Indefinite Quadratic Forms, Closed Geodesics and Spectral Geometry

Yiannis Petridis[1]

[1]University College London

January 10, 2009

Outline

Pell's Equation

- Algorithm for solving $x^2 - 2y^2 = \pm 1$ (Pythagoreans)
- Start with
$$x_1 = 1, \quad y_1 = 1$$
- We get
$$(x_2, y_2) = (3, 2)$$
$$(x_3, y_3) = (7, 5)$$
$$(x_4, y_4) = (17, 12)$$
- Recurrences
$$x_{n+1} = x_n + 2y_n$$
$$y_{n+1} = x_n + y_n$$
- Fundamental solution: $1 + \sqrt{2}$
$$x_n + \sqrt{2} y_n = (1 + \sqrt{2})^n$$

Pell's Equation

- Algorithm for solving $x^2 - 2y^2 = \pm 1$ (Pythagoreans)
- Start with

$$x_1 = 1, \quad y_1 = 1$$

- We get

$$(x_2, y_2) = (3, 2)$$

$$(x_3, y_3) = (7, 5)$$

$$(x_4, y_4) = (17, 12)$$

- Recurrences

$$x_{n+1} = x_n + 2y_n$$

$$y_{n+1} = x_n + y_n$$

- Fundamental solution: $1 + \sqrt{2}$

$$x_n + \sqrt{2}y_n = (1 + \sqrt{2})^n$$

Pell's Equation

- Algorithm for solving $x^2 - 2y^2 = \pm 1$ (Pythagoreans)
- Start with

$$x_1 = 1, \quad y_1 = 1$$

- We get

$$(x_2, y_2) = (3, 2)$$

$$(x_3, y_3) = (7, 5)$$

$$(x_4, y_4) = (17, 12)$$

- Recurrences

$$x_{n+1} = x_n + 2y_n$$

$$y_{n+1} = x_n + y_n$$

- Fundamental solution: $1 + \sqrt{2}$

$$x_n + \sqrt{2} y_n = (1 + \sqrt{2})^n$$

Pell's Equation

- Algorithm for solving $x^2 - 2y^2 = \pm 1$ (Pythagoreans)
- Start with

$$x_1 = 1, \quad y_1 = 1$$

- We get

$$(x_2, y_2) = (3, 2)$$

$$(x_3, y_3) = (7, 5)$$

$$(x_4, y_4) = (17, 12)$$

- Recurrences

$$x_{n+1} = x_n + 2y_n$$

$$y_{n+1} = x_n + y_n$$

- Fundamental solution: $1 + \sqrt{2}$

$$x_n + \sqrt{2}y_n = (1 + \sqrt{2})^n$$

Pell's Equation

- Algorithm for solving $x^2 - 2y^2 = \pm 1$ (Pythagoreans)
- Start with

$$x_1 = 1, \quad y_1 = 1$$

- We get

$$(x_2, y_2) = (3, 2)$$

$$(x_3, y_3) = (7, 5)$$

$$(x_4, y_4) = (17, 12)$$

- Recurrences

$$x_{n+1} = x_n + 2y_n$$

$$y_{n+1} = x_n + y_n$$

- Fundamental solution: $1 + \sqrt{2}$

$$x_n + \sqrt{2} y_n = (1 + \sqrt{2})^n$$

Indefinite Quadratic Forms

- $Q(x,y) = ax^2 + bxy + cy^2, \quad d = b^2 - 4ac > 0$
- $Q(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}^t M \begin{pmatrix} x \\ y \end{pmatrix} \quad M = \begin{pmatrix} a & b/2 \\ b/2 & c \end{pmatrix}$
- Which integers are represented by Q, i.e.

$$Q(x,y) = N, \quad x,y,N \in \mathbb{N}$$

Equivalence of Quadratic Forms

$Q \sim Q' \Leftrightarrow$ they represent the same integers.

- $x^2 - 2y^2 \sim -2x^2 + y^2$
- $x^2 - 2y^2 \sim x^2 + 2xy - y^2$

- $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

- $(x+y)^2 - 2y^2 = x^2 + 2xy - y^2$
  $Q(x+y,y) = Q'(x,y)$ and $Q'(x-y,y) = Q(x,y)$

Indefinite Quadratic Forms

- $Q(x,y) = ax^2 + bxy + cy^2, \quad d = b^2 - 4ac > 0$
- $Q(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}^t M \begin{pmatrix} x \\ y \end{pmatrix} \quad M = \begin{pmatrix} a & b/2 \\ b/2 & c \end{pmatrix}$
- Which integers are represented by Q, i.e.

$$Q(x,y) = N, \quad x, y, N \in \mathbb{N}$$

Equivalence of Quadratic Forms

$Q \sim Q' \Leftrightarrow$ they represent the same integers.

