Groups, Geometry and Combinatorics

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A B S T R A C T S

Rosemary A. Bailey

Generalized wreath products of association schemes

The operation of forming the direct product of two association schemes is commutative but the wreath product is not. Thus these operations can be thought of as defined by the 2-point antichain and the 2-point chain respectively. This can be generalized to any finite poset, mirroring what can be done for permuation groups.

Alexander Baranov

Plain representations of non-semisimple Lie algebras and groups

Jointly with A. Zalesskii we started to study representations of non-semisimple Lie algebras and algebraic (or finite) groups. One of the most important results obtained is the correspondence between so-called "plain" Lie algebras and groups and finite dimensional associative algebras. Moreover, it has been shown that "plain" representation theory is almost equivalent to representation theory of finite dimensional associative algebras. Although the representation theory of finite dimensional non-semisimple associative algebras is quite developed, there were no similar results for non-semisimple Lie algebras.

Barbara Baumeister

Affine dual polar spaces

There is a uniform approach to consider interesting flag-transitive geometries belonging to diagrams close to those of spherical buildings – interesting in the sense that they include many geometies with sporadic automorphism group: The affinization of spherical buildings. In my talk I will focus on the affinization of dual polar spaces. They provide nice examples and it seems to be feasible to classify them.

Jon Brundan

Modular branching rules for the double covers of the symmetric groups

Recently I. Grojnowski has shown in detail how to connect the representation theory of affine Hecke algebras to the Kac-Moody algebra of type $A^{(1)}_{p-1}$. In this talk I will describe the analogues of these results for the twisted Kac-Moody algebra of type $A^{(2)}_{p-1}$. One needs to replace the affine Hecke algebra with the affine Hecke-Clifford superalgebra introduced by Jones and Nazarov. As a consequence, one gets modular branching rules for the double covers of the symmetric group, analogous to Kleshchev's modular branching rules for S_n itself.

Peter J. Cameron

Association schemes and permutation groups

Every permutation group gives rise in a natural way to a coherent configuration. However, not every permutation group preserves a non-trivial association scheme (a symmetric coherent configuration), and it is possible that a group preserves some association scheme but not a minimal association scheme with respect to refinement. I will give some recent results about these conditions and their negations, and how they relate to more familiar concepts from permutation group theory and statistical design such as generous transitivity and stratifiability. This is joint work with P. Alejandro and R. A. Bailey.

Arjeh Cohen

The LUP algorithm for other algebraic groups

The classical LUP algorithm decomposes a matrix into a product LUP of a Lower triangular matrix L, an Upper triangular matrix U and a Permutation matrix P. The matrix U can be chosen so that PUP^{-1} is lower triangular.

If the input matrix A is invertible, the decomposition can be found by elementary row and column operations on A, thereby keeping the column operations to a minimum. The result is then a triple K, N, M of two lower triangular matrices K and M and a monomial matrix N (that is, $N = HP$, the product of diagonal matrix H and a permutation matrix P) such that $A = KNM$ and NMN^{-1} is upper triangular. The triple $L = KH$, $U = PMP^{-1}$, P is then an LUP decomposition of A. The decomposition $A = KNM$ is the so-called Bruhat decomposition of an element of the general linear group.

In this context the question arises whether in any linear (highest weight) representation of an algebraic group we can find the Bruhat decomposition of a group element given by a matrix in the representation. In the talk we shall address this question and interpret the above LUP decomposition as a special case. The work is joint with Scott Murray and Don Taylor.

Robert T. Curtis

Symmetric generation: the way forward

A *progenitor* is defined to be a semi-direct product of a free product of isomorphic cyclic groups by a group of monomial automorphisms. Such a group is written $m^{*n} : N$, where m^{*n} denotes a free copy of n copies of the cyclic group of order m, and N acts on this free product by permuting a set of generators for the cyclic groups and raising them to powers co-prime to m. Thus a monomial representation of degree n of the group N over the field Z_p would enable us to define a progenitor of form $p^{*n} : N$.

Of course, when $m = 2$ the group N simply permutes the set of n involutions which generate the cyclic groups.

It is easy to see that every simple group is a homomorphic image of such progenitors (in many different ways), and over the last few years considerable progress has been made in defining groups in this manner. It turns out that the sporadic simple groups appear to be particularly amenable to this approach, and factoring a suitable progenitor by just one further short relator is usually enough to define the required image. This approach is closely related to the amalgam methods exploited by Parker and Rowley.

