The Energy-Momentum Tensor

Alan Reynolds

Department of Mathematical Sciences
Durham University

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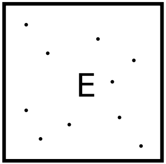
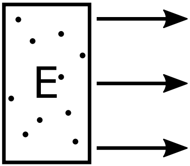
What's in a name?

- Energy-momentum tensor
- Stress-energy tensor
- Stress-energy-momentum tensor
- Energy tensor
- SEM
- ...

Why a tensor?



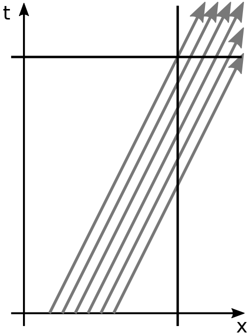
Rest frame



Moving frame

- A particle has energy and momentum.
- If a particle has energy (mass) E in its rest frame, then in a frame moving at relative velocity v, the energy is given by γE, where $\gamma = 1/\sqrt{1 - v^2}$. Energy is a component of a four-vector — the four-momentum.
- Given a field or a fluid, we must consider the energy *density*.
- If we switch from the rest frame to a moving frame, the energy in a volume element increases by a factor of γ, while the volume decreases by the same factor. The energy density increases by a factor of γ^2.
- This suggests that the energy density is an element of a $\binom{2}{0}$ tensor.

What do all the bits mean?



Well,

- T^{00}: Energy density,
- T^{0i}: Energy flux across a surface of constant x^i,
- T^{i0}: Momentum density in i direction,
- T^{ij}: Flux of i momentum across a surface of constant x^j,

or we can be more brief and simply say

- $T^{\alpha\beta}$: Flux of α momentum across a surface of constant x^β.

Note that, for this case at least, the energy-momentum tensor is symmetric — the energy flux across a surface of constant x^i *is* the momentum density in the i direction.[1]

[1] See B. Schutz, " A First Course in General Relativity", chapter 4, for a pedagogical introduction to the energy-momentum tensor, using 'dust' as an example.

Noether current recap

- A continuous 'active' transformation that leaves the equations of motion unchanged leads to a conserved current.
- To leave the equations of motion unchanged we require

$$\Delta S = 0 \;\Leftrightarrow\; \mathcal{L} \to \mathcal{L}' = \mathcal{L} + \epsilon \partial_\mu F^\mu$$

  for some F^μ. If the field transforms as

$$\phi \to \phi' = \phi + \epsilon \Delta \phi,$$

  then the Noether current is given by

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta \phi - F^\mu.$$

- The Noether current is conserved, i.e. $\partial_\mu j^\mu = 0$.

Energy-momentum as a Noether current — A typical presentation

Consider a translation invariant theory and consider the translation

$$x^\mu \to x^\mu - \epsilon a^\mu$$

as an active transformation. Then

$$\phi(x) \to \phi'(x) = \phi(x + \epsilon a) = \phi(x) + \epsilon a^\mu \partial_\mu \phi(x),$$

so that $\Delta\phi = a^\mu \partial_\mu \phi(x)$. Likewise,

$$\mathcal{L} \to \mathcal{L} + \epsilon a^\mu \partial_\mu \mathcal{L},$$

i.e. $F^\mu = a^\mu \mathcal{L}$. Hence, the Noether current is

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - F^\mu = a^\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu \mathcal{L} \right).$$

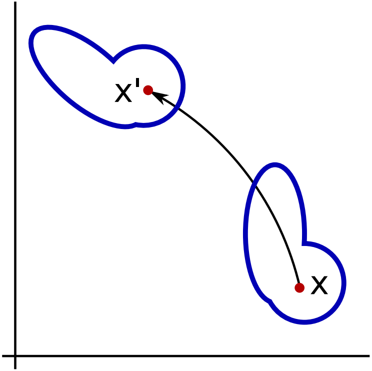
Since a^μ is arbitrary (and constant), we get $\partial_\mu T^\mu{}_\nu = 0$ where

$$T^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu \mathcal{L}.$$

Active transformations: Misconceptions

- It is, perhaps, unhelpful to think of active and passive transformations as being opposites.
- It isn't really the 'activeness' of the transformation that is really important here.
- The kind of active transformation we want *does not change* the coordinates. *Only* the field is transformed. We translate (or rotate) the contents of the universe, but leave everything else — the coordinate system and the 'theory' — alone.