- $x^2 - 2y^2 \sim -2x^2 + y^2$
- $x^2 - 2y^2 \sim x^2 + 2xy - y^2$

- $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

  $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

- $(x+y)^2 - 2y^2 = x^2 + 2xy - y^2$
  $Q(x+y, y) = Q'(x,y)$ and $Q'(x-y, y) = Q(x,y)$

Indefinite Quadratic Forms

- $Q(x,y) = ax^2 + bxy + cy^2, \quad d = b^2 - 4ac > 0$
- $Q(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}^t M \begin{pmatrix} x \\ y \end{pmatrix} \quad M = \begin{pmatrix} a & b/2 \\ b/2 & c \end{pmatrix}$
- Which integers are represented by Q, i.e.

$$Q(x,y) = N, \quad x, y, N \in \mathbb{N}$$

Equivalence of Quadratic Forms

$Q \sim Q' \Leftrightarrow$ they represent the same integers.

- $x^2 - 2y^2 \sim -2x^2 + y^2$
- $x^2 - 2y^2 \sim x^2 + 2xy - y^2$
- $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

- $(x+y)^2 - 2y^2 = x^2 + 2xy - y^2$
$Q(x+y,y) = Q'(x,y)$ and $Q'(x-y,y) = Q(x,y)$

Indefinite Quadratic Forms

- $Q(x,y) = ax^2 + bxy + cy^2, \quad d = b^2 - 4ac > 0$
- $Q(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}^t M \begin{pmatrix} x \\ y \end{pmatrix} \quad M = \begin{pmatrix} a & b/2 \\ b/2 & c \end{pmatrix}$
- Which integers are represented by Q, i.e.

$$Q(x,y) = N, \quad x, y, N \in \mathbb{N}$$

Equivalence of Quadratic Forms

$Q \sim Q' \Leftrightarrow$ they represent the same integers.

- $x^2 - 2y^2 \sim -2x^2 + y^2$
- $x^2 - 2y^2 \sim x^2 + 2xy - y^2$

- $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

  $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

- $(x+y)^2 - 2y^2 = x^2 + 2xy - y^2$
  $Q(x+y, y) = Q'(x,y)$ and $Q'(x-y, y) = Q(x,y)$

Indefinite Quadratic Forms

- $Q(x,y) = ax^2 + bxy + cy^2, \quad d = b^2 - 4ac > 0$
- $Q(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}^t M \begin{pmatrix} x \\ y \end{pmatrix} \quad M = \begin{pmatrix} a & b/2 \\ b/2 & c \end{pmatrix}$
- Which integers are represented by Q, i.e.

$$Q(x,y) = N, \quad x, y, N \in \mathbb{N}$$

Equivalence of Quadratic Forms

$Q \sim Q' \Leftrightarrow$ they represent the same integers.

- $x^2 - 2y^2 \sim -2x^2 + y^2$
- $x^2 - 2y^2 \sim x^2 + 2xy - y^2$

- $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

- $(x+y)^2 - 2y^2 = x^2 + 2xy - y^2$
  $Q(x+y, y) = Q'(x,y)$ and $Q'(x-y, y) = Q(x,y)$

Indefinite Quadratic Forms

- $Q(x,y) = ax^2 + bxy + cy^2, \quad d = b^2 - 4ac > 0$
- $Q(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}^t M \begin{pmatrix} x \\ y \end{pmatrix} \quad M = \begin{pmatrix} a & b/2 \\ b/2 & c \end{pmatrix}$
- Which integers are represented by Q, i.e.

$$Q(x,y) = N, \quad x, y, N \in \mathbb{N}$$

Equivalence of Quadratic Forms

$Q \sim Q' \Leftrightarrow$ they represent the same integers.

