$H_0 = Po(\lambda_i), H_1 = ZIP(p, \lambda_i),$$

- in other words, $H_0: p = 0$.
- Score test statistic:

$$T = S(0, \hat{\boldsymbol{\beta}})^T J(0, \hat{\boldsymbol{\beta}})^{-1} S(0, \hat{\boldsymbol{\beta}})$$

where $S = \partial L / \partial \beta$ and $J = \partial L / \partial \beta \beta^T$ are the score function and Fisher information of model H_1 , evaluated under the model fit β under H_0 .

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- Critical value for $\alpha = 0.05$ is $\chi^2_{1,0.95} = 3.84$.
- Results for 8 data sets (Oliveira et al, 2016):

	Homogenous exposure				Partial exposure			
LET	lo	w	hi	gh	low		high	
id	0.92	18.17	87.72	61.32	2007.39	1418.28	416.20	387.91

(the first two ones are the data sets discussed previously)

All data sets except the first one (X-rays) are zero-inflated!!

New graphical device

- We developed a new graphical tool to detect zero-inflation (and in fact, any-number-inflation).
- ► Effectively, it is demonstrated whether the number of counts, for k = 0, 1, 2, ..., is consistent with the specified count distribution.

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'Christmas tree plot' (Einbeck & Wilson, 2016)

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Christmas tree plot for γ -ray data





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Summary: Model choices

- It can be concluded (Oliveira et al, 2016):
 - Zero-inflation is driven by partial (body) exposure;
 - Overdispersion is driven by densely ionizing radiation, *i.e.* radiation with high linear energy transfer (LET);

Summary: Model choices

- It can be concluded (Oliveira et al, 2016):
 - Zero-inflation is driven by partial (body) exposure;
 - Overdispersion is driven by densely ionizing radiation, *i.e.* radiation with high linear energy transfer (LET);
 - the absence of partial exposure/ densely ionizing radiation does not guarantee the absence of zero-inflation/overdispersion.
- Recommended model choices

exposure		whole body	partial
LET low		Poisson/NB	ZIP
	high	$NB/Neyman\ A$	ZINB

 Some work has been done to extend dose (uncertainty) estimation routines to NB/ZIP models; further research required (Higueras et al, 2015)

Strength and limitations of dicentric assay

Strength:

- very little inter-individual (and inter-laboratory) variation;
- dicentric chromosome aberrations can be identified still several weeks after exposure;
- internationally accepted 'gold-standard'.
- Limitations:
 - needs 3–4 days after irradiation before dicentric chromosomes become visible (need to reach metaphase);

- relatively work-intensive and expensive methodology;
- not viable for large-scale radiation accidents.
- ► Hence, alternative biomarkers have recently been considered.

$\gamma-$ H2AX data

- Relatively new technology: Protein biomarker
- ► Double strand breaks lead to 'phosphorylation' of the H2AX protein, yielding γ-H2AX foci.
- ► γ-H2AX foci are counted using flow cytometers or immunofluorescence microscopy.
- Quicker, cheaper, less invasive than dicentric array.



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 - only visible up to 24 hours after exposure;
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- Quicker, cheaper, less invasive than dicentric array.
- Problems:
 - only visible up to 24 hours after exposure;
 - large inter-individual variation, hence requires further calibration steps, or models which can capture this variation.
- Some ad-hoc dose estimation methods available (Ainsbury et al, 2016).
- Critical gap remains: Fast (< 24h) dose assessment with samples that taken > 24h after the radiation incident.

- Recently, it has been demonstrated that certain genes respond to ionizing radiation with a change in their gene expression level.
- Genes are expressed by production of mRNAs from DNA, and protein from mRNAs.
- mRNA is a linear molecule which carries a copy of the gene to be expressed from the nucleus.



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 - Hybridization based methods (e.g. microarrays, usually continuous data);
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- Several radiation-responsive genes are known.
- Again, substantial inter–individual variation.
- No reliable statistical methodology for dose estimation yet.

References

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