

**LMS Durham Symposium:  
Representations of Finite Groups and Related Algebras  
1-11 July 2002  
Schedule of talks**

There are brief abstracts at the back for talks marked [*Abstract*].

**Monday 1 July**

*Arrival*

**Tuesday 2 July**

10.00

*Opening announcements*

10.15-11.15

Dave Benson

**Modules with injective cohomology** [*Abstract*]

11.15

*coffee*

11.45-12.45

Raphaël Rouquier

**A survey of splendid equivalences (1/3)**

12.45

*lunch*

3.30

*tea*

4.00-5.00

Kai Meng Tan

**On Rouquier blocks of symmetric groups and Schur algebras** [*Abstract*]

5.15-6.15

Gerhard Hiss

**Some news on Donovan's conjecture**

6.30

*sherry reception*

7.00

*dinner*

## Wednesday 3 July

9.15-10.15

Susumu Ariki

**LLT and representation type of Hecke algebras (1/2)**

10.20-11.20

Gabriel Navarro

**More on the McKay conjecture**

11.20

*coffee*

11.45-12.45

Raphaël Rouquier

**A survey of splendid equivalences (2/3)**

12.45

*lunch*

3.30

*tea*

4.00-4.45

Katsuhiko Uno

**Modifications and applications of reduction theorems for conjectures on character degrees [Abstract]**

4.50-5.20

Alexander Zimmermann

**From tilting complexes to unstable modules**

5.30-6.30

Markus Linckelmann

*title to be announced*

7.00

*dinner*

## Thursday 4 July

9.15-10.15

Susumu Ariki

**LLT and representation type of Hecke algebras (2/2)**

10.20-11.20

Jon Carlson

**My last talk on the classification of endotrivial modules (I hope)**

11.20

*coffee*

11.45-12.45

Raphaël Rouquier

**A survey of splendid equivalences (3/3)**

12.45

*lunch*

3.30

*tea*

4.00-4.45

Leonard Scott

**Some empirical and empirically-inspired results in Lie-type modular representation theory [Abstract]**

4.50-5.20

Thorsten Holm

**Auslander's representation dimension [Abstract]**

5.30-6.30

Jørn Olsson

**Generalized blocks for symmetric groups**

7.00

*dinner*

## Friday 5 July

9.15-10.15

Alexander Kleshchev

**Representations of symmetric groups and Kac-Moody algebras (1/2)**

10.20-11.20

Lluís Puig

**Frobenius systems**

11.20

*coffee*

11.45-12.45

Ian Grojnowski

*title to be announced (1/2)*

12.45

*lunch*

Afternoon

*Cathedral visit/photograph*

7.00

*dinner*

## Saturday 6 July

9.15-10.15

Alexander Kleshchev

**Representations of symmetric groups and Kac-Moody algebras (2/2)**

10.20-11.20

Everett Dade

**Reduction theorems for conjectures of Uno and of Navarro [Abstract]**

11.20

*coffee*

11.45-12.45

Ian Grojnowski

*title to be announced (2/2)*

12.45

*lunch*

3.30

*tea*

4.00-4.45

Nicole Snashall

**Support varieties and Hochschild cohomology rings [Abstract]**

4.50-5.20

Naoko Kunugi

**An observation on lifting equivalences**

5.30-6.30

Christine Bessenrodt

**Cartan Matrices for symmetric groups**

7.00

*dinner*

## Sunday 7 July

*Excursion (Whitby and Robin Hood's Bay)*

## Monday 8 July

9.15-10.15

Gunter Malle

**Springer correspondence for disconnected groups** [*Abstract*]

10.20-11.20

Meinolf Geck

**On the decomposition numbers of Hecke algebras**

11.20

*coffee*

11.45-12.45

Gordon James

**Representations of general linear groups**

12.45

*lunch*

3.30

*tea*

4.30-6.30

Graduate students' lectures to be announced (so far we have had offers of talks from:

Frank Himstedt

**On the decomposition numbers of Steinberg's triality groups** [*Abstract*]

Max Neunhöffer

**Idempotents in Symmetric Algebras** [*Abstract*]

Robert Hartmann

**Endo-monomial modules** [*Abstract*]

7.00

*dinner*

## Tuesday 9 July

9.15-10.15

Andrew Mathas

**Tilting modules for cyclotomic Hecke algebras**

10.20-11.20

Joe Chuang

**Equivalences of derived categories for blocks of symmetric groups**

11.20

*coffee*

11.45-12.45

Robert Boltje

**Alperin's weight conjecture and chain complexes**

12.45

*lunch*

3.30

*tea*

4.00-5.00

Shigeo Koshitani

**On Broué's abelian defect group conjecture** [*Abstract*]

5.15-6.15

Will Turner

**RoCK blocks of symmetric groups of nonabelian defect** [*Abstract*]

6.30

*reception*

7.00

*Conference Dinner*

## Wednesday 10 July

9.25-9.55

Hyohe Miyachi

**Uno's conjecture on 1-parameter Iwahori-Hecke algebras** [*Abstract*]

10.05-10.35

Cedric Bonnafé;

**Around the Mackey formula for Lusztig functors** [*Abstract*]

10.45-11.15

Christakis Pallikaros

**Kazhdan-Lusztig cells and parabolic elements in finite Coxeter groups**  
[*Abstract*]

11.15

*coffee*

11.45

Steve Donkin

**The higher decomposition numbers for  $SL(3)$**  [*Abstract*]

12.45

*lunch*

3.30

*tea*

4.00-5.00

Jon Alperin

**Counting conjectures, past, present, future: an overview**

5.15-6.15

Michel Broué

*title to be announced*

7.00

*dinner*

## Thursday 11 July

*Departure.*

## Abstracts

Dave Benson

### **Modules with injective cohomology**

This talk will describe joint work with Henning Krause. We classify the injective modules over the Tate cohomology ring of a finite group, and we show how to construct  $kG$ -modules whose Tate cohomology is equal to a given injective. The construction produces pure injective modules which are (almost) characterized by their cohomology. We conjecture that these are (up to an explicitly described degree shift) the infinite dimensional kappa modules of Benson, Carlson and Rickard. The conjecture is true for groups whose cohomology is Cohen-Macaulay, but is still open in general. This is related to a conjectural way of calculating the variety of an infinite dimensional module by forming an injective resolution of its Tate cohomology.

Cedric Bonnafé;

### **Around the Mackey formula for Lusztig functors**

The Mackey formula for Lusztig induction and restriction (for finite reductive groups) is analogous to the Mackey formula for usual induction and restriction (for abstract finite groups). But it is not proved in full generality. We explain in this talk our recent progress in this direction : one of our main result is that the Mackey formula holds for classical groups.

In this talk, we also explain a related result in modular representation theory of finite reductive groups : with R. Rouquier, we proved that some Morita equivalence (“Jordan decomposition”) given by the Lusztig functor associated to a Borel subgroup does not depend on the choice of the Borel subgroup.

Everett Dade

### **Reduction theorems for conjectures of Uno and of Navarro**

A strengthened form of Uno’s new conjecture can be reduced by standard methods to the case of decorated simple groups. But Navarro’s conjecture requires some new methods and structures for a similar reduction. I’ll try to explain exactly what must be verified for each simple group in order to prove these conjectures by induction.

Steve Donkin

### **The higher decomposition numbers for $SL(3)$**

For each pair of integers  $\lambda$  and  $i = 0, 1, 2, 3$  there is a naturally occurring module  $H^i(\lambda)$  for the algebraic group  $SL(3)$ , with coefficients in an algebraically closed field of positive characteristic  $p$ . (The cohomology group of the line bundle  $L_\lambda$  on the flag variety  $G/B$ .) The modules  $H^0(\lambda)$  are the duals of the Weyl modules. The irreducible modules for the algebraic group  $SL(3)$  are well known and are indexed by pairs  $\mu$  of nonnegative integers. The composition multiplicities of the Weyl modules are also well known. We here describe the composition multiplicities  $[H^i(\lambda) : L(\mu)]$ , for all  $i, \lambda, \mu$ , in terms of the base  $p$ -expansion of  $\lambda$ . The proof involves some of the partial tilting modules for  $SL(3)$ .