- $x^2 - 2y^2 \sim -2x^2 + y^2$
- $x^2 - 2y^2 \sim x^2 + 2xy - y^2$

- $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

  $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

- $(x+y)^2 - 2y^2 = x^2 + 2xy - y^2$
  $Q(x+y, y) = Q'(x,y)$ and $Q'(x-y, y) = Q(x,y)$

Class Numbers

$h(d)$ = number of inequivalent forms of discriminant d.

h(d)	d=
1	5, 8, 12, 13, 17, 21, 24, 28, 29, 33, 37, 41, 44, 53
2	40, 60, 65, 85, 104, 105, 120, 136, 140, 156, 165
3	229, 257, 316, 321, 469, 473, 568, 733, 761, 892
4	145, 328, 445, 505, 520, 680, 689, 777, 780, 793
5	401, 817, 1093, 1393, 1429, 1641, 1756, 1897, ...

- $\mathcal{D} = \{d \mid d > 0, d \equiv 0, 1 \pmod 4), d \neq \square\}$
- $x^2 - dy^2 = 4$, fundamental solution (t, u)

$$\epsilon_d = \frac{t + u\sqrt{d}}{2}$$

- **Gauss, Siegel (1944)**

$$\sum_{d \in \mathcal{D}, d \leq x} h(d) \log \epsilon_d = \frac{\pi^2 x^{3/2}}{18\zeta(3)} + O(x \log x).$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \Re(s) > 1.$$

Class Numbers

$h(d) =$ number of inequivalent forms of discriminant d.

h(d)	d=
1	5, 8, 12, 13, 17, 21, 24, 28, 29, 33, 37, 41, 44, 53
2	40, 60, 65, 85, 104, 105, 120, 136, 140, 156, 165
3	229, 257, 316, 321, 469, 473, 568, 733, 761, 892
4	145, 328, 445, 505, 520, 680, 689, 777, 780, 793
5	401, 817, 1093, 1393, 1429, 1641, 1756, 1897, ...

- $\mathcal{D} = \{d | d > 0, d \equiv 0, 1 \pmod 4), d \neq \square\}$
- $x^2 - dy^2 = 4$, fundamental solution (t, u)

$$\epsilon_d = \frac{t + u\sqrt{d}}{2}$$

- **Gauss, Siegel (1944)**

$$\sum_{d \in \mathcal{D}, d \leq x} h(d) \log \epsilon_d = \frac{\pi^2 x^{3/2}}{18\zeta(3)} + O(x \log x).$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \Re(s) > 1.$$

Class Numbers

$h(d)$ = number of inequivalent forms of discriminant d.

h(d)	d=
1	5, 8, 12, 13, 17, 21, 24, 28, 29, 33, 37, 41, 44, 53
2	40, 60, 65, 85, 104, 105, 120, 136, 140, 156, 165
3	229, 257, 316, 321, 469, 473, 568, 733, 761, 892
4	145, 328, 445, 505, 520, 680, 689, 777, 780, 793
5	401, 817, 1093, 1393, 1429, 1641, 1756, 1897, ...

- $\mathcal{D} = \{d \mid d > 0, d \equiv 0, 1 \pmod 4), d \neq \square\}$
- $x^2 - dy^2 = 4$, fundamental solution (t, u)

$$\epsilon_d = \frac{t + u\sqrt{d}}{2}$$

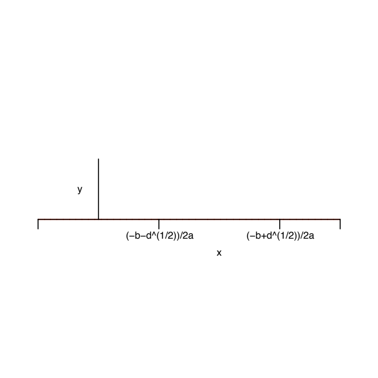
- **Gauss, Siegel (1944)**

$$\sum_{d \in \mathcal{D}, d \leq x} h(d) \log \epsilon_d = \frac{\pi^2 x^{3/2}}{18\zeta(3)} + O(x \log x).$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \Re(s) > 1.$$

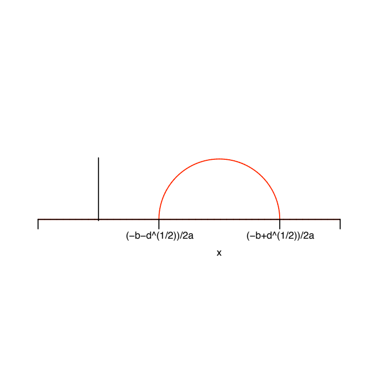
From Quadratic Forms to Hyperbolic Geometry

$$Q(x,y) = ax^2 + bxy + cy^2$$
$$az^2 + bz + c = 0 \Leftrightarrow z_1 = \frac{-b+\sqrt{d}}{2a}, \quad z_2 = \frac{-b-\sqrt{d}}{2a}$$



