Twistor Strings and Gravity

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- Amplitudes for YM, Gravity have elegant twistor space structure: Twistor Geometry

- N=4 SYM, N=8 SUGR: Structure of amplitudes suggest string theory Nair

- QCD String? QCD is stringy at large N

- Dual string for N=4 SYM:
  Strong coupling: IIB in AdS
  Weak coupling: String in twistor space?

SU(2,2/4) acts on targets: Twistor space, AdSxS

Penrose Transform

Space-Time M ⟷ **Twistor Space PT**

Field, helicity h ⟷ Cohomology class

Solution of massless field equn

$H^1(PT, \mathcal{O}(n))$

$\mathcal{O}(n)$ Homogeneous functions of weight n=2h-2

Topological String: Physical states are cohomology classes

Witten: Topological B String on SuperTwistor Space

N=4 SUSY: PT is Calabi-Yau

Twistor Strings

Witten: Topological B-model. Open and closed strings
Berkovits: Open strings
Witten; Mason, Skinner: Heterotic

All give same theory:

N=4 SYM + Conformal Supergravity

$$\frac{1}{g^2}\int F^2 + W^2$$

W= Weyl tensor

- Can't decouple YM, can't extract YM loop amplitudes as gravitons etc propagate in loops

- Anomaly in SU(4) R-symmetry: only cancels if YM gauge group has dimension D_{Gauge}=4 e.g. SU(2)xU(1). Don't yet know how this arises in twistor strings.

- Don't yet know how to calculate loops

New Twistor Strings

- Einstein supergravity (or chiral version) + SYM
- Has decoupling limit giving SYM
- World-sheet anomalies cancel
- Interesting spectra. Interactions?

$$\int \frac{1}{g^2} F^2 + \frac{1}{\kappa^2} R$$

3 Classes of twistor strings. Spectra same as those of following theories

1) **N=4 SYM + SUGRA**, any gauge group

2) N=4 SUGRA + four N=4 gravitino multiplets + D_{Gauge} vector multiplets.

N=8 SUGRA spectrum if D_{Gauge}=6

3) N=0 Self-dual gravity $R^- = 0$ + Self-dual YM $F^- = 0$ + scalar

4) Any N. $(\mathbf{1}, \mathrm{N}, \mathrm{N}(\mathrm{N}-\mathbf{1})/\mathbf{2}, ..., \mathrm{N}', \mathbf{1})$

SD graviton multiplet, helicities 2,3/2....,2-(N/2)

Scalar multiplet, helicities 0,-1/2,-1,....,-N/2

SD YM multiplet, helicities 1,1/2,....,1-(N/2)

Yang-Mills $F = dA + A \wedge A$ $G = *G$ 2-form

$$\int d^4x \mathrm{Tr}\left(G^{\mu\nu}F_{\mu\nu} - \frac{\epsilon}{2}G^{\mu\nu}G_{\mu\nu}\right)$$

Siegel, Chalmers

Eliminate auxiliary 2-form

$$\frac{1}{2\epsilon}\int d^4x \mathrm{Tr}\left(F^{(+)}_{\mu\nu}F^{(+)\mu\nu}\right)$$

$$F^{(\pm)} = \frac{1}{2}(F \pm *F)$$

Yang-Mills with Chiral Interactions $\epsilon \to 0$

$$\int \mathrm{Tr}\,(G \wedge F) = \int \mathrm{Tr}\,(G \wedge dA + G \wedge A \wedge A)$$

Field Equations $F^{(+)} = 0$ $D^\mu G_{\mu\nu} = 0$

Helicities +1, -1. Chiral Interactions: $(+ + -)$ not $(+ - -)$

Einstein Action, 1st Order Form

$$\int e^a \wedge e^b \wedge R^{cd}(\omega)\varepsilon_{abcd} = \int e^a \wedge e^b \wedge R^{(+)}_{ab}(\omega) + d(T^a \wedge e_a)$$

$$R^{(\pm)}_{bc} \equiv \frac{1}{2}\left(R_{bc} \pm \frac{1}{2}\varepsilon_{bc}{}^{de}R_{de}\right)$$

Field Equns $\quad \omega_{\mu ab} = \Omega_{\mu ab}(e) \qquad Ricci = 0$

$$\Omega_\mu{}^{ab}(e) \equiv e^{\nu a}\partial_{[\mu}e_{\nu]}{}^b - e^{\nu b}\partial_{[\mu}e_{\nu]}{}^a - e^{\rho a}e^{\sigma b}\partial_{[\rho}e_{\sigma]c}e_\mu{}^c$$

Write in terms of SD connection $\qquad \omega \equiv \omega^{(+)}$

$$\int e^a \wedge e^b \wedge \left(d\omega_{ab} + \kappa^2\omega_{ac} \wedge \omega^c{}_b\right)$$

Chiral Limit: $\qquad \kappa \to 0$

Chiral Gravity

Siegel
Abou Zeid, Hull

$$\int e^a \wedge e^b \wedge d\omega_{ab} = -\int e^a \wedge e^b \wedge \omega_{ac} \wedge \Omega^{(+)c}{}_b(e)$$

