Durham University

North-East and Midlands Stochastic Analysis Seminar

supported by *the London Mathematical Society and Isaac Newton Institute for Mathematical Sciences*

Organisers: *Zdzislaw Brzezniak (York), Horatio Boedihardjo (Warwick), David Elworthy (Warwick), Chunrong Feng (Durham), Maximiliano Gubinelli (Oxford), Zhongmin Qian (Oxford), Roger Tribe (Warwick), Huaizhong Zhao (Durham)*

Programme (Venue: MCS2068, Department of Mathematical Sciences, Teams Meeting ID: 346 831 660 340 4 Passcode: 2G8gq9W6)

Thursday 25 Sept. 2025

14:00-15:00: **Arnaud Debussche (Ecole Normale Supérieure de Rennes)** *From correlated to white transport noise in fluid models*

15:00-15:30: Break

15:30-16:30: **Khoa Le (Leeds)** *McKean-Vlasov equations with rough common noise*

16:30-17:30: **Yuzhao Wang (Birmingham)** *Non-triviality of the radial Φ^4_d-model for $d \ge 4$*

Friday 26 Sept. 2025

9:10-10:10: **Ana Bela Cruzeiro (Lisbon)** *On some pathwise stochastic control problems*

10:10-11:10: **Mu Niu (Glasgow)** *Data-Driven Riemannian Geometry for Statistical Modelling and Machine* Learning

11:10-11:30: Break

11:30-12:30: **Samuel Cohen (Oxford)** *Mean-field generalization error for optimal control in stochastic environments*

12:30-14:00: Lunch

14:00-14:30: **Jinxiang Yao (Durham)** *Pathwise dynamics of cooperative time-inhomogeneous SDEs and McKean-Vlasov SDEs*

14:30-15:00: **Hugo Chu (Imperial College)** *Rigorous enclosure of Lyapunov exponents of stochastic flows*

15:00-15:30: **Danyang Zhao (Shandong)** (online) *The existence of invariant sublinear expectations for G-SDEs*

15:30-16:00: **Tong Lu (Shandong)** (online) *Invariant and periodic measures in classical spin systems on infinite lattices with highly degenerate noise*

If you have any queries, please contact Chunrong Feng (chunrong.feng@durham.ac.uk) or Huaizhong Zhao (huaizhong.zhao@durham.ac.uk).

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Titles and Abstracts

Arnaud Debussche (Ecole Normale Supérieure de Rennes)
Title: From correlated to white transport noise in fluid models
Abstract: *Stochastic fluid models with transport noise are popular, the transport noise models unresolved small scales. The main assumption in these models is a very strong separation of scales allowing this representation of small scales by white - ie fully decorrelated - noise. It is therefore natural to investigate whether these models are limits of models with correlated noises. Also, an advantage of correlated noises is that they allow classical calculus. In particular, it allows to revisit the derivation of stochastic models from variational principle and allows to derive equation for the evolution of the noise components. The advantage of having such equations is that in most works, the noise components are considered as given and stationary with respect to time which is non realistic. Coupling stochastic fluid models with these gives a more realistic system.*

Khoa Le (Leeds)
Title: McKean-Vlasov equations with rough common noise
Abstract: *We show well-posedness for McKean--Vlasov equations with rough common noise and progressively measurable coefficients. Our results are valid under natural regularity assumptions on the coefficients, in agreement with the respective requirements of Ito and rough path theory. To achieve these goals, we work in the framework of rough stochastic differential equations recently developed earlier.*

Yuzhao Wang (Birmingham)
Title: Non-triviality of the radial Φ^4_d-model for $d \ge 4$
Abstract: *The Φ^4_d-measure is the central object in constructive quantum field theory. Its construction for $d = 2, 3$ is one of the early achievements in the field in 1970s. For $d \ge 4$, the Φ^4_d-measure is known to be trivial with a recent breakthrough work for the 4-d case by Aizenman and Duminil-Copin (2021).*

In this talk, by imposing the radial assumption, I will present the construction of the Φ^4_d-measure for $d \ge 4$ on the unit ball by considering the limit of the Φ^4_d-measures on punctured balls. In order to control the singularity at the origin, we exploit the Markov property of the Gaussian free field over disjoint sets and sharp Green's function estimates on a punctured ball. We also show that the resulting Φ^4_d-measure has a sub-Gaussian tail and that it is singular with respect to the base Gaussian free field.

This is based on a joint work with Tadahiro Oh (the University of Edinburg), Leonardo Tolomeo (the University of Edinburgh), and Nikolay Tzvetkov (ENS Lyon).