The smaller sporadic groups, including the Mathieu groups J_1, J_2, J_3, and HS, can be defined and constructed by hand in this way. Other groups, such as the Held group and the Harada-Norton group, at present require computation to verify that a certain progenitor factored by a single rather natural relator yields the desired group. For the largest sporadic groups, including the Monster, suitable progenitors are known, and relators exist which are conjectured to be sufficient to define the group. However, mechanical verification of these conjectures will remain out of range of computers for the foreseeable future.

It is anticipated that modern geometric techniques, and in particular the approach made famous by Conway, Ivanov and Norton in connection with Y-diagrams and the Monster, will provide the way forward in proving these conjectures by hand for all the outstanding cases.

Persi Diaconis

The mathematics of shuffling cards

The well known result that seven shuffles suffice to mix up 52 cards has its roots in the descent theory of Coxeter groups. There have been many refinements, and extensions and applications; to buildings and hyperplane arrangements, cyclic homology, ergodic theory, symmetric function theory and fairly applied computer science. My plan is to review the results (old and new) in some detail. There are also many lovely conjectures.

Peter Fleischmann

Constructive modular invariant theory of finite groups

Let G be a finite group, V a finite dimensional FG- module over the field F and $A := Sym(V^*)$ the symmetric algebra over the dual of V. The ring of invariants $A^G := \{a \in A | ga = a \text{ for all } g \in G\}$ is the main object of study in invariant theory. Classically the case where F is the field of complex numbers and G is a (possibly infinite) reductive group has been studied and the theory here is well developed. In particular it is known that A^G is a Cohen - Macaulay ring which, in the finite group case, is generated in degree less or equal to $|G|$ (called the 'Noether bound').

New applications, e.g. in algebraic geometry and group cohomology, ask for results about invariant rings of finite groups over fields of characteristic $p > 0$.

Here the theory is much less developed and has found new interest during the last twenty years or so. For example, if p divides $|G|$, the invariant ring is in general not Cohen - Macaulay and the computation of the depth of A^G is a serious challenge. Moreover Noether's degree bound also fails in this case and there is no replacement known, except in special cases. Even if $0 < p$ does not divide $|G|$, it has been a long standing conjecture that the Noether bound is still valid, but this has been proved only quite recently.

In my talk I will give an account on recent results, current developments and conjectures in modular invariant theory of finite groups. This includes new interesting connections between the depth of A^G, the cohomology of G and certain constructible 'trace ideals' of A^G and their varieties in the quotient space.

Beth Holmes

Computing in the Monster

Recent constructions of the Monster as implicit 196882×196882 matrices over $GF(2)$ and $GF(3)$ have led to effective computational capabilities and opened up the possibility of solving several previously intractable problems concerning this group. For example, we have shown that the Monster is a $(2,3,7)$-group, and that it contains a (previously unknown) maximal subgroup $L_2(29):2$, as well as making further inroads into the maximal subgroup problem.

Alexander Kleshchev

Irreducible restrictions of linear and projective representations of symmetric and alternating groups

We will report on the problem of describing the pairs (G, D) where G is a subgroup of S_n (resp. A_n) and D is a representation of S_n (resp. A_n), which is irreducible upon restriction to G. This is of importance for the problem on maximal subgroups in finite classical groups.

Ross Lawther

Elements of specified order in simple algebraic groups

Let Φ be an irreducible root system, and r be a natural number. If G is a simple algebraic group with root system Φ, the variety of elements $g \in G$ satisfying $g^r = 1$ is considered. A lower bound for the codimension in G of this variety is obtained, which depends only on Φ and r (and in particular is independent of characteristic), and is attained if G is of adjoint type.

Caiheng Li

The finite primitive permutation groups containing abelian regular subgroups

In this talk, some results regarding permutation groups containing regular subgroups and 2-arc-transitive Cayley graphs will be reported:

A complete classification is given of finite primitive permutation groups which contain abelian regular subgroups, solving a problem initiated by Burnside (1911).

This classification is then applied to give a classification of 2-arc-transitive Cayley graphs of abelian groups, which was a recent open problem in graph theory.

Alex Lubotzky

Groups which are expanders and non expanders

In response to a question asked a decade ago by Weiss and the speaker, we present an infinite family of finite groups whose Cayley graphs are expanders w.r.t. to one chice of (bounded number of) generators and are not w.r.t. another such chice. (Joint work with Avi Wigderson).