From Quadratic Forms to Hyperbolic Geometry

$$Q(x,y) = ax^2 + bxy + cy^2$$
$$az^2 + bz + c = 0 \Leftrightarrow z_1 = \frac{-b+\sqrt{d}}{2a}, \quad z_2 = \frac{-b-\sqrt{d}}{2a}$$



Linear Fractional Transformations and $\mathrm{SL}_2(\mathbb{R})$

$$T(z) = \frac{az+b}{cz+d}, \quad ad - bc \neq 0$$

maps circles to circles

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \to T(z)$$

$$\mathrm{SL}_2(\mathbb{R}) = \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, ad - bc = 1 \right\}$$

acts on

Upper-half space

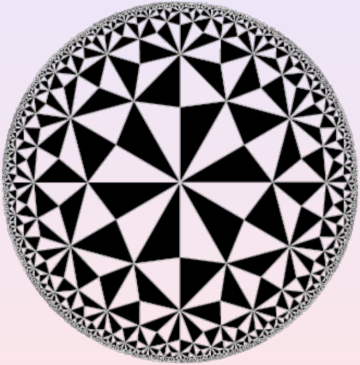
$$\mathbb{H} = \{z = x + iy, y > 0\} \qquad ds^2 = \frac{dx^2 + dy^2}{y^2}$$

$$z(t) = x(t) + iy(t), \quad t \in [a, b], \quad L = \int_a^b \frac{\sqrt{x'(t)^2 + y'(t)^2}}{y(t)}\, dt$$

- $z \to z + 1$
- $z \to -\frac{1}{z}$

$$\Gamma = \mathrm{SL}_2(\mathbb{Z}) = \{\gamma \in \mathrm{SL}_2(\mathbb{R}), a, b, c, d \in \mathbb{Z}\}$$

The Hyperbolic Disc



- Model of hyperbolic geometry

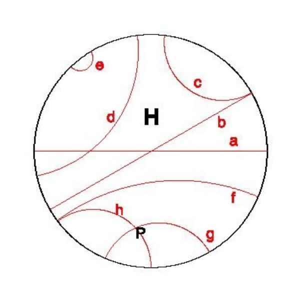
$$\mathbb{H} = \{z = x + iy, |z| < 1\}$$

- Hyperbolic metric

$$ds^2 = \frac{dx^2 + dy^2}{(1 - (x^2 + y^2))^2}$$

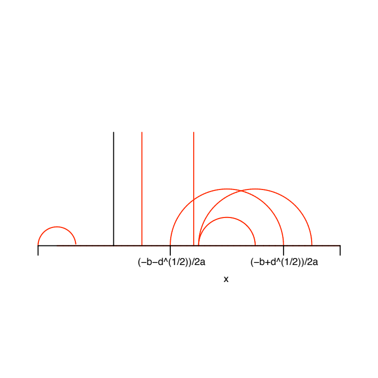
$$d(0, z) = \ln \frac{1 + |z|}{1 - |z|},$$

Geodesics in Hyperbolic Disc



- Semicircles perpendicular to boundary
- Diameters

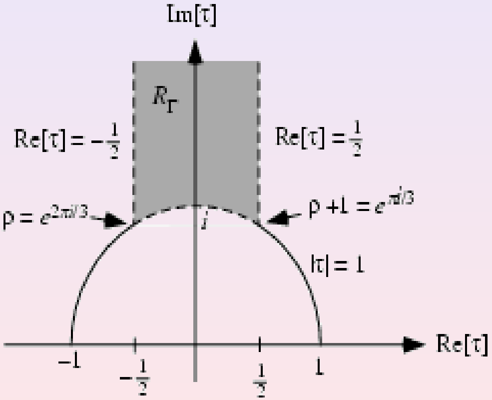
Geodesics in the Upper Half Plane



Fundamental Domains

$z \sim w \Leftrightarrow w = T(z), \quad T \in \Gamma$

$\mathbb{H}/\Gamma$ the modular surface



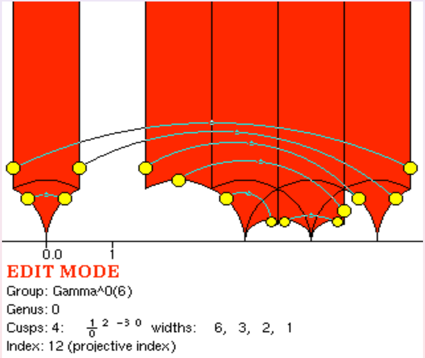
Arithmetic subgroups of $\mathrm{SL}_2(\mathbb{Z})$