Active transformations: Scalar field



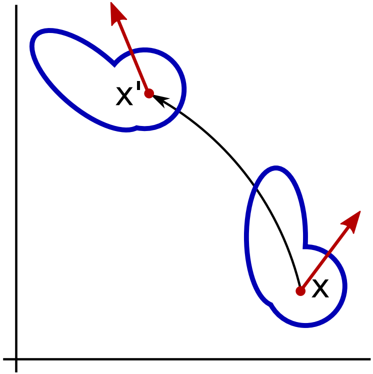
- An active transformation is one "in which the field is truly shifted" (Tong).
- But it is *only* the field that is changed.
- We want the value of the new field at x' to be equal to that for the old field at x, i.e.

$$\phi'(x') = \phi(x).$$

- If $x' = Rx$ then we need $\phi'(Rx) = \phi(x)$ and hence

$$\phi'(x) = \phi(R^{-1}x).$$

Active transformations: Vector field



- A rotation of a vector field results in a rotation of each individual vector, as well as the change of location.
- We get

$$\phi'(x') = R\phi(x).$$

and hence

$$\phi'(x) = R\phi(R^{-1}x).$$

- Of course, other fields might undergo different transformations, i.e.

$$\phi'(x) = L_R\phi(R^{-1}x),$$

where the L_R are elements of a representation of the group of rotations.

Energy-momentum as a Noether current, revisited

Consider a transformation of the field, $\phi \to \phi'$, such that

$$\phi'(x') = \phi(x),$$

where $x'^{\mu} = x^{\mu} - \epsilon a^{\mu}$ corresponds to a translation of the field. Then

$$\phi'(x) = \phi(x + \epsilon a) = \phi(x) + \epsilon a^{\mu} \partial_{\mu} \phi(x)$$

and $\Delta\phi = a^{\mu} \partial_{\mu} \phi(x)$.

Energy-momentum as a Noether current, revisited

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Energy-momentum as a Noether current, revisited

Consider a transformation of the field, $\phi \to \phi'$, such that

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$$\phi'(x) = \phi(x + \epsilon a) = \phi(x) + \epsilon a^\mu \partial_\mu \phi(x)$$

and $\Delta\phi = a^\mu \partial_\mu \phi(x)$.

We do not change the form of the Lagrangian, $\mathcal{L}(\phi, \partial_\mu \phi)$, but its value at position x will be different for the new field:

$$\begin{aligned} \mathcal{L}(\phi'(x), \partial_\mu \phi'(x)) &= \mathcal{L}(\phi(x + \epsilon a), \partial_\mu \phi(x + \epsilon a)) \\ &= \mathcal{L}(\phi(x), \partial_\mu \phi(x)) + \epsilon a^\mu \partial_\mu \mathcal{L}(\phi(x), \partial_\mu \phi(x)). \end{aligned}$$

Hence $F^\mu = a^\mu \mathcal{L}$. As a result, the Noether current is

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - F^\mu = a^\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu \mathcal{L} \right).$$

Since a^μ is arbitrary (and constant), we get $\partial_\mu T^\mu{}_\nu = 0$ where

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Energy-momentum as a Noether current, revisited

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Hence $F^{\mu} = a^{\mu}\mathcal{L}$. As a result, the Noether current is

$$j^{\mu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)}\Delta\phi - F^{\mu} = a^{\nu}\left(\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)}\partial_{\nu}\phi - \delta^{\mu}_{\nu}\mathcal{L}\right).$$

Since a^{μ} is arbitrary (and constant), we get $\partial_{\mu} T^{\mu}{}_{\nu} = 0$ where

$$T^{\mu}{}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)}\partial_{\nu}\phi - \delta^{\mu}_{\nu}\mathcal{L}.$$

A non-translation invariant Lagrangian

Consider the Lagrangian

$$\mathcal{L}(\phi(x), \partial_\mu \phi(x), x^\mu) = x^\mu x_\mu \phi^*(x)\phi(x).$$

If we perform the same active transformation, i.e. translate the field *only*, then the value of the Lagrangian at point x becomes

$$\begin{aligned}\mathcal{L}(\phi'(x), \partial_\mu \phi'(x), x^\mu) &= x^\mu x_\mu \phi^*(x+\epsilon a)\phi(x+\epsilon a) \\ &\neq \mathcal{L}(\phi(x), \partial_\mu \phi(x)) + \epsilon a^\mu \partial_\mu \mathcal{L}(\phi(x), \partial_\mu \phi(x)).\end{aligned}$$

So, we now know where the derivation of the energy-momentum tensor fails for non-translation invariant theories.