Kay Magaard

Imprimitve representations of quasi-simple groups

Let G be a quasi-simple finite group, K an algebraically closed field and M an irreducible KG-module. We say that M is imprimitive with block stabilizer $H \subset G$ if there exists some KH-module M_0 such that $M = Ind_H^G(M_0)$. If no such H exists we call M a primitive KG-module. Seitz proved that if G is of Lie type of characteristic p and if $char(k) = p$, then with four exceptions every irreducible KG-module is primitive. Djorkovic and Malzan proved a similar result for characteristic zero modules of alternating and symmetric groups. I will present joint work with Gerhard Hiss and William Husen that shows that the situation is very different when G is of Lie type of characteristic p and if $char(K) \neq p$. In fact in our case most irreducible KG-modules are imprimitive. I will also discuss how our results fit into the program of classifying maximal subgroups of classical groups.

Gunter Malle

$2F$-modules

I report on joint work in progress with R.M.Guralnick on the classification of $2F$-modules for quasi-simple groups and their automorphism groups. These modules play a crucial role in the recent second revision of parts of the classification by Meier-Frankenfeld Stellmacher, Stroth et al.

Igor Pak

The product replacement algorithm

I will survey recent progress on the product replacement algorithm. The algorithm is designed to generate random group elements. The results cover connectivity, polynomial mixing and the universality theorem.

Antonio Pasini

Embeddings and expansions

Let $\mathcal{G}$ be a geometry belonging to a string diagram of rank $n > 1$ where the nodes of the diagram are labelled by the integers $1, 2, ..., n$ from left to right, as usual. So, $\mathcal{G}$ can also be regarded as a poset, where we write $x < y$ for two elements x, y when x and y are incident and the type of x is less then the type of y. Denoted the set of 1-elements of $\mathcal{G}$ by P, I will write $P(x)$ for the set of 1-elements incident to a given element x. I also write $x \in \mathcal{G}$ to say that x is an element of $\mathcal{G}$. Having stated these notations, we can define embeddings.

An *embedding* of $\mathcal{G}$ in a group G is an injective mapping e from the poset $\mathcal{G}$ to the poset of non-trivial proper subgroups of G such that:

(1) $x \leq y$ iff $e(x) \leq e(y)$, for any two $x, y \in \mathcal{G}$;

(2) $e(x) = \langle e(p) \rangle_{p \in P(x)}$ for any $x \in \mathcal{G}$;

(3) $G = \langle e(p) \rangle_{p \in P}$.

We denote by $A(e)$ the amalgam $e(x)_{x \in \mathcal{G}}$ and we define the *expansion* $E(e)$ of $\mathcal{G}$ to G via e as the geometry over the set of types $\{0, 1, ..., n\}$ having the elements of G as 0-elements and the right cosets of the subgroups $e(x)$ ($x \in \mathcal{G}$ of type i) as i-elements, the incidence relation being defined via inclusion.

Clearly, denoted by $G(e)$ the universal completion of the amalgam $A(e)$, an embedding $\tilde{e}$ is naturally given in $G(e)$. We call it the *universal hull* of e. The following is the main result of my talk:

Theorem 1. The expansion $E(\tilde{e})$ is the universal cover of $E(e)$.

Morphisms of embeddings can also be defined: if $e_1 : \mathcal{G} \to G_1$ is another embedding of $\mathcal{G}$, a morphism $f : e_1 \to e$ is a homorphism $f : G_1 \to G$ such that, for every $x \in \mathcal{G}$, f induces on $e_1(x)$ an isomorphism to $e(x)$. (Note that f_1 induces a covering from $E(e_1)$ to $E(e)$.) In particular, with $G(e)$ and $\tilde{e}$ as above, the natural projection $\pi : G(e) \to G$ is a morphism from $\tilde{e}$ to e.

Proposition 2. The pair $(\tilde{e}, \pi)$ is characterized (up to isomorphism) by the following 'universal' property: for every morphism of embeddings $f : e_1 \to e$, there exists a morphism $g : \tilde{e} \to e_1$ such that $fg = \pi$.

Finally, I will turn to projective embeddings (where $\mathcal{G}$ has rank 2, G is the additive group of a vector space V and the subgroups corresponding to points and lines are 1- and 2-dimensional subspaces of V), revisiting some known results and constructions on projective embeddings in the light of the above general framework.

Cheryl Praeger

Overgroups of finite quasiprimitive permutation groups

A finite transitive permutation group is quasiprimitive if each if its non-trivial normal subgroups is transitive. Such groups often arise as automorphism groups of combinatorial or geometric structures, and a natural problem is to determine the full automorphism group. The group theoretic equivalent is the problem of determining the overgroups of a given quasiprimitive permutation group in the symmetric group. The heart of this problem is to find all quasiprimitive overgroups. I will discuss the extent to which this can be done in terms of the 'O'Nan Scott type' of a quasiprimitive group.