Anna Bela Cruzeiro (Lisbon)
Title: On some pathwise stochastic control problems
Abstract: *I will describe some pathwise stochastic optimal control problems, as initially proposed by Lions and Souganidis in 1998, and derive the corresponding stochastic Hamilton-Jacobi-Bellman equations. This is joint work with N. Bhauryal and C. Oliveira.*

Mu Niu (Glasgow)
Title: Data-Driven Riemannian Geometry for Statistical Modelling and Machine Learning
Abstract:
In this talk, I will present a data-driven framework for incorporating Riemannian geometry into statistical modelling, with a particular focus on Gaussian process (GP) regression. High-dimensional data encountered in practice—such as point clouds, remote sensing measurements, or image collections—often concentrate near lower-dimensional manifolds with non-Euclidean geometry. Standard Euclidean GPs ignore this structure, leading to poor predictive performance and misleading uncertainty quantification. Our approach constructs GPs on complex or unknown manifolds by first learning a probabilistic atlas of the latent geometry, using tools such as autoencoders and latent variable models, and then defining stochastic processes that respect this geometry. This perspective connects ideas from stochastic differential equations on manifolds with statistical learning, allowing principled modelling of manifold-valued data. I will illustrate the method through simulations on the torus and applications to remote sensing of chlorophyll concentration in the Aral Sea, WiFi signal localisation, and image point clouds. The talk will give an overview of how data-driven Riemannian geometry can inform statistical modelling more broadly, including directions towards generative modelling and the study of animal movement, while highlighting the role of stochastic processes in bridging geometry, statistics, and machine learning.

Samuel Cohen (Oxford)
Title: Mean-field generalization error for optimal control in stochastic environments
Abstract: *In many problems in stochastic control, we ultimately have to begin with some training data, and use this to make decisions. In this talk we will talk about how mean-field analysis can be used to understand the out-of-sample error this introduces, in particular in a context where controls are parameterized by neural networks, and hence are extremely overparameterized.*

Jinxiang Yao (Durham)
Title: Pathwise dynamics of cooperative time-inhomogeneous SDEs and McKean-Vlasov SDEs
Abstract: *In this talk, by analyzing the structure of attractors for monotone non-autonomous random dynamical systems, we show that weakly dissipative cooperative stochastic differential equations (SDEs) with external forcing admit a unique globally attracting random path, which is (quasi-)periodic whenever the forcing is (quasi-)periodic. Building on these results, we further obtain pathwise conclusions for cooperative McKean-Vlasov SDEs, including the existence of order-related stationary solutions and the occurrence of connecting paths. This is a joint work with Chunrong Feng, Baoyou Qu and Huaizhong Zhao.*

Hugo Chu (Imperial College)
Title: Rigorous enclosure of Lyapunov exponents of stochastic flows
Abstract: *I will present a new method for obtaining rigorous and accurate upper and lower bounds for Lyapunov exponents of stochastic flows. Our approach is based on computer-assisted tools, the adjoint method, and established results on the ergodicity of diffusion processes. We do not require any structural assumptions on the stochastic system and work under mild hypoellipticity conditions and outside of perturbative regimes. Therefore, our*

method allows for the treatment of systems that were so far out of reach from existing mathematical tools. I will illustrate this method on different examples, for which we establish the positivity of the (top) Lyapunov exponent. This is joint work with Maxime Breden (École Polytechnique), Jeroen Lamb and Martin Rasmussen (Imperial College London).

Danyang Zhao (Shandong)
Title: The existence of invariant sublinear expectations for G-SDEs
Abstract: *We study the existence of invariant sublinear expectations of Markovian semigroups on sublinear expectation spaces. To achieve this, we establish a complete metric space of sublinear expectations, on which we extend Harris' method to the nonlinear setting on the convergence of sublinear semigroups. We then explore two cases of G-diffusions by studying the Lyapunov function and the local Doeblin condition. One is the G-Brownian motion on the unit circle. Another is the multidimensional G-SDEs on the whole space. We establish, for the first time in the literature, the existence of the invariant sublinear expectation for G-SDEs under the non-degenerate and weakly dissipative assumption. For this, we prove that for a class of G-SDEs, the G-expectation can be represented as the supremum of the semigroup of a family of SDEs, of which the regularity is obtained by considering the Bismut-Elworthy-Li formula and the Denis-Hu-Peng representation for the distribution of G-Brownian motions. This is a joint work with Huaizhong Zhao.*

Tong Lu (Shandong)
Title: Invariant and periodic measures in classical spin systems on infinite lattices with highly degenerate noise
Abstract: *We consider the classical spin systems on unbounded lattices given by infinite-dimensional stochastic differential equations (SDEs). We assume that the stochastic forcing acts only on one particle. The other particles are not subject to stochastic forcing directly but interact with their nearest neighbouring particles. Under the above highly degenerate noise setting, with some mild assumptions on the local interaction of each particle such as weak dissipation, we obtain the existence, uniqueness and the Markovian property of weak martingale solutions. We prove that the one-dimensional noise can propagate to any spin particle in the system in the sense that there exists a unique invariant/periodic measure, and geometric ergodicity holds for the Markovian system when restricted to any finite volume. We then prove the finite-dimensional invariant measure and the average of lifted periodic measure are tight, and weak convergent subsequence gives an invariant and periodic measures of the infinite spin systems, respectively, in the time-homogeneous or time-periodic cases. This is a joint work with Huaizhong Zhao.*