The mean field opinion model

Conrado da Costa

Department of Mathematical Sciences (Durham University)

email: conrado.da-costa@durham.ac.uk

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LONDON MATHEMATICAL SOCIETY
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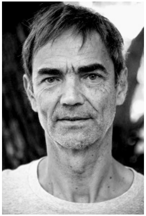
The project

The project

Joint with:

Inés Armendáriz

Monia Capanna



Pablo Ferrari

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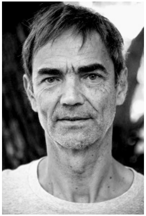

Pablo Ferrari

University of Buenos Aires,

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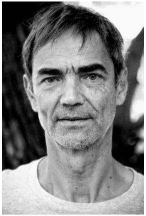

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University of Buenos Aires,
Leiden University,
Durham University.

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Group of individuals that must act as a team or committee.
Revision of opinions by an individual: $O_{i,n+1} = \sum_j p_{ij}^n O_{j,n}$

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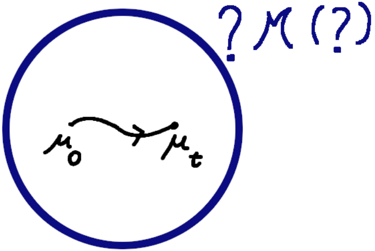
Goal: to design efficient fault-tolerant and distributed algorithms for computations in networks.

Approach



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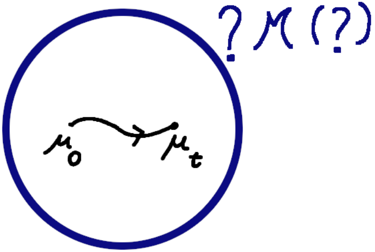

$$\frac{d}{dt}\mu_t = L^* \mu_t$$

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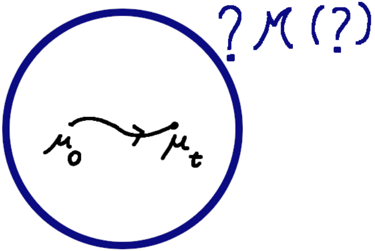

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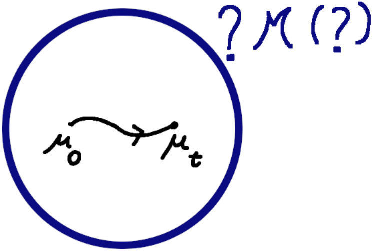

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The infinitesimal generator L is the heart of the process.

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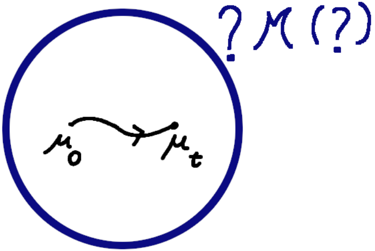

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Goal: To obtain a qualitative description of the evolution.

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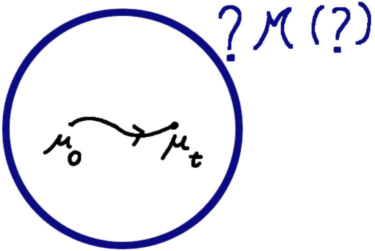

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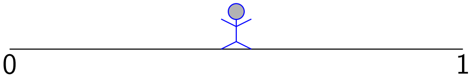
The infinitesimal generator L is the heart of the process.
Goal: To obtain a qualitative description of the evolution.
Method: ajust scales and compute distances.