An example: Electromagnetism

For electromagnetism[2], without sources, the action is

$$S = \int d^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right),$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Now

$$\frac{\partial F_{\mu\nu}}{\partial(\partial_\rho A_\sigma)} = \delta^\rho_\mu \delta^\sigma_\nu - \delta^\rho_\nu \delta^\sigma_\mu, \qquad \frac{\partial F^{\mu\nu}}{\partial(\partial_\rho A_\sigma)} = \eta^{\rho\mu}\eta^{\sigma\nu} - \eta^{\rho\nu}\eta^{\sigma\mu},$$

so

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial(\partial_\rho A_\sigma)} &= -\frac{1}{4}\left(\delta^\rho_\mu \delta^\sigma_\nu - \delta^\rho_\nu \delta^\sigma_\mu\right) F^{\mu\nu} - \frac{1}{4} F_{\mu\nu}\left(\eta^{\rho\mu}\eta^{\sigma\nu} - \eta^{\rho\nu}\eta^{\sigma\mu}\right) \\ &= -\frac{1}{4}F^{\rho\sigma} + \frac{1}{4}F^{\sigma\rho} - \frac{1}{4}F^{\rho\sigma} + \frac{1}{4}F^{\sigma\rho} = F^{\sigma\rho}. \end{aligned}$$

The energy-momentum tensor is therefore

$$T^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)} \partial_\nu A_\rho - \delta^\mu_\nu \mathcal{L} = F^{\rho\mu}\partial_\nu A_\rho + \frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} \delta^\mu_\nu.$$

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[2] Peskin and Schroeder, exercise 2.1

A symmetric tensor?

- For fluids or dust, the energy-momentum tensor defined in terms of mometum flux is symmetric.
- We would like a symmetric tensor for many reasons:
  - General relativity *requires* a symmetric tensor,
  - Symmetric tensors are simpler to manipulate,
  - etc.
- But

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi - \eta^{\mu\nu} \mathcal{L}$$

  is not obviously symmetric.
- The energy-momentum tensor for electromagnetism,

$$T^{\mu\nu} = F^{\rho\mu} \partial^\nu A_\rho + \frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} \eta^{\mu\nu}$$

  certainly doesn't look symmetric.

Creating a symmetric energy-momentum tensor — the Belinfante tensor

- If $K^{\lambda\mu\nu}$ is antisymmetric in the first two indices, then adding $\partial_\lambda K^{\lambda}{}_{\mu\nu}$ to the energy-momentum tensor does not change its conservation properties, since if
$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu},$$
then
$$\partial_\mu \tilde{T}^{\mu\nu} = \cancel{\partial_\mu T^{\mu\nu}} + \cancel{\partial_\mu \partial_\lambda K^{\lambda\mu\nu}} = 0.$$
- For electromagnetism[3], choose $K^{\lambda\mu\nu} = F^{\mu\lambda} A^{\nu}$. After a few calculations, we get
$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu} = \frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} \eta^{\mu\nu} + F^{\lambda\mu} F^{\nu}{}_{\lambda} + A^{\nu} \partial_\lambda F^{\mu\lambda}.$$
- But we can use the equations of motion, $\partial_\lambda F^{\mu\lambda}$, to eliminate the last term to leave
$$\frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} \eta^{\mu\nu} + F^{\lambda\mu} F^{\nu}{}_{\lambda},$$
which is manifestly symmetric.

[3] See Wikipedia (!) for the general case.

Symmetric 'on shell'

- We used the equations of motion to show that the new energy-momentum tensor is symmetric.
- In other words, we have shown that we can convert the canonical energy-momentum tensor to an alternative form that is equal, *classically* to an identically symmetric energy-momentum tensor.
- Classically, this distinction is of no importance.
- Quantum mechanically, the two energy-momentum tensors are different and result in different Ward identities.
- Apparently, though, the change in the Ward identities is of no physical significance.[4]

[4] See Di Francesco et al., "Conformal Field Theory", section 2.5.