Hecke subgroups $\Gamma_0(N)$

$$\frac{az+b}{cz+d} \in \mathrm{SL}_2(\mathbb{Z}), N|c$$

Example

Fundamental Domain for $\Gamma_0(6)$



Tesselations

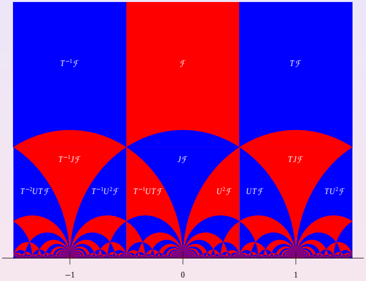


Figure: Translates of the fundamental domain of $\mathrm{SL}_2(\mathbb{Z})$

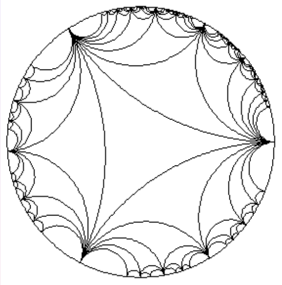


Figure: Triangles in the disc

Equivalence of Quadratic forms and $\mathrm{SL}_2(\mathbb{R})$

$$Q' \sim Q \Leftrightarrow M' = \gamma^t M \gamma, \quad \gamma \in \Gamma.$$

$$Q \to g = \begin{pmatrix} \frac{t-bu}{2} & -cu \\ au & \frac{t+bu}{2} \end{pmatrix} \in \Gamma$$

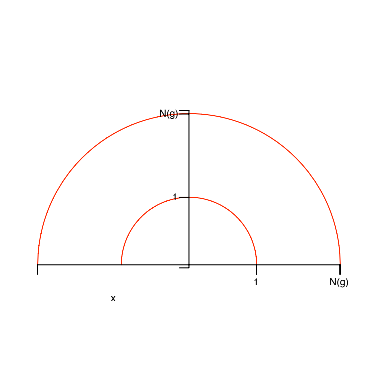
where $t^2 - du^2 = 4$ and (t, u) is the fundamental (smallest solution).

Remarks: 1. g has eigenvalue $\epsilon_d = \dfrac{t + u\sqrt{d}}{2}$

2. Most $g \in \mathrm{SL}_2(\mathbb{R})$ can be diagonalised

$$g \sim \begin{pmatrix} N(g)^{1/2} & 0 \\ 0 & N(g)^{-1/2} \end{pmatrix}.$$

Pell's Equation and Lengths of Closed Geodesics



$$\int_1^{N(g)} \frac{1}{y}\, dy = \ln N(g) = \ln(\epsilon_d^2)$$

Theorem

The lengths of the closed geodesics for the hyperbolic surface $\mathbb{H}/\mathrm{SL}_2(\mathbb{Z})$ are $2\log\epsilon_d$ with multiplicity $h(d)$, $d \in \mathcal{D}$.

Distribution Closed geodesics of $\mathbb{H}/\Gamma$

Closed geodesics γ.

Prime Geodesic Theorem

- $\pi(x) = \{\gamma, \text{length } (\gamma) \leq e^x\}$
  $$\pi(x) \sim \frac{x}{\ln x}, \quad x \to \infty$$

Prime Number Theorem

- $\pi(x) = \{p \text{ prime}, p \leq x\}$
  $$\pi(x) \sim \frac{x}{\ln x}, \quad x \to \infty$$

- Class number distribution (Sarnak, 1982)

$$\sum_{d \in \mathcal{D}, \epsilon_d \leq x} h(d) \sim \frac{x^2}{2 \ln x}$$

Distribution Closed geodesics of $\mathbb{H}/\Gamma$

Closed geodesics γ.

Prime Geodesic Theorem

- $\pi(x) = \{\gamma, \text{length } (\gamma) \leq e^x\}$
  $\pi(x) \sim \frac{x}{\ln x}, \quad x \to \infty$

Prime Number Theorem

- $\pi(x) = \{p \text{ prime}, p \leq x\}$
  $\pi(x) \sim \frac{x}{\ln x}, \quad x \to \infty$

- Class number distribution (Sarnak, 1982)

$$\sum_{d \in \mathcal{D}, \epsilon_d \leq x} h(d) \sim \frac{x^2}{2 \ln x}$$

Distribution Closed geodesics of $\mathbb{H}/\Gamma$

Closed geodesics γ.