The model: single individual



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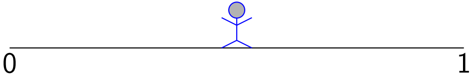

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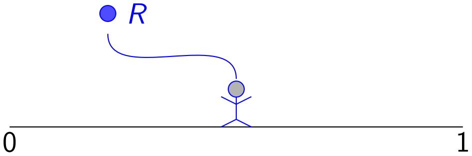

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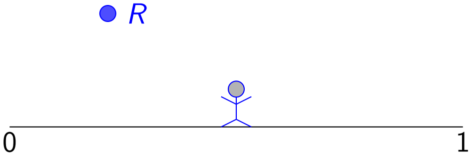
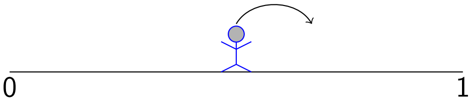


The model: single individual

$$\eta \to \eta^+$$

● *R*



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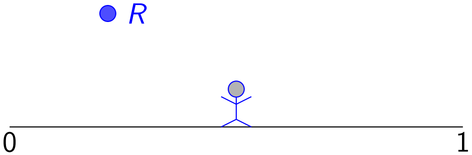
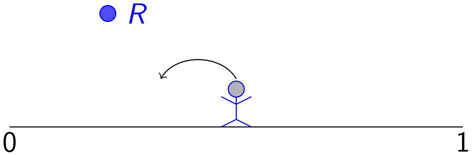


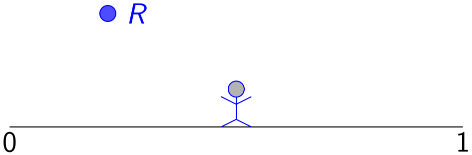
The model: single individual

$$\eta \to \eta^-$$



The model: single individual

$$\eta \to \begin{cases} \eta^{+} = (1-\alpha)\eta + \alpha \cdot 1 & \text{if} \\ \eta^{-} = (1-\alpha)\eta + \alpha \cdot 0 & \text{if} \end{cases}$$



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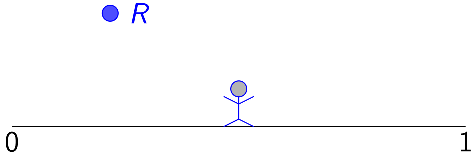
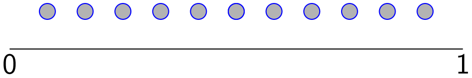


$$Lf(\eta) = R\,[f(\eta^+) - f(\eta)] + (1-R)\,[f(\eta^-) - f(\eta)]$$

The model: our case

$$\eta \to \begin{cases} \eta^{x,+}(y) := \begin{cases} (1-\alpha)\eta(x) + \alpha \cdot 1 & \text{if } y = x \\ \eta(y) & \text{if } y \neq x \end{cases} & \nabla_{x,+} f(\eta) = [f(\eta^{x,+}) - f(\eta)] \\ \\ \eta^{x,-}(y) := \begin{cases} (1-\alpha)\eta(x) + \alpha \cdot 0 & \text{if } y = x \\ \eta(y) & \text{if } y \neq x \end{cases} & \nabla_{x,-} f(\eta) = [f(\eta^{x,-}) - f(\eta)] \end{cases}$$



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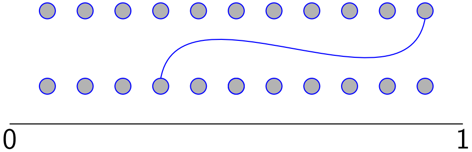

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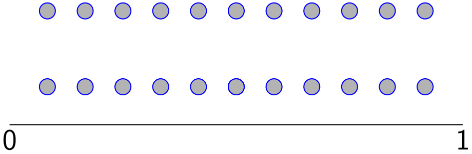

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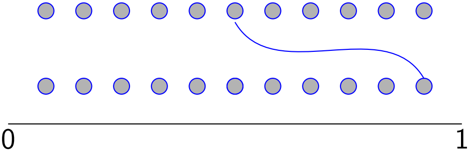

$$L_N f(\eta) = \sum_x \frac{1}{N} \sum_y \eta(y) \nabla_{x,+} f(\eta) + (1 - \eta(y)) \nabla_{x,-} f(\eta)$$

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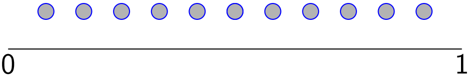