Alan Prince

A search for a projective plane of order 20

It is an open problem whether or not the order of a finite projective plane must be a prime-power. It was suggested by R. H. Bruck about 40 years ago that it might be possible to construct a projective plane of order $q(q+1)$, for some value of q, by extending the point-line geometry of PG(3,q). For certain values of q, such as $q = 2$, the construction is ruled out by the Bruck-Ryser theorem. Of the remaining values, the construction has been shown to be impossible only for $q = 3$. In this talk, I shall discuss a search for a projective plane of order 20, which extends the point-line geometry of PG(3,4).

Colva Roney-Dougal

Affine Groups with Two Self-Paired Orbitals

In this talk we define the *self-paired rank* of a group to be the number of nondiagonal self-paired orbitals. We give a number of reduction theorems for groups with self-paired rank 1, before constructing some infinite families of soluble affine groups with this property. We finish with an insoluble affine example.

Peter Rowley

A Monster graph

This talk will be about the structure of the point-line collinearity graph of the maximal 2-local geometry of the Monster simple group.

Yoav Segev

Finite quotients of the multiplicative group of a finite dimensional division algebra are solvable

We prove the theorem of the title, using techniques developed recently to obtain "congruence subgroup type" theorems over arbitrary fields; and using a property of the commuting graph of minimal nonsolvable groups (i.e., nonsolvable groups all of whose proper quotients are solvable) which is stronger than having diameter ≥ 3 and weaker than having diameter ≥ 4 (these graphs have diameter 3 and a half "so to speak"). The theorem of the title is closely related to the Margulis-Platonov conjecture for anisotropic algebraic groups of type A_n. This work is joint with A.S. Rapinchuk and G.M. Seitz.

Johannes Siemons

Representations from simplicial homology

Standard homology, defined via the usual boundary map $\beta : [x_1, x_2, .., x_n] \mapsto \sum (-1)^i \, [x_1, .., \hat{x_i}, .., x_n]$, is fundamental for understanding the topology of simplicial complexes. However, there are other kinds of homology, and some yield a rich representations theory for the groups associated to such complexes. In the lecture I shall describe the homology defined by the map

$$\partial : \{x_1, x_2, .., x_n\} \mapsto \sum_i \, \{x_1, .., \hat{x_i}, .., x_n\}$$

and discuss some of its properties in relation to standard homology.

The first observation is that $\partial^p = 0$ if $p = 0$ as an element of the coefficient domain. In particular, if coefficients are taken from a field of characteristic $p > 0$ then homology modules may be defined as quotients of $\mathrm{Ker}\, \partial^i$ over $\mathrm{Im}\, \partial^{p-i}$ with regards to modules belonging to a suitably arranged sequence. Already the ordinary n-simplex shows interesting features: Its homology modules are trivial in all but one position of every such sequence. The non-trivial homology yields irreducible $\mathrm{Sym}(n)$-modules and these form the simplest inductive systems of modular representations for the symmetric groups. In addition, standard techniques from algebraic topology such as the Hopf-Lefschetz trace theorem give ready character formulae and we shall discuss applications to orbits of permutation groups whose order is co-prime to p.

Having vanishing homology in all but one position is an important property. For instance, the order complex of every Cohen-Macauley poset has this property and there the non-trivial homology is commonly regarded as the Steinberg representation of the poset. In general the modular homology of such order complexes turns out to be much richer. Nevertheless, certain constructions carry over from ordinary homology and so we have a reasonably detailed description for the modular homology of all simplicial complexes which can be built up inductively from spheres. These include Coxeter complexes and finite buidings. For more details please go to my homepage http://www.mth.uea.ac.uk/~h260/ .

Steve Smith

Final steps in the classification of quasithin groups

Aschbacher and Smith are revising the final two chapters of their work on quasithin groups. The talk will indicate some of the ideas used in the "endgame" cases of the work, such as a new version of large-extraspecial theory, and some ideas related to N-group type configurations.

Leonard Soicher

Computing cliques in G-graphs, with applications to finite geometry

A *G-graph* is a triple (Γ, G, ψ), where Γ is a graph, G is a group, and ψ is a homomorphism from G into $\mathrm{Aut}(\Gamma)$. Many classification problems in finite geometry reduce to the problem of classifying certain G-orbits of cliques in a given G-graph. I will talk about some advances in search algorithms for computing representatives of the G-orbits of cliques (possibly satisfying certain conditions) in a G-graph, as well as some applications of these algorithms to problems in finite geometry. The first problem is the classification of maximal partial spreads P of lines in a given projective space $PG(n, q)$, such that P is invariant under a given subgroup of $PGL(n + 1, q)$. The second problem is the discovery and classification of certain SOMA(k,n)s, which are a class of combinatorial designs generalizing the concept of k mutually orthogonal Latin squares of order n.