Prime Geodesic Theorem

- $\pi(x) = \{\gamma, \text{length } (\gamma) \leq e^x\}$
  $\pi(x) \sim \dfrac{x}{\ln x}, \quad x \to \infty$

Prime Number Theorem

- $\pi(x) = \{p \text{ prime}, p \leq x\}$
  $\pi(x) \sim \dfrac{x}{\ln x}, \quad x \to \infty$

- Class number distribution (Sarnak, 1982)

$$\sum_{d \in \mathcal{D}, \epsilon_d \leq x} h(d) \sim \frac{x^2}{2 \ln x}$$

The Laplace Operator

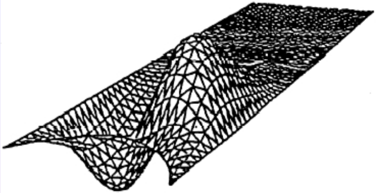
$$\Delta = -y^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$$

$$\Delta f = 0 \Leftrightarrow f \quad \text{is harmonic}$$

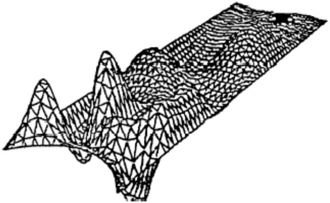
Eigenvalue problem: Solve

$$\Delta f = \lambda f$$

Infinite Matrix, no determinant to compute eigenvalues.
I require $f(\gamma z) = f(z)$, $\gamma \in \Gamma$ (automorphic form)



$\lambda_1 = 91.12\ldots$



$\lambda_{10} = 379.90\ldots$

$\lambda_{17} = 541.27\ldots$

$\lambda_{33} = 916.52\ldots$

Contour plots of eigenfunctions of $\mathbb{H}/\Gamma_0(7)$

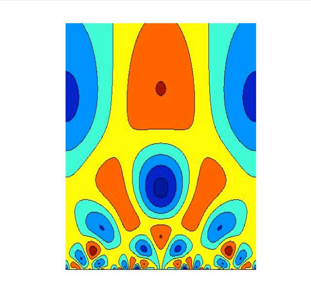
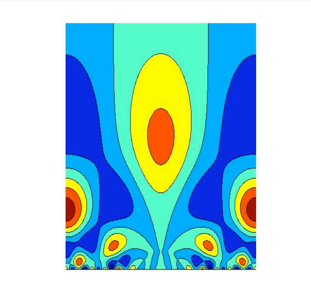


Figure: $\lambda = 37.08033$ $\lambda = 692.7292$

Contour plots of eigenfunctions of $\mathbb{H}/\Gamma_0(3)$



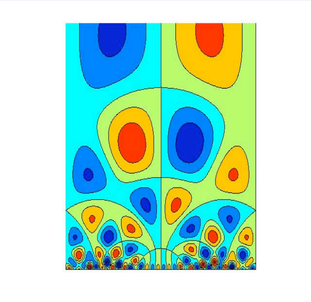


Figure: $\lambda = 26.3467$ $\qquad$ $\lambda = 60.4397$

The Riemann Hypothesis

- Think of $\sum_{n=0}^{\infty} z^n = \dfrac{1}{1-z}$.
  Series: $|z| < 1, \quad 1/(1-z), z \neq 1$.
- $\zeta(s) = \sum_{n=1}^{\infty} \dfrac{1}{n^s}, \quad \Re(s) > 1$.
  Extend to $1/2 \leq \Re(s)$, then to $\mathbb{C}$ using functional equation

  $$\zeta(s) = \pi^{-1+s} \frac{\Gamma((1-s)/2)}{\Gamma(s/2)} \zeta(1-s).$$

  Zeros: ρ, $1-\rho$ symmetric around $\Re(s) = 1/2$.

