The Energy-Momentum Tensor

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What’s in a name?

- Energy-momentum tensor
- Stress-energy tensor
- Stress-energy-momentum tensor
- Energy tensor
- SEM
- ...

• Energy-momentum tensor
• Stress-energy tensor
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• Energy tensor
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• ...
Why a tensor?

- A particle has energy and momentum.
- If a particle has energy (mass) $E$ in its rest frame, then in a frame moving at relative velocity $v$, the energy is given by $\gamma E$, where $\gamma = 1 / \sqrt{1 - v^2}$. Energy is a component of a four-vector — the four-momentum.
- Given a field or a fluid, we must consider the energy density.
- If we switch from the rest frame to a moving frame, the energy in a volume element increases by a factor of $\gamma$, while the volume decreases by the same factor. The energy density increases by a factor of $\gamma^2$.
- This suggests that the energy density is an element of a $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ tensor.
What do all the bits mean?

Well,

- $T^{00}$: Energy density,
- $T^{0i}$: Energy flux across a surface of constant $x^i$,
- $T^{i0}$: Momentum density in $i$ direction,
- $T^{ij}$: Flux of $i$ momentum across a surface of constant $x^j$,

or we can be more brief and simply say

- $T^{\alpha\beta}$: Flux of $\alpha$ momentum across a surface of constant $x^\beta$.

Note that, for this case at least, the energy-momentum tensor is symmetric — the energy flux across a surface of constant $x^i$ is the momentum density in the $i$ direction.¹

¹ See B. Schutz, "A First Course in General Relativity", chapter 4, for a pedagogical introduction to the energy-momentum tensor, using ‘dust’ as an example.
Noether current recap

- A continuous ‘active’ transformation that leaves the equations of motion unchanged leads to a conserved current.
- To leave the equations of motion unchanged we require
  \[ \Delta S = 0 \iff \mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L} + \epsilon \partial_\mu F^\mu \]
  for some \( F^\mu \). If the field transforms as
  \[ \phi \rightarrow \phi' = \phi + \epsilon \Delta \phi, \]
  then the Noether current is given by
  \[ j^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \Delta \phi - F^\mu. \]
- The Noether current is conserved, i.e. \( \partial_\mu j^\mu = 0 \).
Energy-momentum as a Noether current — A typical presentation

Consider a translation invariant theory and consider the translation

$$x^\mu \rightarrow x^\mu - \epsilon a^\mu$$

as an active transformation. Then

$$\phi(x) \rightarrow \phi'(x) = \phi(x + \epsilon a) = \phi(x) + \epsilon a^\mu \partial_\mu \phi(x),$$

so that $\Delta \phi = a^\mu \partial_\mu \phi(x)$. Likewise,

$$L \rightarrow L + \epsilon a^\mu \partial_\mu L,$$

i.e. $F^\mu = a^\mu L$. Hence, the Noether current is

$$j^\mu = \frac{\partial L}{\partial (\partial_\mu \phi)} \Delta \phi - F^\mu = a^\nu \left( \frac{\partial L}{\partial (\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu L \right).$$

Since $a^\mu$ is arbitrary (and constant), we get $\partial_\mu T^\mu_\nu = 0$ where

$$T^\mu_\nu = \frac{\partial L}{\partial (\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu L.$$
Active transformations: Misconceptions

- It is, perhaps, unhelpful to think of active and passive transformations as being opposites.
- It isn’t really the ’activeness’ of the transformation that is really important here.
- The kind of active transformation we want does not change the coordinates. Only the field is transformed. We translate (or rotate) the contents of the universe, but leave everything else — the coordinate system and the ‘theory’ — alone.
Active transformations: Scalar field

- An active transformation is one “in which the field is truly shifted” (Tong).
- But it is only the field that is changed.
- We want the value of the new field at $x'$ to be equal to that for the old field at $x$, i.e.
  $$\phi'(x') = \phi(x).$$
- If $x' = Rx$ then we need $\phi'(Rx) = \phi(x)$ and hence
  $$\phi'(x) = \phi(R^{-1}x).$$
Active transformations: Vector field