Irina D. Suprunenko

Properties of unipotent elements in modular irreducible representations of the classical algebraic groups

Some properties of unipotent elements in modular irreducible representations of the classical algebraic groups specific for positive characteristic are discussed. The bulk of the talk will be concerned with computing the minimal polynomials of the images of unipotent elements of nonprime order in irreducible representations of the classical algebraic groups in odd characteristic. Actually, for a fixed such element of order p^s and a fixed representation of a classical algebraic group in characteristic $p > 2$ the problem is reduced to computing the minimal polynomials of s certain unipotent elements in a certain irreducible representation of a certain semisimple algebraic group in characteristic 0 with classical simple components. For the latter question explicit algorithms are available.

Some other problems on the behavior of unipotent elements in representations can be considered as well (this depends upon the length of my talk). These problems are connected with the number of Jordan blocks of the maximal size in the image of a unipotent element in an irreducible representation with highest weight large enough with respect to the (positive) characteristic and with recognizing the classical algebraic groups with the help of matrices with big Jordan blocks with respect to the degree of a group.

Balas Szegedy

On the characters of the Borel and Sylow subgroups of classical groups

M. Isaacs has proved that every irreducible complex character of the group of all upper unitriangular $GF(q)$ matrices has q-power degree. The analogous question for the Sylow subgroups of all classical groups will be discussed. These results have also important consequences for the Borel subgroups.

Paul Terwilliger

Leonard pairs associated with the classical geometries

Let V denote a complex vector space with finite positive dimension, and let $A : V \to V$ and $B : V \to V$ denote linear transformations. The pair A, B is said to be a *Leonard pair* whenever both conditions (i), (ii) hold below:

1. There exists a basis for V with respect to which the matrix representing A is diagonal, and the matrix representing B is irreducible tridiagonal.
2. There exists a basis for V with respect to which the matrix representing B is diagonal, and the matrix representing A is irreducible tridiagonal.

It is known that Leonard pairs are closely related to the q-Racah polynomials and related polynomials from the Askey Scheme. In this talk, we show how Leonard pairs are naturally related to the following classical geometries: The subset lattice, the subspace lattice, the Hamming semi-lattice, the attenuated spaces, and the classical polar spaces.

Pham Huu Tiep

Cross-characteristic representations of finite groups of Lie type

Let G be a finite group of Lie type defined in characteristic p. Then the degree of any nontrivial irreducible (projective) representation of G in characteristic $\neq p$ is bounded below by the Landazuri-Seitz-Zalesskii bound, $\ell_{LSZ}(G)$. In a number of applications it is important to know all the cross-characteristic representations of G of degree less than $(\ell_{LSZ}(G))^{2-\epsilon}$. This problem has been solved for $G = SL_n(q)$ by R. M. Guralnick and P. H. Tiep, and for $G = SU_n(q)$ by G. Hiss and G. Malle. We solve the problem for the case $G = Sp_{2n}(q)$ with q odd. We also improve the results of Hiss and Malle for the case $G = SU_n(q)$. Applications of our results to the minimal polynomial problem, the quadratic pair problem and to submodule structure of various rank 3 permutation modules will also be given. This talk is mostly based on joint work with R. M. Guralnick, K. Magaard and J. Saxl.

Franz Timmesfeld

Classifications of Lie-type groups

Let $\mathcal{B}$ be an irreducible spherical building, $\mathcal{A}$ an apartment of $\mathcal{B}$. Then the subgroup of Aut($\mathcal{B}$) generated by a root-subgroup corresponding to roots in $\mathcal{A}$, will be called the group of Lie-type $\mathcal{B}$.

I will speak about general classifications of such groups of Lie-type $\mathcal{B}$. For example a classification of groups with a " local BN-pair ", which generalizes the notion of split BN-pair.

Vladimir I. Trofimov

Track method for amalgams with groups of Lie type

The track method of investigation of amalgams with groups of Lie type (including $PSL_n(q)$) is set forth. A brief survey of results obtained by the method is also given, and perspectives are discussed.

Robert A. Wilson

Black box algorithms for recognising simple groups

Algorithms for black box groups have been developed over a number of years to the point where Monte Carlo polynomial time algorithms exist for many problems, in particular for recognising a group if it is know to be simple. The problem of recognising a simple group if it is not known in advance to be simple is much harder, the main stumbling block being to distinguich between a group of Lie type in characteristic p, and the same group acting on a module in defining characteristic. We solve this problem for p odd.