Riemann Hypothesis

$\Re(\rho) = 1/2$ ALWAYS

The Riemann Hypothesis

- Think of $\sum_{n=0}^{\infty} z^n = \dfrac{1}{1-z}$.
  Series: $|z| < 1, \qquad 1/(1-z), z \neq 1.$
- $\zeta(s) = \sum_{n=1}^{\infty} \dfrac{1}{n^s}, \quad \Re(s) > 1.$
  Extend to $1/2 \leq \Re(s)$, then to $\mathbb{C}$ using functional equation

$$\zeta(s) = \pi^{-1+s} \frac{\Gamma((1-s)/2)}{\Gamma(s/2)} \zeta(1-s).$$

Zeros: ρ, $1-\rho$ symmetric around $\Re(s) = 1/2$.

Riemann Hypothesis

$\Re(\rho) = 1/2$ ALWAYS

The Riemann Hypothesis

- Think of $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$.
  Series: $|z| < 1, \quad 1/(1-z), z \neq 1$.
- $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \Re(s) > 1$.
  Extend to $1/2 \leq \Re(s)$, then to $\mathbb{C}$ using functional equation

$$\zeta(s) = \pi^{-1+s} \frac{\Gamma((1-s)/2)}{\Gamma(s/2)} \zeta(1-s).$$

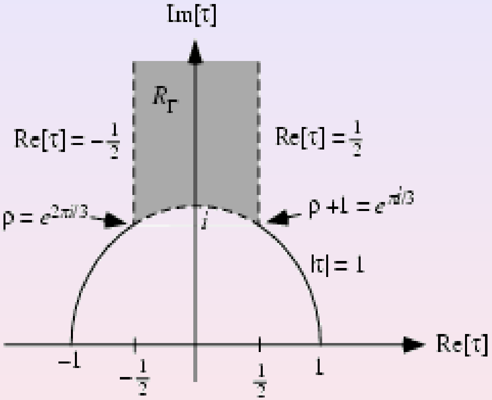
  Zeros: ρ, $1-\rho$ symmetric around $\Re(s) = 1/2$.

Riemann Hypothesis

$\Re(\rho) = 1/2$ ALWAYS

Relations of RH with spectral theory

Scattering of waves on $\mathbb{H}/\Gamma$



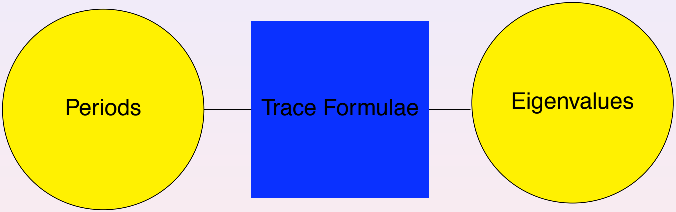
$$\phi(s) = \sqrt{\pi}\frac{\Gamma(s-1/2)\zeta(2s-1)}{\Gamma(s)\zeta(2s)}$$

Poles of $\phi(s)$ at $\rho/2$.

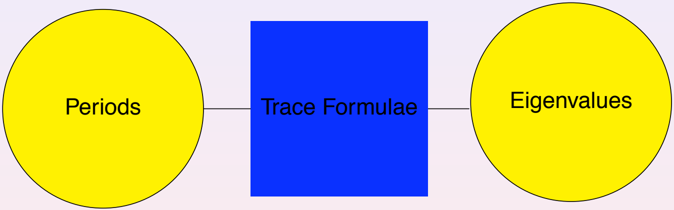
RH⇔Poles of $\phi(s)$, have real part 1/4.

Duality between periods and eigenvalues

Duality between periods and eigenvalues



Duality between periods and eigenvalues



Lengths of closed geodesics —— Selberg Trace formula — Laplace eigenvalues

Simple trace formula

$$Tr(A) = \sum_{i=1}^{n} a_{ii} = \sum_{j=1}^{n} \lambda_j$$

What is and how do I compute the trace of an operator?

Credits for the pictures

1. V. Golovshanski, N. Motrov: preprint, Inst. Appl. Math. Khabarovsk (1982)
2. D. Hejhal, B. Rackner: On the topography of Maass waveforms for $\mathrm{PSL}(2, Z)$. Experiment. Math. 1 (1992), no. 4, 275–305.
3. A. Krieg: http://www.matha.rwth-aachen.de/forschung/fundamentalbereich.html
4. http://mathworld.wolfram.com/
5. F. Stromberg: http://www.math.uu.se/ fredrik/research/gallery/
6. H. Verrill: http://www.math.lsu.edu/ verrill/

Where can you study my research interests? (apart from UCL)

Analytic Aspects of Automorphic Forms, Analytic Number Theory, Spectral Theory of hyperbolic manifolds, Quantum Chaos