Robert Hartmann

### **Endo-monomial modules**

For a complete discrete valuation ring  $\mathcal{O}$  and a  $p$ -group  $P$ , we generalize the concept of endo-permutation modules by considering  $\mathcal{O}P$ -modules whose endomorphism algebra forms a monomial  $\mathcal{O}P$ -module. We introduce the analog of the Dade group and show that for abelian  $P$  every indecomposable endo-monomial  $\mathcal{O}P$ -module with vertex  $P$  is already an endo-permutation module.

Frank Himstedt

### **On the decomposition numbers of Steinberg's triality groups**

In 1991, M. Geck has determined the decomposition numbers of Steinberg's triality groups  ${}^3D_4(q)$ ,  $q$  odd, in characteristics not dividing  $q$ , leaving some ambiguities in the decomposition numbers of the unipotent characters.

In this talk, new results are presented about one of these ambiguities, which have been obtained using techniques similar to those introduced by T. Okuyama and K. Waki in their determination of the decomposition numbers of the symplectic groups  $Sp(4, q)$  in 1996 and 1998.

First, using computers, the character table of a maximal parabolic subgroup of  ${}^3D_4(q)$  has been calculated. Then, an analysis of restrictions of suitable modules from  ${}^3D_4(q)$  to this subgroup using Green correspondence has given new information on one of the ambiguities in the decomposition numbers of the unipotent characters.

Thorsten Holm

### **Auslander's representation dimension**

(joint work with K. Erdmann and J. Schröer)

M. Auslander introduced the representation dimension of an algebra around 1970 as a possible way of measuring how far an algebra is from being of finite representation type. In fact, he proved that being representation finite is equivalent to having representation dimension 2. Apart from this result the notion remained mysterious and only very recently new progress emerged. Firstly, O. Iyama showed that the representation dimension is always finite. This is obtained by showing that there exists a certain module having quasi-hereditary endomorphism ring. Secondly, C. Xi proved that for selfinjective algebras the representation dimension is invariant under derived equivalence.

In the talk we report on recent progress on the representation dimension for group algebras and related algebras. Two main results will be discussed:

- 1) All special biserial algebras (e.g. blocks of finite groups with dihedral defect group) have representation dimension 3.
- 2) All blocks of finite groups with tame representation type (and more generally, all algebras of dihedral, semidihedral and quaternion type) have representation dimension 3.

Shigeo Koshitani

### **On Broué's abelian defect group conjecture**

There is a well-known and important conjecture called “Broué's abelian defect group conjecture”. By this we here mean his conjecture on splendid Rickard equivalences. We are going to discuss by taking some examples of non-principal blocks. Hopefully, some of them should be new.

Gunter Malle

### **Springer correspondence for disconnected groups**

The Springer correspondence is an injective map from the set of unipotent classes of an algebraic group to the set of characters of its Weyl group. It plays a crucial role in Lusztig's algorithm for the computation of character values of finite groups of Lie type. In her PhD thesis, Karin Sorlin has extended the Springer correspondence to the case of disconnected groups. In joint work we have now determined this correspondence in the case of disconnected classical groups. It is expected that this will eventually allow to compute character values for these disconnected groups.

Hyohe Miyachi

### **Uno's conjecture on 1-parameter Iwahori-Hecke algebras**

In the early 90's, K.Uno determined the representation type of the (1-parameter) Iwahori-Hecke algebras for the Coxeter groups with rank 2 and the finite Weyl groups of type  $A$ . Uno conjectured that a given Iwahori-Hecke algebra  $H$  is not semisimple and of finite type if and only if the parameter of  $H$  is a simple root of the Poincare polynomial for the Weyl group of  $H$ . Recently, by Ariki-Mathas (*math.RT/0106185*) and Ariki (*math.QA/0108176*) we can know all the representation types of classical (1-parameter) Hecke algebras. Motivated by these works, we determine the representation type of the (1-parameter) Iwahori-Hecke algebras for the (crystallographic) exceptional Weyl groups over a splitting field with a good characteristic.