Passive transformations

- A passive transformation is one "in which the mapping $x \to x'$ is viewed simply as a coordinate transformation"[5].
- Wikipedia (!) suggests that, given a rotation matrix R, a passive transformation consists of rotating the basis vectors of our coordinate system using R. Hence with initial basis vectors $\mathbf{e}_\mu$ we obtain new basis vectors $\mathbf{e}'_\mu = R\mathbf{e}_\mu$. In the usual way, we find coordinates transform as

$$x'^\mu = (R^{-1})^\mu_\nu x^\nu,$$

or simply $x \to x' = R^{-1}x$. Hence, in a sense, we obtain the opposite transformation to that in the active case. But...

[5] Di Francesco et al.

Opposites?

- As physicists, we tend to consider coordinate transformations without thinking a great deal about basis vectors. Our starting point is usually something like $x \rightarrow x' = Rx$. If the same coordinate transformation is also used to generate the active transformation, then the field transforms *the same way* in both cases.
- An active transformation *transforms the fields only*. Any explicit dependence the Lagrangian has on x is unchanged. However, if we perform a mere change of coordinates, then the explicit dependence of the Lagrangian on x' will be different.

Variation with respect to the metric

Just when you think you might be getting the hang of the energy-momentum tensor, someone will say that it is defined as something like

$$T_{ab} = -\frac{\alpha_M}{8\pi}\frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{ab}}$$

or

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta \mathcal{L}_{\text{matter}}}{\delta g^{\mu\nu}}$$

or

$$T_{\alpha\beta} = -\frac{4\pi}{\sqrt{-g}}\frac{\partial S}{\partial g^{\alpha\beta}}.$$

They might also describe this as the 'gravitational' or 'relativists' definition of energy-momentum. But where on earth did these come from?

An intermediate, new definition[6]

- "Promote the constant parameter ϵ that appears in the symmetry to a function of the spacetime coordinates".
- The change in the action should then be of the form

$$\delta S = \int d^4x\, J^{\alpha} \partial_{\alpha} \epsilon,$$

  or in the case of a translation

$$\delta S = \int d^4x\, J^{\alpha\beta} \partial_{\alpha} \epsilon_{\beta}.$$

- Terms in ϵ (undifferentiated) should cancel — these are the terms that occur when ϵ is a constant and we have assumed that the theory is symmetric under such transformations.
- But, on-shell, equations of motion are found by assuming that δS is zero for *any* infinitesimal variation of the fields.
- Integrate by parts to get $\partial_{\alpha} J^{\alpha} = 0$, or $\partial_{\alpha} J^{\alpha\beta} = 0$.

[6] Here we attempt to follow David Tong's Part III lecture notes for String Theory.

A dynamical background metric

- Now consider the same theory, but coupled to a dynamical background metric, and view $x' = x + \epsilon$ as a "diffeomorphism".
- The idea is that any sensible theory should not depend on the choice of coordinates.
- We will need to use methods from general relativity, since the metric (or its dependence on the coordinates) will change.
- We must introduce, temporarily at lease, the usual $\sqrt{-g}$ into the measure, and replace partial derivatives with covariant ones.
- The *assumption* is that the various terms of δS_{diff} should come from two different sources — the change in the field and the change in the metric.

A dynamical background metric

- So

$$0 = \delta S_{\text{diff}} = \int d^4x\, J^{\alpha\beta}\partial_\alpha \epsilon_\beta + \int d^4x\, \frac{\delta S}{\delta g_{\alpha\beta}} \delta g_{\alpha\beta}.$$

- Evaluating $\delta g_{\mu\nu}$:

$$\begin{aligned} g'_{\mu\nu} &= \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta} \\ &= (\delta^\alpha_\mu - \partial_\mu \epsilon^\alpha)(\delta^\beta_\nu - \partial_\nu \epsilon^\beta) g_{\alpha\beta} \\ &= g_{\mu\nu} - \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu, \end{aligned}$$

  i.e. $\delta g_{\mu\nu} = -(\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu)$.
- Hence we get

$$\int d^4x\, J^{\alpha\beta}\partial_\alpha \epsilon_\beta = 2 \int d^4x\, \frac{\delta S}{\delta g_{\alpha\beta}} \partial_\alpha \epsilon_\beta.$$

  This means that $J^{\alpha\beta}$ and $2\frac{\delta S}{\delta g_{\alpha\beta}}$ must have equal divergences.
- Since we know $J^{\alpha\beta}$ is conserved, we can define

$$T^{\alpha\beta} = \frac{\delta S}{\delta g_{\alpha\beta}}$$

  and know that this is also conserved.