$\omega_{\mu ab}$ Lagrange multiplier:

$$\Omega^{(+)a}{}_b(e) = 0 \longrightarrow R_{\mu\nu}{}^{ab}(\Omega^{(+)}) = 0$$

Vierbein e: Anti-SD curvature, helicity +2

$e_b \wedge d\omega^{ab} = 0$ Lin. Einstein, helicity -2

Helicities +2, -2. Chiral Interactions: $(++-)$ not $(+--)$

Twistor Strings

Naively give chiral YM $\int \mathrm{Tr}\,(G \wedge F)$

Instanton corrections give GG term:

$$\int \mathrm{Tr}\left(G \wedge F - \frac{\epsilon}{2} G \wedge G\right)$$

New Twistor Strings

N=4, N=8 have right 3-graviton interaction $(+ + -)$

Non-trivial ineraction! Chiral or full Einstein?

Need to find $(+ - -)$

If vanishes, chiral gravity. Still good for N=4 SYM!

4) Any N.

SD graviton multiplet, helicities 2,3/2....,2-(N/2)

Scalar multiplet, helicities 0,-1/2,-1,....,-N/2

SD YM multiplet, helicities 1,1/2,....,1-(N/2)

Interactions, higher spin?

N<4: Self-dual supergravity and super Yang-Mills

N=4: Spectrum of N=4 supergravity and super YM

N>4: helicities < - 2

Helicity -n/2 $\Phi_{A'_1 A'_2 \dots A'_n}$

$\nabla^{BA'_1}\Phi_{A'_1 A'_2 \dots A'_n} = 0$ Consistent for ASD gravity, YM

$$F_{A'B'} = 0, \quad W_{A'B'C'D'} = 0$$

Higher spin fields linear, but gravity, YM can be chiral

Twistor space T $Z^\alpha = (\omega^A, \pi_{A'})$ C^4

Projective Twistor space PT $Z \sim \lambda Z$ CP^3

$$\omega^A = x^{AA'} \pi_{A'}$$

Super Twistor space $Z^I = (Z^\alpha, \psi^a)$ $C^{4|N}$

$a = 1, ..., N$ ψ^a anti-commuting variables

Projective Super Twistor space $CP^{3|N}$

Superconformal group SU(2,2/N) acts linearly on Z

Signature 2+2: Superconformal group SL(4,R/N)
Can take Z real, real twistor spaces $R^{4|N}$ $RP^{3|N}$

Penrose-Ferber Twistor Particle

Particle moving in super twistor space $Z^I(\tau)$

$$S = \int d\tau \left(Y_I \partial_\tau Z^I - A(J - n) \right)$$

Y_I momentum conjugate to $Z^I(\tau)$

$J = Y_I Z^I$ n constant, A gauge field

Gauge symmetry $Z \to \lambda Z,\ Y \to \lambda^{-1} Y$

Gauging: theory on projective space PT

$$Y_I \sim \frac{\partial}{\partial Z^I} \qquad J \sim Z^I \frac{\partial}{\partial Z^I}$$

Constraint J-n =0 imposed on wave-functions $\Phi(Z)$

$$J \sim Z^I \frac{\partial}{\partial Z^I} \qquad \left(Z^I \frac{\partial}{\partial Z^I} - n \right) \Phi = 0$$

$\Phi(Z)$ Homogeneous weight n

$$\Phi(Z^\alpha, \psi^a) = \phi(Z^\alpha) + \psi^a \phi_a(Z^\alpha) + \psi^a \psi^b \phi_{ab}(Z^\alpha) + \dots$$

Weight w=n
Helicity h=n/2+1

w=n-1
Helicity h-1/2

w=n-2
Helicity h-1

Supermultiplet, superspin n

Berkovits Twistor String

$$S_0 = \int d^2\sigma \left(Y_I \tilde{\partial} Z^I - \tilde{A} J \right) \qquad \tilde{S}_0 = \int d^2\sigma \left(\tilde{Y}_J \partial \tilde{Z}^J - A \tilde{J} \right)$$

$$S = S_0 + \tilde{S}_0 + S_C$$

$$J = Y_I Z^I, \qquad \tilde{J} = \tilde{Y}_I \tilde{Z}^I$$

$$Z^I = (\omega^A, \pi_{A'}, \psi^a), \qquad \tilde{Z}^I = (\tilde{\omega}^A, \tilde{\pi}_{A'}, \tilde{\psi}^a)$$

Independent real coordinates $RP^{3|N} \times RP^{3|N}$

Real twistors for signature 2+2

$Y, \tilde{Y}$ conjugate momenta $A, \tilde{A}$ Gauge fields

World-sheet: real coordinates $\sigma, \tilde{\sigma}$

Lorentzian metric $ds^2 = 2 d\sigma d\tilde{\sigma}$

Anomaly cancellation $Q^2 = 0$

Kac-Moody $J, \tilde{J}$ N=4 (or N=D)

Virasoro $-26 - 2 + c_C = 0$

Extra CFT S_C has central charge $c_C = 28$

Open Strings

$Y = \tilde{Y}, \quad Z = \tilde{Z}$ on boundary

Extra CFT S_C assumed to have Kac-Moody currents $j^r, \ \tilde{j}^r$ for some group G