- A rotation of a vector field results in a rotation of each individual vector, as well as the change of location.
- We get
  \[ \phi'(x') = R\phi(x). \]
  and hence
  \[ \phi'(x) = R\phi(R^{-1}x). \]
- Of course, other fields might undergo different transformations, i.e.
  \[ \phi'(x) = L_R\phi(R^{-1}x), \]
  where the \( L_R \) are elements of a representation of the group of rotations.
Energy-momentum as a Noether current, revisited

Consider a transformation of the field, \( \phi \rightarrow \phi' \), such that

\[
\phi'(x') = \phi(x),
\]

where \( x'^\mu = x^\mu - \epsilon a^\mu \) corresponds to a translation of the field. Then

\[
\phi'(x) = \phi(x + \epsilon a) = \phi(x) + \epsilon a^\mu \partial_\mu \phi(x)
\]

and \( \Delta \phi = a^\mu \partial_\mu \phi(x) \).
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and \( \Delta \phi = a^\mu \partial_\mu \phi(x) \).

We do not change the form of the Lagrangian, \( \mathcal{L}(\phi, \partial_\mu \phi) \), but its value at position \( x \) will be different for the new field:
\[
\mathcal{L}(\phi'(x), \partial_\mu \phi'(x)) = \mathcal{L}(\phi(x + \epsilon a), \partial_\mu \phi(x + \epsilon a))
= \mathcal{L}(\phi(x), \partial_\mu \phi(x)) + \epsilon a^\mu \partial_\mu \mathcal{L}(\phi(x), \partial_\mu \phi(x)).
\]
Hence \( F^\mu = a^\mu \mathcal{L} \). As a result, the Noether current is
\[
j^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \Delta \phi - F^\mu = a^\nu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu \mathcal{L} \right).
\]
Since \( a^\mu \) is arbitrary (and constant), we get \( \partial_\mu T^\mu_\nu = 0 \) where
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T^\mu_\nu = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu \mathcal{L}.
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We do not change the form of the Lagrangian, $\mathcal{L}(\phi, \partial_\mu \phi)$, but its value at position $x$ will be different for the new field:

$$\mathcal{L}(\phi'(x), \partial_\mu \phi'(x)) = \mathcal{L}(\phi(x + \epsilon a), \partial_\mu \phi(x + \epsilon a)$$

$$= \mathcal{L}(\phi(x), \partial_\mu \phi(x)) + \epsilon a^\mu \partial_\mu \mathcal{L}(\phi(x), \partial_\mu \phi(x)).$$

Hence $F^\mu = a^\mu \mathcal{L}$. As a result, the Noether current is

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Since $a^\mu$ is arbitrary (and constant), we get $\partial_\mu T^\mu_{\nu} = 0$ where

$$T^\mu_{\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \partial_\nu \phi - \delta_\mu^{\nu} \mathcal{L}.$$
A non-translation invariant Lagrangian

Consider the Lagrangian

$$\mathcal{L}(\phi(x), \partial_\mu \phi(x), x^\mu) = x^\mu x_\mu \phi^*(x) \phi(x).$$

If we perform the same active transformation, i.e. translate the field only, then the value of the Lagrangian at point $x$ becomes

$$\mathcal{L}(\phi'(x), \partial_\mu \phi'(x), x'^\mu) = x'^\mu x_\mu \phi^*(x + \epsilon a) \phi(x + \epsilon a)$$

$$\neq \mathcal{L}(\phi(x), \partial_\mu \phi(x)) + \epsilon a^\mu \partial_\mu \mathcal{L}(\phi(x), \partial_\mu \phi(x)).$$

So, we now know where the derivation of the energy-momentum tensor fails for non-translation invariant theories.
An example: Electromagnetism