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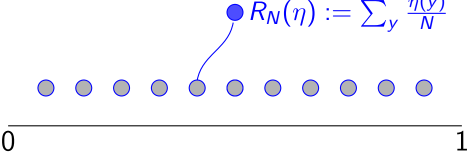

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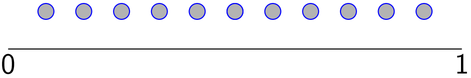

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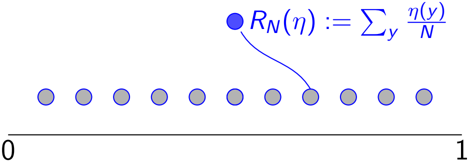

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Results: hydrodynamic limit

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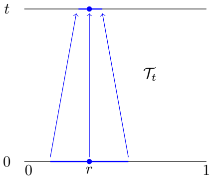

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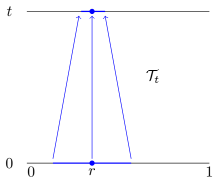

Intuition: Contraction

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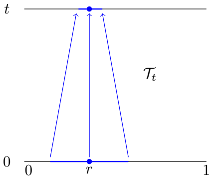

Intuition: Contraction

$$\eta^{\pm} - \zeta^{\pm} = (1 - \alpha_N)(\eta - \zeta)$$

Results: hydrodynamic limit

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$$\alpha_N := \frac{1}{\sqrt{N}} \qquad \eta_t^N := (\eta_t^N(1), \ldots, \eta_t^N(N))$$

$$\pi_t^N := \frac{1}{N}\sum_x \delta_{\eta_t^N(x)} \quad d_{\mathrm{V}}(\pi^N, \pi) \text{ (Vague distance)}$$

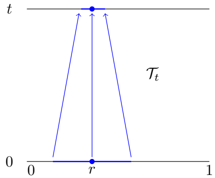
Theorem

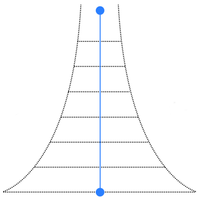
$d_{\mathrm{V}}(\pi_0^N, \pi^*) \to 0$ *then for each* $T > 0$

$$\sup_{t\in[0,T]} d_{\mathrm{V}}(\pi^N_{\sqrt{N}t}, \mathcal{T}_t(\pi^*)) \to 0, \quad \textit{with}$$

$$\mathcal{T}_t\pi^*(A) = \int 1_A(\mathcal{T}_t(x))\, d\pi^*(x), \ \textit{and}$$

$$\mathcal{T}_t(x) := r + (x-r)e^{-t}, r = \int x\, d\pi^*(x).$$





Intuition: Contraction

$$\eta^{\pm} - \zeta^{\pm} = (1-\alpha_N)(\eta - \zeta)$$

Results: long term behavior

$$R_t^N := \sum_x \frac{\eta_t^N(x)}{N}$$

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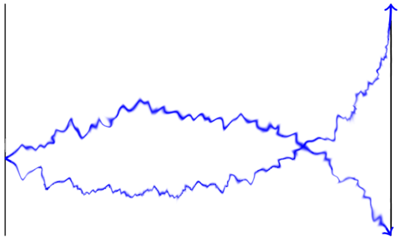

Results: motion of the mean

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Theorem

If $R_0^N \to r$ then $(R^N_{N^2 t}, t \in [0, T])$ converges in law to the unique solution of

$$\begin{cases} dR_t &= \sqrt{R_t(1-R_t)}dB_t \\ R_0 &= r, \end{cases}$$

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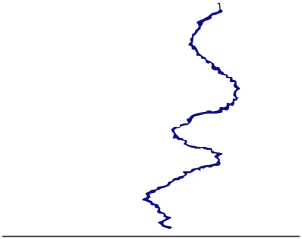

