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Optimal Control with Imprecise Gain through Dynamic Programming

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Abstract

We generalise the optimisation technique of dynamic programming for discrete-time systems with an uncertain gain function. The main objective in optimal control is to find out how a system can be influenced, or controlled, in such a way that it its behaviour satisfies certain requirements, while at the same time maximising a given gain function. A very effective method for solving such problems, is the well-known recursive *dynamic programming* method, introduced by Richard Bellman [1].

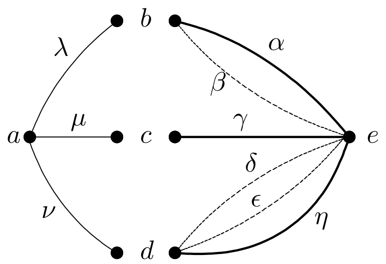

Figure 1: A Simple Example

To explain the ideas behind this method, we refer to Figure 1. If the optimal paths from b, c and d to the final state e are known to be α, γ and η, respectively, then to find the optimal path from a to e, we only need to compare the paths $\lambda\alpha$, $\mu\gamma$ and $\nu\eta$. This follows from Bellman's *principle of optimality*, by which $\lambda\beta$, $\nu\delta$ and $\nu\epsilon$ cannot be optimal, since in that case β, δ and ϵ would be optimal. Based on these observations, an efficient recursive algorithm can be constructed to calculate optimal paths.

We now wish to weaken the assumption that the gain associated with every path is exactly known. This problem is most often treated by modelling the uncertainty about the gain function by means of a probability measure, and by maximising the *expected gain* under this probability measure, rather than the (unknown) gain itself—we could call this the Bayesian approach. It turns out that, due to the linearity of the expectation operator, this approach does not change the nature of the optimisation problem, and the usual dynamic programming method can therefore still be applied to find the 'optimal' controls.

But it has often been argued that uncertainty cannot always be modelled adequately by probability measures, because, roughly speaking, there will in certain cases not be enough information in order to identify a single probability measure. In those cases, the available information can be represented through so-called *imprecise probability models* (see [3] and references therein), such as comparative probability orderings, Choquet capacities, belief functions, possibility measures, lower previsions, sets of desirable gambles, or convex sets of probability distributions.

This approach naturally gives rise to a strict preference order on paths. But, in contradistinction to the Bayesian approach, this order is only partial. This means that two paths will not always be comparable and that there may be no maximally preferred path, i.e., there may be no path that is strictly preferred or equivalent to all other paths. However, we have shown [2] that the principle of optimality still holds, if we look for *undominated paths*, these are paths for which there is no other path that is strictly preferred to it. An efficient recursive dynamic programming-like algorithm follows. It turns out that as imprecision increases, more paths become undominated, and consequently, decisions based on the model also become more indeterminate. As imprecision decreases, we recover the classical theory of dynamic programming as a special case.

References

[1] Richard Bellman. *Dynamic Programming*. Princeton University Press, 1957.

[2] Gert de Cooman and Matthias C. M. Troffaes. Dynamic programming for discrete-time systems with uncertain gain. In preparation.

[3] Peter Walley. *Statistical Reasoning with Imprecise Probabilities*. Chapman and Hall, London, 1991.