For electromagnetism\(^2\), without sources, the action is

\[
S = \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right),
\]

where \( F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \). Now

\[
\frac{\partial F_{\mu\nu}}{\partial (\partial_{\rho} A_{\sigma})} = \delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} - \delta_{\nu}^{\rho} \delta_{\mu}^{\sigma}, \quad \frac{\partial F^{\mu\nu}}{\partial (\partial_{\rho} A_{\sigma})} = \eta^{\rho \mu} \eta^{\sigma \nu} - \eta^{\rho \nu} \eta^{\sigma \mu},
\]

so

\[
\frac{\partial L}{\partial (\partial_{\rho} A_{\sigma})} = -\frac{1}{4} \left( \delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} - \delta_{\nu}^{\rho} \delta_{\mu}^{\sigma} \right) F_{\mu\nu} - \frac{1}{4} F_{\mu\nu} \left( \eta^{\rho \mu} \eta^{\sigma \nu} - \eta^{\rho \nu} \eta^{\sigma \mu} \right)
\]

\[
= -\frac{1}{4} F^{\rho \sigma} + \frac{1}{4} F^{\sigma \rho} - \frac{1}{4} F^{\rho \sigma} + \frac{1}{4} F^{\sigma \rho} = F^{\sigma \rho}.
\]

The energy-momentum tensor is therefore

\[
T^{\mu}_{\nu} = \frac{\partial L}{\partial (\partial_{\mu} A_{\rho})} \partial_{\nu} A_{\rho} - \delta^{\mu}_{\nu} L = \eta^{\rho \mu} \partial_{\nu} A_{\rho} + \frac{1}{4} F_{\rho \sigma} F^{\rho \sigma} \delta^{\mu}_{\nu}.
\]

\(^2\text{Peskin and Schroeder, exercise 2.1}\)
A symmetric tensor?

- For fluids or dust, the energy-momentum tensor defined in terms of momentum flux is symmetric.
- We would like a symmetric tensor for many reasons:
  - General relativity requires a symmetric tensor,
  - Symmetric tensors are simpler to manipulate,
  - etc.
- But

\[ T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \phi)} \partial^{\nu} \phi - \eta^{\mu\nu} \mathcal{L} \]

is not obviously symmetric.
- The energy-momentum tensor for electromagnetism,

\[ T^{\mu\nu} = F^{\rho\mu} \partial^{\nu} A_\rho + \frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} \eta^{\mu\nu} \]

certainly doesn’t look symmetric.
Creating a symmetric energy-momentum tensor — the Belinfante tensor

- If $K^\lambda_{\mu\nu}$ is antisymmetric in the first two indices, then adding $\partial_\lambda K^\lambda_{\mu\nu}$ to the energy-momentum tensor does not change its conservation properties, since if

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu},$$

then

$$\partial_\mu \tilde{T}^{\mu\nu} = \partial_\mu T^{\mu\nu} + \overline{\partial_\mu \partial_\lambda K^{\lambda\mu\nu}} = 0.$$

- For electromagnetism$^3$, choose $K^{\lambda\mu\nu} = F^{\mu\lambda} A^{\nu}$. After a few calculations, we get

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu} = \frac{1}{4} F^{\rho\sigma} F^{\rho\sigma} \eta^{\mu\nu} + F^{\lambda\mu} F^{\nu}_{\lambda} + A^{\nu} \partial_\lambda F^{\mu\lambda}.$$

- But we can use the equations of motion, $\partial_\lambda F^{\mu\lambda}$, to eliminate the last term to leave

$$\frac{1}{4} F^{\rho\sigma} F^{\rho\sigma} \eta^{\mu\nu} + F^{\lambda\mu} F^{\nu}_{\lambda},$$

which is manifestly symmetric.

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$^3$See Wikipedia (!) for the general case.
Symmetric ‘on shell’

- We used the equations of motion to show that the new energy-momentum tensor is symmetric.
- In other words, we have shown that we can convert the canonical energy-momentum tensor to an alternative form that is equal, *classically* to an identically symmetric energy-momentum tensor.
- Classically, this distinction is of no importance.
- Quantum mechanically, the two energy-momentum tensors are different and result in different Ward identities.
- Apparently, though, the change in the Ward identities is of no physical significance.⁴

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⁴See Di Francesco et al., "Conformal Field Theory", section 2.5.
A passive transformation is one “in which the mapping $x \to x'$ is viewed simply as a coordinate transformation”\textsuperscript{5}.

Wikipedia (!) suggests that, given a rotation matrix $R$, a passive transformation consists of rotating the basis vectors of our coordinate system using $R$. Hence with initial basis vectors $e_\mu$ we obtain new basis vectors $e'_\mu = Re_\mu$. In the usual way, we find coordinates transform as

$$x'^\mu = (R^{-1})^\mu_\nu x^\nu,$$

or simply $x \to x' = R^{-1}x$. Hence, in a sense, we obtain the opposite transformation to that in the active case. But...