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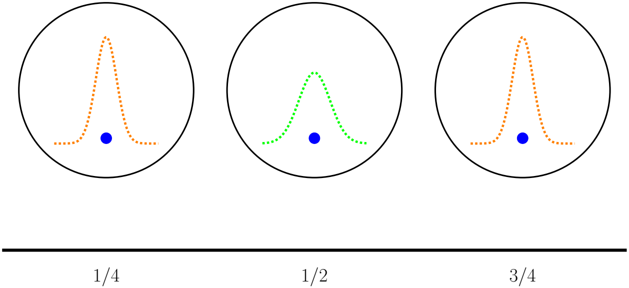
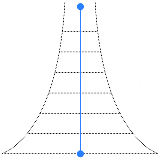


Summary



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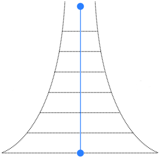

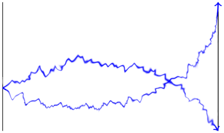

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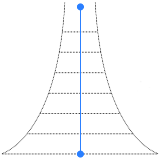

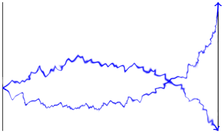

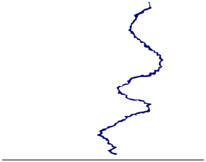

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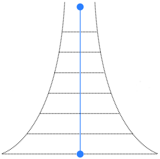

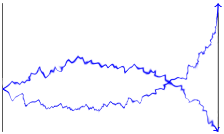

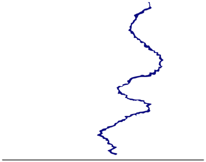

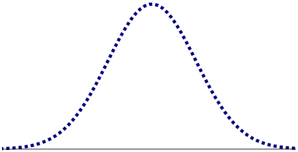

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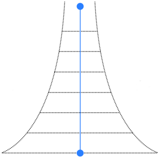

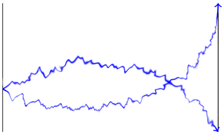

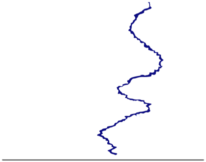

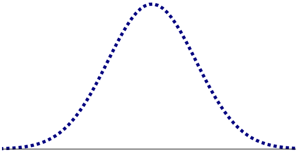

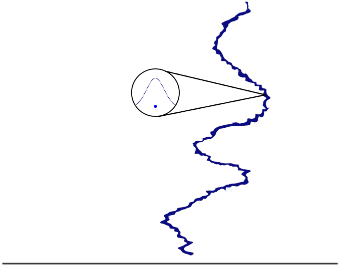

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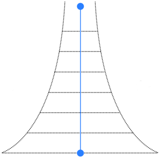
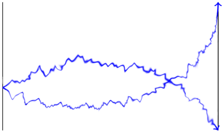
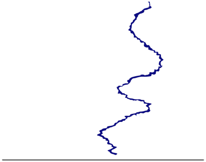
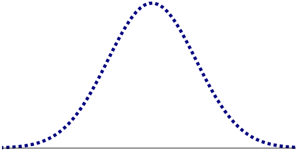
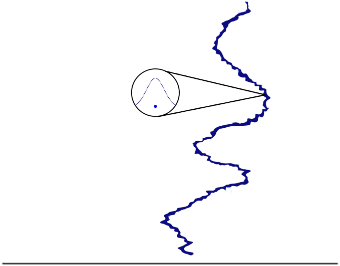


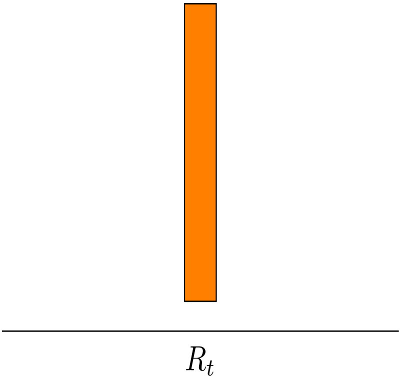








Fluctuation of the mean opinion



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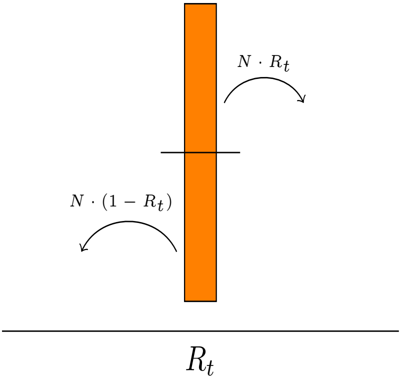