Max Neunhöffer

### **Idempotents in symmetric algebras**

Let  $H$  be a non-semisimple symmetric algebra over a field  $K$ . In this talk explicit formulae for primitive idempotents of  $H$  are presented, which involve only the matrix coefficients of certain matrix representations corresponding to projective indecomposable modules.

Christakis Pallikaros

### **Kazhdan-Lusztig cells and parabolic elements in finite Coxeter groups**

The Hecke algebra  $H$  of a Coxeter group  $W$  has certain ideals associated with the longest elements of the parabolic subgroups of  $W$  which allow a straightforward analysis of the corresponding cell representations. These ideals have the form  $CwH$  where  $C$  refers to the  $C$ -basis introduced by Kazhdan and Lusztig. We show that the isomorphism classes of these ideals depend only on the Coxeter classes of the corresponding parabolic subgroups, as do the 2-sided cells containing their longest elements. In the special case of Hecke algebras of type  $A$ , we show that this correspondence is bijective.

Leonard Scott

**Some empirical and empirically-inspired results in Lie-type modular representation theory**

I will discuss a constructive, though recursive, construction of the radical quotient of the maximal submodule of a standard (Verma, Weyl, baby Verma, ...) module, in the presence of a valid Kazhdan-Lusztig or Lusztig conjecture. The result was found empirically, though verified theoretically, and I will discuss some of the related computer calculations. Some of these calculations led to verification of new small cases of the Lusztig conjecture in characteristic  $p$ , while others led to counterexamples to a conjecture of some standing on dimensions of 1-cohomology groups for finite groups with faithful absolutely irreducible coefficients.

Nicole Snashall

**Support varieties and Hochschild cohomology rings**

This is joint work with Ø. Solberg.

We define a support variety for a finitely generated module over any artin algebra  $\Lambda$  in terms of the maximal ideal spectrum of the Hochschild cohomology ring of  $\Lambda$ . This is modelled on what is done in modular representation theory, and the varieties defined in this way are shown to have many of the same properties as for group rings. Information is also given on nilpotent elements of  $HH^*(\Lambda)$  and I will discuss the question of whether the quotient algebra  $HH^*(\Lambda)$  modulo the ideal generated by homogeneous nilpotent elements is finitely generated as an algebra.

Kai Meng Tan

**On Rouquier blocks of symmetric groups and Schur algebras**

I will present explicit formulas for the radical layers of the principal indecomposable, Weyl, Young and Specht modules of these blocks, and address some conjectures of James, Martin, Lascoux-Leclerc-Thibon-Rouquier for these blocks.

Will Turner

**RoCK blocks of symmetric groups of nonabelian defect**

Rouquier has defined a family of symmetric group blocks which possess a very symmetric structure. Chuang and Kessar have proved, for example, that these blocks are Morita equivalent to principal blocks of wreath products  $S_p \text{wr} S_w$  of symmetric groups, so long as their defect group is abelian. I would like to describe some theory for “RoCK” blocks of nonabelian defect. For example, I shall state a formula for the decomposition matrices of these blocks, given in terms of Littlewood-Richardson coefficients and decomposition numbers of “small” Schur algebras. This generalises a formula obtained by Chuang and Tan in the abelian defect case.

Katsuhiko Uno

**Modifications and applications of reduction theorems for conjectures on character degrees**

Let  $G$  be a finite group and  $p$  a prime. We consider a conjecture on the alternating sum of the numbers of complex irreducible characters in certain blocks of chain normalizers having fixed defect and  $\pm$  fixed residue mod  $p$ . Here, the defect  $d$  of a character  $\chi$  of a subgroup  $H$  of  $G$  is defined by  $(|H|/\chi(1))_p = p^d$  and its residue is defined by  $(|H|/\chi(1))_{p'}$ . This conjecture is a refinement of Dade's, and moreover, its projective form implies the Isaacs-Navarro conjecture, just like that form of Dade's implies the Alperin-McKay. Dade proved a reduction theorem for the projective form of his conjecture, and it turns out that it is also valid for ours. We give some modifications of it and apply them in several cases. In particular, finite groups with small Sylow  $p$ -subgroups and sporadic simple groups will be treated.