\textsuperscript{5}Di Francesco et al.
Opposites?

- As physicists, we tend to consider coordinate transformations without thinking a great deal about basis vectors. Our starting point is usually something like $x \rightarrow x' = Rx$. If the same coordinate transformation is also used to generate the active transformation, then the field transforms the same way in both cases.

- An active transformation transforms the fields only. Any explicit dependence the Lagrangian has on $x$ is unchanged. However, if we perform a mere change of coordinates, then the explicit dependence of the Lagrangian on $x'$ will be different.
Variation with respect to the metric

Just when you think you might be getting the hang of the energy-momentum tensor, someone will say that it is defined as something like

\[ T_{ab} = -\frac{\alpha_M}{8\pi} \frac{1}{\sqrt{-\bar{g}}} \frac{\delta S_M}{\delta g^{ab}} \]

or

\[ T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_{\text{matter}}}{\delta g^{\mu\nu}} \]

or

\[ T_{\alpha\beta} = -\frac{4\pi}{\sqrt{-g}} \frac{\partial S}{\partial g^{\alpha\beta}}. \]

They might also describe this as the ‘gravitational’ or ‘relativists’ definition of energy-momentum. But where on earth did these come from?
An intermediate, new definition\textsuperscript{6}

- "Promote the constant parameter $\epsilon$ that appears in the symmetry to a function of the spacetime coordinates".
- The change in the action should then be of the form

$$\delta S = \int d^4x \ J^\alpha \partial_\alpha \epsilon,$$

or in the case of a translation

$$\delta S = \int d^4x \ J^{\alpha\beta} \partial_\alpha \epsilon_\beta.$$

- Terms in $\epsilon$ (undifferentiated) should cancel — these are the terms that occur when $\epsilon$ is a constant and we have assumed that the theory is symmetric under such transformations.
- But, on-shell, equations of motion are found by assuming that $\delta S$ is zero for \textit{any} infinitesimal variation of the fields.
- Integrate by parts to get $\partial_\alpha J^\alpha = 0$, or $\partial_\alpha J^{\alpha\beta} = 0$.

\textsuperscript{6}Here we attempt to follow David Tong’s Part III lecture notes for String Theory.
A dynamical background metric

- Now consider the same theory, but coupled to a dynamical background metric, and view $x' = x + \epsilon$ as a “diffeomorphism”.
- The idea is that any sensible theory should not depend on the choice of coordinates.
- We will need to use methods from general relativity, since the metric (or its dependence on the coordinates) will change.
- We must introduce, temporarily at lease, the usual $\sqrt{-g}$ into the measure, and replace partial derivatives with covariant ones.
- The *assumption* is that the various terms of $\delta S_{\text{diff}}$ should come from two different sources — the change in the field and the change in the metric.
A dynamical background metric

- So
  \[ 0 = \delta S_{\text{diff}} = \int d^4x \, J^{\alpha\beta} \partial_\alpha \epsilon_\beta + \int d^4x \, \frac{\delta S}{\delta g_{\alpha\beta}} \delta g_{\alpha\beta}. \]

- Evaluating \( \delta g_{\mu\nu} \):
  \[
  g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^{\mu}} \frac{\partial x^\beta}{\partial x'^{\nu}} g_{\alpha\beta}
  = (\delta^\alpha_\mu - \partial_\mu \epsilon^\alpha_\nu)(\delta^\beta_\nu - \partial_\nu \epsilon^\beta_\mu) g_{\alpha\beta}
  = g_{\mu\nu} - \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu,
  \]
  i.e. \( \delta g_{\mu\nu} = - (\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) \).

- Hence we get
  \[
  \int d^4x \, J^{\alpha\beta} \partial_\alpha \epsilon_\beta = 2 \int d^4x \, \frac{\delta S}{\delta g_{\alpha\beta}} \partial_\alpha \epsilon_\beta.
  \]
  This means that \( J^{\alpha\beta} \) and \( 2 \frac{\delta S}{\delta g_{\alpha\beta}} \) must have equal divergences.