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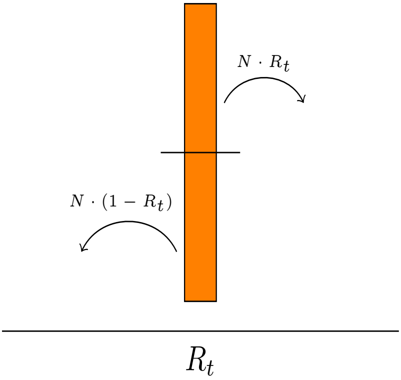

$$R_{t^+} - R_t = \frac{1}{N}\left[\quad N(1 - R_t) \quad \left(R_t - \frac{R_t}{\sqrt{N}}\right) + \left(N \cdot R_t\right) \quad \left(R_t + \frac{(1-R_t)}{\sqrt{N}}\right)\right] - R_t$$

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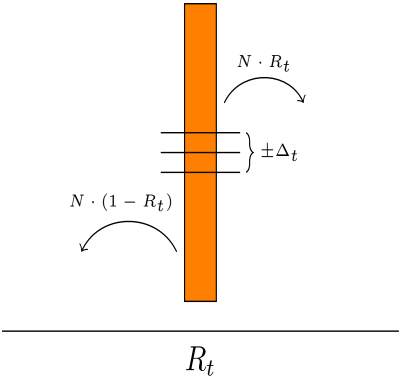

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$$\Delta_t = \sqrt{N}\sqrt{R_t(1 - R_t)}e_t$$

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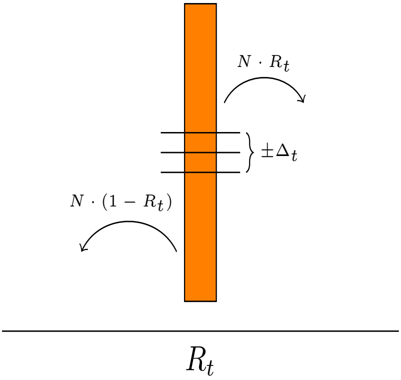

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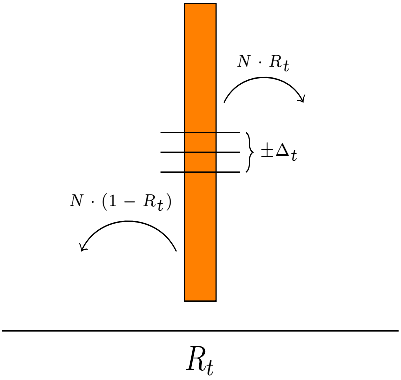

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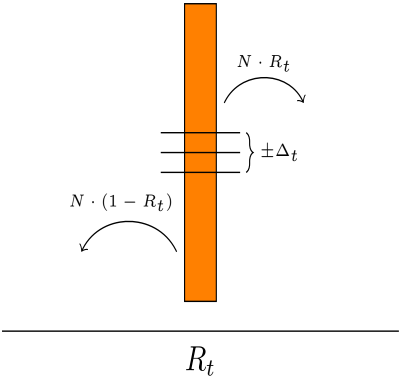

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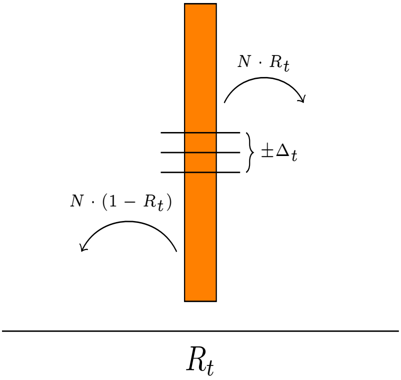

