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Optimal Control under Imprecision

Matthias Troffaes
Onderzoeksgroep SYSTeMS
Universiteit Gent
Technologie Park – Zwijnaarde 9,
B-9052 Zwijnaarde, Belgie
Email: matthias.troffaes@rug.ac.be

Gert de Cooman
Onderzoeksgroep SYSTeMS
Universiteit Gent
Technologie Park – Zwijnaarde 9,
B-9052 Zwijnaarde, Belgie
Email: gert.decooman@rug.ac.be

Dirk Aeyels
Onderzoeksgroep SYSTeMS
Universiteit Gent
Technologie Park – Zwijnaarde 9,
B-9052 Zwijnaarde, Belgie
Email: dirk.aeyels@rug.ac.be

1 Introduction and Description of the Problem

A solution to a problem is called optimal when it minimizes a given cost. In some situations this cost depends on some parameter which is not precisely known. One approach is to propose a stochastic distribution for this parameter, and then minimize the expected value of the cost. This approach has the disadvantage that the stochastic distribution must be known before we can solve the problem. In some situations this is not the case: a class of distributions may be compatible with the given information, and different choices within this class can lead to completely different results.

A possible solution is to use a more general model for lack of knowledge. We use a multi-valued random map to combine the effects of probability and imprecision. Every compatible single-valued random map (i.e. probability distribution) produces a unique expectation. The set of these expectations has a supremum and an infimum: these are called the upper and lower prevision [1]. In addition, this approach gives rise to an upper prevision which is 2-alternating (or equivalently, a lower prevision which is 2-monotone) and which can be calculated by a Choquet integral [2].

Instead of minimizing a compatible expectation of the cost, we try to minimize the worst expectation (i.e. the upper prevision) of the cost. We expect that this will lead to more conservative results.

2 Consumption/Pollution Tradeoff under Uncertainty

We apply this criterium to a consumption/pollution model [3]. Consumption increases pollution and leads to welfare. Pollution has a constant rate of decay and reduces welfare. If pollution exceeds some unknown treshold x, a terrible catastrophe could happen and welfare reduces to zero. The problem is to maximize welfare without exceeding the maximum pollution level, which is not exactly known.

We assume that the treshold x lies in a closed interval $[a, b]$ and that the consumption is constant. The solution of the problem is then, under appropriate assumptions, the solution we would get if we assumed $x = a$ (no uncertainty): a very conservative result is obtained.

3 Further Investigations

Our goal is to generalise this model for piecewise continuous controls (not constant) and stochastic multi-valued mappings (larger probability space). We hope to find an equivalent for the minimum principle [4] that can be applied in case of imprecision.

References

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