- Since we know \( J^{\alpha\beta} \) is conserved, we can define
  \[
  T^{\alpha\beta} = \frac{\delta S}{\delta g_{\alpha\beta}}
  \]
  and know that this is also conserved.
Electromagnetism revisited

- Returning to the electromagnetism example, we adapt the action to curved spacetime.

\[ S = \int d^4x \sqrt{-g} \left( -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right), \]

where \( F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu. \)

- A bunch of calculations then gives us

\[ \frac{\delta A}{\delta g_{\alpha\beta}} = -\frac{1}{8} \eta^{\alpha\beta} F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} F^{\alpha\sigma} F_{\sigma\beta}. \]

- Di Francesco et al. suggest a normalization factor of -2, giving us

\[ T^{\alpha\beta} = \frac{1}{4} \eta^{\alpha\beta} F_{\mu\nu} F_{\mu\nu} + F^{\alpha\sigma} F_{\sigma\beta}, \]

which is precisely what we obtained for the Belinfante tensor.
Issues

- Remember, we assumed that $\delta S_{\text{diff}}$ consisted of two parts — one from varying the fields and the other from varying the metric. If these are the only contributions, then the above analysis is correct. But are they?
- Consider electromagnetism again. Calculating $\delta S_{\text{diff}}$, we get

$$
\delta S_{\text{diff}} = \int d^4x \, \sqrt{-g} \left( -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right) \\
+ \int d^4x \, \sqrt{-g} \left( -\frac{1}{4} \delta g^{\mu\rho} g^{\nu\sigma} F^{\mu\nu} F_{\rho\sigma} \right) \\
+ \int d^4x \, \sqrt{-g} \left( -\frac{1}{4} g^{\mu\rho} \delta g^{\nu\sigma} F^{\mu\nu} F_{\rho\sigma} \right) \\
+ \int d^4x \, \sqrt{-g} \left( -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} \delta F^{\mu\nu} F_{\rho\sigma} \right) \\
+ \int d^4x \, \sqrt{-g} \left( -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F^{\mu\nu} \delta F_{\rho\sigma} \right) .
$$
Issues

- At first sight, this looks fine — the first three lines come from varying the metric, while the last two appear to come from varying the field.
- However, the field we vary to find the equation of motion is $A_\mu$, not $F_{\mu\nu}$.
- Moreover, as $F_{\mu\nu}$ is defined in terms of derivatives of $A_\mu$, we must consider the possibility that we get contributions from the change, under the coordinate transformation, of the derivative.
- Under the active transformation, the derivative operator does not change. Looking at just $\nabla_\mu A_\nu$, we get
  \[ \nabla_\mu A'_\nu = \partial_\mu A_\nu - \partial_\mu \epsilon^\rho \partial_\rho A_\nu - \epsilon^\rho \partial_\mu \partial_\rho A_\nu - \partial_\mu A_\rho \partial_\nu \epsilon^\rho - A_\rho \partial_\mu \partial_\nu \epsilon^\rho. \]
- Under the passive transformation, the derivative operator (or its dependence on the coordinates) changes. We get
  \[ \nabla'_\mu A'_\nu = \partial_\mu A_\nu - \partial_\mu \epsilon^\rho \partial_\rho A_\nu - \epsilon^\rho \partial_\mu \partial_\rho A_\nu - \partial_\mu A_\rho \partial_\nu \epsilon^\rho. \]
- It looks like we might get a third contribution to $\delta S_{\text{diff}}$. But, in electromagnetism we get a stroke of luck. The derivatives of $A_\mu$ are combined antisymmetrically to obtain $F_{\mu\nu}$, which results in this third contribution cancelling. (Or is there some deeper reason for this ‘luck’?)
Conclusions

- Active and passive transformations are often presented in a less than clear manner, yet an understanding of precisely what sort of transformation is taking place is often necessary.
- Rather than being one object, there are multiple definitions of the energy-momentum tensor.
- These different definitions lead to different tensors — for instance, the canonical energy-momentum tensor need not be symmetric.
- A ‘relocalization procedure’ can be used to convert between forms (though I haven’t described the details.)