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Typical concentration: $$\frac{1}{N}\sum_x (\eta_t^N(x) - R_t^N)^2 \sim \frac{1}{\sqrt{N}}$$

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$$\mu_t^N := \mathrm{OU}_{R_0^N,t}^{\nu_0^N} - \nu_{\sqrt{N}t}^N$$

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Zoom-in: $D_t^N(x) := N^{1/4}(\eta_t^N(x) - R_t^N)$

Integral expressions: for $\nu_t^N := \frac{1}{N}\sum_x \delta_{D_t^N(x)}$ scale $t \rightsquigarrow \sqrt{N}t$

$$\left\langle \nu_{\sqrt{N}t}^N, G \right\rangle = \left\langle \nu_0^N, G \right\rangle + \int_0^t \left\langle \nu_{\sqrt{N}s}^N, -xG' + \frac{R_0^N\,(1-R_0^N)}{2} G'' \right\rangle ds + \mathcal{M}_{\sqrt{N}t}^{N,G},$$

$$\left\langle \mathrm{OU}_{R_0^N,t}^{\nu_0^N}, G \right\rangle = \left\langle \nu_0^N, G \right\rangle + \int_0^t \left\langle \mathrm{OU}_{R_0^N,s}^{\nu_0^N}, -xG' + \frac{R_0^N\,(1-R_0^N)}{2} G'' \right\rangle ds.$$

$\mu_t^N := \mathrm{OU}_{R_0^N,t}^{\nu_0^N} - \nu_{\sqrt{N}t}^N$ $\sup_{t\in[0,T]} \mathcal{M}_{\sqrt{N}t}^{N,G} \to 0$

Dispersion around the mean

Typical concentration: $\frac{1}{N}\sum_x (\eta_t^N(x) - R_t^N)^2 \sim \frac{1}{\sqrt{N}}$

Zoom-in: $D_t^N(x) := N^{1/4}(\eta_t^N(x) - R_t^N)$

Integral expressions: for $\nu_t^N := \frac{1}{N}\sum_x \delta_{D_t^N(x)}$ scale $t \rightsquigarrow \sqrt{N}t$

$$\left\langle \nu^N_{\sqrt{N}t}, G \right\rangle = \left\langle \nu_0^N, G \right\rangle + \int_0^t \left\langle \nu^N_{\sqrt{N}s}, -xG' + \frac{R_0^N(1-R_0^N)}{2} G'' \right\rangle ds + \mathcal{M}^{N,G}_{\sqrt{N}t},$$

$$\left\langle \mathrm{OU}^{\nu_0^N}_{R_0^N,t}, G \right\rangle = \left\langle \nu_0^N, G \right\rangle + \int_0^t \left\langle \mathrm{OU}^{\nu_0^N}_{R_0^N,s}, -xG' + \frac{R_0^N(1-R_0^N)}{2} G'' \right\rangle ds.$$

$\mu_t^N := \mathrm{OU}^{\nu_0^N}_{R_0^N,t} - \nu^N_{\sqrt{N}t}$ $\quad \sup_{t\in[0,T]} \mathcal{M}^{N,G}_{\sqrt{N}t} \to 0$ $\quad \mu_t^N \to \mu_t^*$

Dispersion around the mean

Typical concentration: $\frac{1}{N}\sum_x (\eta_t^N(x) - R_t^N)^2 \sim \frac{1}{\sqrt{N}}$

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$$\left\langle \mathrm{OU}_{R_0^N,t}^{\nu_0^N}, G \right\rangle = \left\langle \nu_0^N, G \right\rangle + \int_0^t \left\langle \mathrm{OU}_{R_0^N,s}^{\nu_0^N}, -xG' + \frac{R_0^N(1-R_0^N)}{2} G'' \right\rangle ds.$$