- Bristol
  A. Booker, T. Browning, B. Conrey, R. Dietmann, A. Gorodnik, H. Helfgott, J. Keating, J. Marklof, J. Pila, N. Snaith, L. Walling, T. Wooley
- Cardiff
  Martin Huxley, N. Watt
- Durham
  N. Peyerimhoff
- Nottingham
  N. Diamantis
- Oxford
  Roger Heath-Brown
- Royal Holloway
  Glen Harman

Department of Mathematics, UCL

18 pure mathematicians, 24 applied mathematicians
42 postgraduate students, half in pure, half in applied, mostly funded.
We offer Msc in Mathematical Modeling, MPhil, Ph.D.
Information on postgraduate admissions:

Pure Mathematics

Professor Keith Ball
Tel: 020-7679-2843
Email: kmb@math.ucl.ac.uk

Applied Mathematics

Professor Frank T Smith
Tel: 020-7679-2837
Email: frank@math.ucl.ac.uk

Research Interests of UCL Staff in Pure Mathematics

- Analysis: Keith Ball
  Marianna Csörnyei (Leverhume Prize)
  Rod Halburd
  Ya. Kurylev (Inverse Problems)
  Miklos Laczkovich
  David Larman
  Leonid Parnovski (Spectral Theory)
  Y. Petridis (Spectral Theory, Number Theory)
  Nadia Sidorova (Probability)
  Alex Sobolev (Spectral Theory)
  Alan Sokal
  Dima Vassiliev (Spectral Theory)
- Number Theory, Algebraic Geometry:
  Keith Ball (Irrationality of zeta values)
  Richard Hill (Automorphic Forms)
  Minhyong Kim (Arithmetic Algebraic Geometry)
  Y. Petridis
  Andrei Yafaev (Leverhume Prize, Algebraic Geom.)

- Algebra, Topology: Frank Johnson
- Combinatorics, Geometry:
  Imre Bárány
  David Larman
  Miklos Laczkovich
  Alan Sokal
  John Talbot
  Keith Ball

Analysis Seminar (joint with Imperial College, King's College and QMW) organised by
Crisan (Imperial), B. Davies (King's), Goldshein (QMW),
Ya. Kurylev (UCL), A. Laptev (Imperial),
L. Parnovski (UCL), Pushnitski (King's),
Ruzhansky (Imperial), Yu. Safarov (King's),
Eu. Shargorodsky (King's), A. Sobolev (UCL),
Zegarlinski (Imperial).
London-Paris Analysis Seminar

Number Theory:

- Activities: Basic Notions Seminar, Study group and Seminar (joint with Imperial College and King's College), London-Paris Number Theory Seminar
- Members:
  King's: M. Breuning, D. Burns, C. Bushnell, F. Diamond, P. Kassaei, D. Solomon.
  Imperial: K. Buzzard, A. Pal
  UCL: R. Hill, M. Kim, Y. Petridis, A. Yafaev.

Pure Colloquium, Applied Colloquium
Lighthill Institute for Mathematical Sciences
EU research training network (Marie-Curie): 'Phenomena in High-Dimensions'.
Origins: Interdisciplinary (Physics, Astronomy etc.)

London Taught Course Center

Participants: UCL, King's, QMW, Brunel, Kent, Imperial (statistics)

Pure Mathematics Courses

Finite Simple Groups
Measure and Category
Graph Theory
C*-algebras
Algebraic number theory
Measure-theoretic Probability
Cryptography and Mathematics
Introduction to Random Matrix Theory
Modular Group and Automorphic Forms
Non-Commutative Geometry
Representation Theory
Applications of Differential Geom. to Math. Physics
Mathematical Physics Topics

Research Interests of UCL Staff in Applied Mathematics

- Fluid Dynamics:
  Rob Bowles
  Rod Halburd
  Sergei Timoshin
  Jean-Marc Vanden-Broeck
  Helen Wilson
- Industry Oriented
  Steven Bishop
  J Curtis
  A Ellis
  N Ovenden
- Environmental Flows
  M Davey
  Gavin Esler
  Ted Johnson
  Rob McDonald
  Frank Smith

- Cross Disciplinary
  R Ahmad
  Steven Baigent
  M Banaji
  Christian Böhmer
  Ya. Kurylev
  J McGlade
  Karen Page
  M Ramsza
  RM Seymour
  A Zaikin

+ CORU (Clinical Operations Research Group)
+ Complex