Electromagnetism revisited

- Returning to the electromagnetism example, we adapt the action to curved spacetime.

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right),$$

  where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$.
- A bunch of calculations then gives us

$$\frac{\delta A}{\delta g_{\alpha\beta}} = -\frac{1}{8} \eta^{\alpha\beta} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} F^{\alpha\sigma} F_\sigma{}^\beta.$$

- Di Francesco et al. suggest a normalization factor of -2, giving us

$$T^{\alpha\beta} = \frac{1}{4} \eta^{\alpha\beta} F^{\mu\nu} F_{\mu\nu} + F^{\alpha\sigma} F_\sigma{}^\beta,$$

  which is precisely what we obtained for the Belinfante tensor.

Issues

- Remember, we assumed that δS_{diff} consisted of two parts — one from varying the fields and the other from varying the metric. If these are the only contributions, then the above analysis is correct. But are they?
- Consider electromagnetism again. Calculating δS_{diff}, we get

$$\begin{aligned}\delta S_{\text{diff}} = &\int d^4x\, \delta\sqrt{-g}\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right)\\ &+\int d^4x\,\sqrt{-g}\left(-\frac{1}{4}\delta g^{\mu\rho}g^{\nu\sigma}F^{\mu\nu}F_{\rho\sigma}\right)\\ &+\int d^4x\,\sqrt{-g}\left(-\frac{1}{4}g^{\mu\rho}\delta g^{\nu\sigma}F^{\mu\nu}F_{\rho\sigma}\right)\\ &+\int d^4x\,\sqrt{-g}\left(-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}\delta F^{\mu\nu}F_{\rho\sigma}\right)\\ &+\int d^4x\,\sqrt{-g}\left(-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F^{\mu\nu}\delta F_{\rho\sigma}\right).\end{aligned}$$

Issues

- At first sight, this looks fine — the first three lines come from varying the metric, while the last two appear to come from varying the field.
- However, the field we vary to find the equation of motion is A_μ, not $F_{\mu\nu}$.
- Moreover, as $F_{\mu\nu}$ is defined in terms of derivatives of A_μ, we must consider the possibility that we get contributions from the change, under the coordinate transformation, of the *derivative*.
- Under the active transformation, the derivative operator does not change. Looking at just $\nabla_\mu A_\nu$, we get

$$\nabla_\mu A'_\nu = \partial_\mu A_\nu - \partial_\mu \epsilon^\rho \partial_\rho A_\nu - \epsilon^\rho \partial_\mu \partial_\rho A_\nu - \partial_\mu A_\rho \partial_\nu \epsilon^\rho - A_\rho \partial_\mu \partial_\nu \epsilon^\rho .$$

- Under the passive transformation, the derivative operator (or its dependence on the coordinates) changes. We get

$$\nabla'_\mu A'_\nu = \partial_\mu A_\nu - \partial_\mu \epsilon^\rho \partial_\rho A_\nu - \epsilon^\rho \partial_\mu \partial_\rho A_\nu - \partial_\mu A_\rho \partial_\nu \epsilon^\rho .$$

- It looks like we might get a third contribution to δS_{diff}. But, in electromagnetism we get a stroke of luck. The derivatives of A_μ are combined antisymmetrically to obtain $F_{\mu\nu}$, which results in this third contribution cancelling. (Or is there some deeper reason for this 'luck'?)

Conclusions

- Active and passive transformations are often presented in a less than clear manner, yet an understanding of precisely what sort of transformation is taking place is often necessary.
- Rather than being one object, there are multiple definitions of the energy-momentum tensor.
- These different definitions lead to different tensors — for instance, the canonical energy-momentum tensor need not by symmetric.
- A 'relocalization procedure' can be used to convert between forms (though I haven't described the details.)