$\mu_t^N := \mathrm{OU}_{R_0^N,t}^{\nu_0^N} - \nu_{\sqrt{N}t}^N$ $\quad \sup_{t\in[0,T]} \mathcal{M}_{\sqrt{N}t}^{N,G} \to 0$ $\quad \mu_t^N \to \mu_t^*$

$$\langle \mu_t^*, G \rangle = \int_0^t -\langle \mu_s^*, xG' \rangle + \frac{R_0^*(1-R_0^*)}{2} \langle \mu_s^*, G'' \rangle \, ds$$

Dispersion around the mean

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$$\left\langle \mathrm{OU}_{R_0^N,t}^{\nu_0^N}, G \right\rangle = \left\langle \nu_0^N, G \right\rangle + \int_0^t \left\langle \mathrm{OU}_{R_0^N,s}^{\nu_0^N}, -xG' + \frac{R_0^N(1-R_0^N)}{2} G'' \right\rangle ds.$$

$\mu_t^N := \mathrm{OU}_{R_0^N,t}^{\nu_0^N} - \nu_{\sqrt{N}t}^N$ $\sup_{t\in[0,T]} \mathcal{M}_{\sqrt{N}t}^{N,G} \to 0$ $\mu_t^N \to \mu_t^*$

$$\langle \mu_t^*, G \rangle = \int_0^t -\langle \mu_s^*, xG' \rangle + \frac{R_0^*(1-R_0^*)}{2} \langle \mu_s^*, G'' \rangle \, ds$$

Unique solution:

Dispersion around the mean

Typical concentration: $\frac{1}{N}\sum_x (\eta_t^N(x) - R_t^N)^2 \sim \frac{1}{\sqrt{N}}$

Zoom-in: $D_t^N(x) := N^{1/4}(\eta_t^N(x) - R_t^N)$

Integral expressions: for $\nu_t^N := \frac{1}{N}\sum_x \delta_{D_t^N(x)}$ scale $t \rightsquigarrow \sqrt{N}t$

$$\left\langle \nu_{\sqrt{N}t}^N, G \right\rangle = \left\langle \nu_0^N, G \right\rangle + \int_0^t \left\langle \nu_{\sqrt{N}s}^N, -xG' + \frac{R_0^N(1-R_0^N)}{2} G'' \right\rangle ds + \mathcal{M}_{\sqrt{N}t}^{N,G},$$

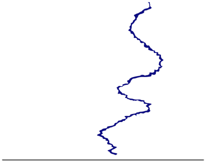
$$\left\langle \mathrm{OU}_{R_0^N,t}^{\nu_0^N}, G \right\rangle = \left\langle \nu_0^N, G \right\rangle + \int_0^t \left\langle \mathrm{OU}_{R_0^N,s}^{\nu_0^N}, -xG' + \frac{R_0^N(1-R_0^N)}{2} G'' \right\rangle ds.$$

$\mu_t^N := \mathrm{OU}_{R_0^N,t}^{\nu_0^N} - \nu_{\sqrt{N}t}^N$ $\quad \sup_{t\in[0,T]} \mathcal{M}_{\sqrt{N}t}^{N,G} \to 0$ $\quad \mu_t^N \to \mu_t^*$

$$\langle \mu_t^*, G \rangle = \int_0^t -\langle \mu_s^*, xG' \rangle + \frac{R_0^*(1-R_0^*)}{2} \langle \mu_s^*, G'' \rangle \, ds$$

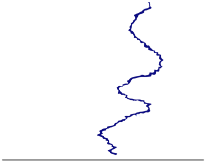
Unique solution: $\langle \mu_t^*, G \rangle = 0.$

Zoom-in on top of a fluctuation

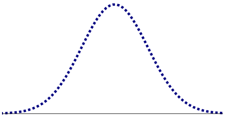


$t \mapsto R^N_{N^2 t}$

Zoom-in on top of a fluctuation

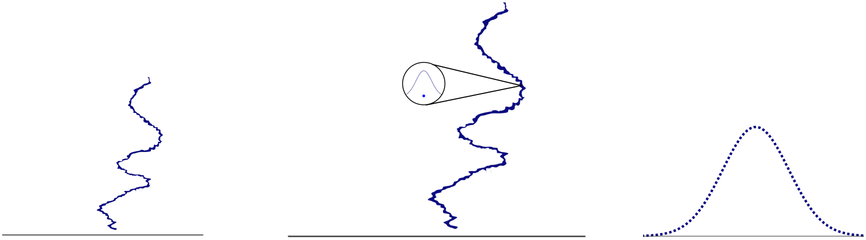


$t \mapsto R^N_{N^2 t}$



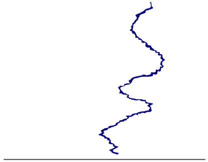
$t \mapsto \nu^N_{\sqrt{N} t}$

Zoom-in on top of a fluctuation

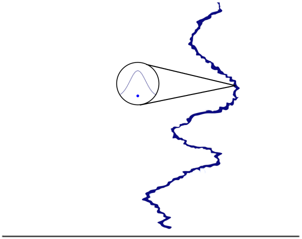


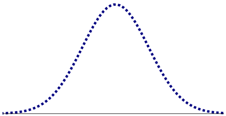
$t \mapsto R^N_{N^2 t}$ $t \mapsto \nu^N_{\sqrt{N} t}$

Zoom-in on top of a fluctuation



$t \mapsto R^N_{N^2 t}$

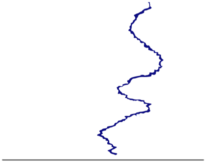




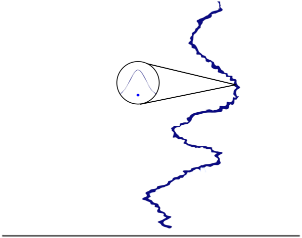
$t \mapsto \nu^N_{\sqrt{N} t}$

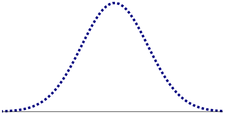
Two scales

Zoom-in on top of a fluctuation



$t \mapsto R^N_{N^2 t}$



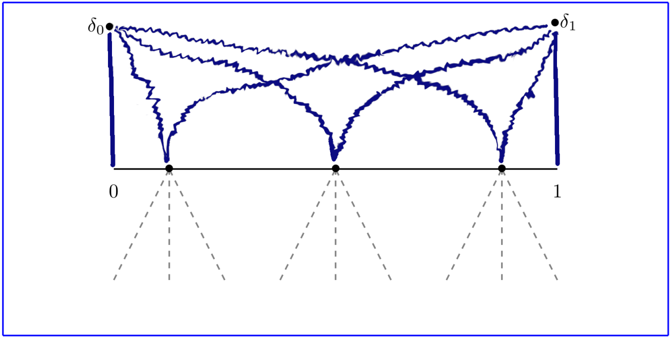


$t \mapsto \nu^N_{\sqrt{N} t}$

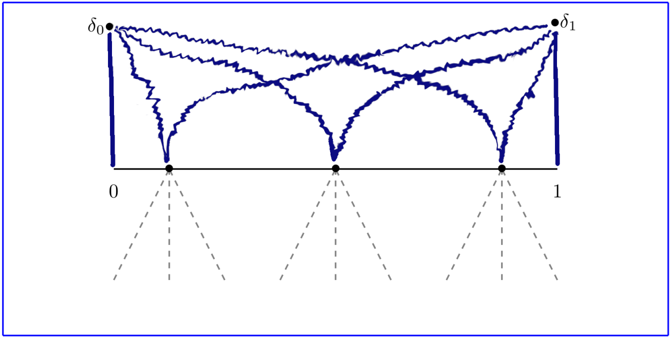
Two scales

$$\left(R^N_{N^2 t}, \nu^N_{N^2 t}\right) \xrightarrow[N\to\infty]{\mathrm{d}} \left(\mathrm{WF}_t, \mathsf{OU}_{\mathrm{WF}_t,\mathsf{eq}}\right),$$

Resumo da obra



Resumo da obra



Atypical example of metaestability.

Thank you!

Thank you!

Questions?