Vertex Operators

On open string boundary, primary wrt J,T

$$V_\phi = j_r \phi^r(Z)$$

ϕ^r weight 0, in lie algebra of G

Helicities $(1, \frac{1}{2}, 0, -\frac{1}{2}, -1)$ SYM multiplet for G

$$V_f = Y_I f^I(Z)$$

f^I Weight 1.

$$V_g = g_I(Z) \partial Z^I$$

g_I Weight -1

Conformal Supergravity

$$V_f = Y_I f^I(Z) \quad V_g = g_I(Z)\partial Z^I$$

Subject to constraints $\partial_I f^I = 0, \quad Z^I g_I = 0.$

Gauge equivalences $\delta f^I = Z^I \Lambda, \quad \delta g_I = \partial_I \chi$

(Changes vertices by BRST trivial operators)

$f^A(Z),\ f^{A'}(Z)$ each give a graviton multiplet, helicities $(+2, +\frac{3}{2}, +1, +\frac{1}{2}, 0)$

f^a Four gravitino multiplets $(+\frac{3}{2}, +1, +\frac{1}{2}, 0, -\frac{1}{2})$

$g_I = (g_A, g_{A'}, g_a)$

2 graviton multiplets $(0, -\frac{1}{2}, -1, -\frac{3}{2}, -2)$

Four gravitino multiplets $(+\frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2})$

Non-Linear Graviton

Curved Twistor Spaces PT ⟷ Conformally anti-self-dual spaces

$W^+ = 0$

Deform Complex Structure ⟷ New C.A.S.D. spaces

An infinitesimal deformation: holomorphic vector field of weight 1 $f^I(Z)$ cf Berkovits vertex

In overlap $U_i \cap U_j$ transition fns $Z_i^I - Z_j^I = f_{ij}^I$

Restriction to Ricci Flat Space-Times

PT fibred over ⟷ Ricci-Flat ASD

Holomorphic 1-form on CP^1 pulls back to1-form on PT

$$k = I_{\alpha\beta} Z^\alpha \wedge dZ^\beta \sim \epsilon_{A'B'} \pi^{A'} d\pi^{B'}$$

Deformation preserves fibration if hamiltonian:

$$f^\alpha = I^{\alpha\beta} \partial_\beta h$$

h: weight 2

Poisson structure $I^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} I_{\gamma\delta}$

INFINITY TWISTOR

New Theories

- Restrict to twistor spaces fibred over $CP^{1|0}$ with 1-form k
- New Current $K \propto I_{IJ} Z^I \partial Z^J$
- Gauge: set K~0
- New constraints give $f^I = I^{IJ} \partial_J h$
- Conformal gravity → gravity

More New Theories

- Restrict to twistor spaces fibred over $CP^{1|N}$ with 1-forms k, k^a
- New Currents $K \propto I_{IJ} Z^I \partial Z^J \quad K^a$
- Gauge: set $K, K^a \sim 0$
- New constraints give $f^I = I^{IJ} \partial_J h$
- Conformal gravity ⟶ gravity

New String Theories

$$S = \int d^2\sigma \left(Y_I \tilde{\partial} Z^I + \tilde{Y}_J \partial \tilde{Z}^J - \tilde{A} J - A \tilde{J} - B_i \tilde{K}^i - \tilde{B}_i K^i \right) + S_C$$

Gauging currents $K^i = (K, K^a)$ or $K^i = K$

Ghosts:
T: b,c
J: u,v
K: r,s

BRST charges $Q, \tilde{Q}$

$$Q = \oint \left(cT + vJ + s_i K^i + cu\partial v + cb\partial c + cr^i \partial s_i - \sum_i v h_i s_i r^i \right)$$

Physical states primary wrt T,J,K

Seek theories without anomalies, $Q^2 = 0$

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2) N=4 SUGRA + four N=4 gravitino multiplets + D_{Gauge} vector multiplets.

N=8 SUGRA spectrum if D_{Gauge}=6

3) N=0 Self-dual gravity (R^- =0) + Self-dual YM (F^- =0) + scalar

4) Any N. $(\mathbf{1}, \mathrm{N}, \mathrm{N}(\mathrm{N}-\mathbf{1})/\mathbf{2}, ..., \mathrm{N}', \mathbf{1})$

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3) N=0 Self-dual gravity ($R^-=0$) + Self-dual YM ($F^-=0$) + scalar

Standard N=0 curved twistor space fibred over CP^1

$$K = I_{IJ} Z^I \partial Z^J$$

4) Any N. PT has dimension 3/N fibred over $CP^{1|N}$
SD graviton multiplet, Scalar multiplet, SD YM multiplet,

$$K^i = (K, K^a)$$

2) N=4 SUGRA + four N=4 gravitino multiplets + D_{Gauge} vector multiplets.

N=8 SUGRA spectrum if D_{Gauge}=6

N=4 PT has dimension 3/4 fibred over $CP^{1|0}$

$$K = w(Z) I_{IJ} Z^I \partial Z^J$$

w has weight -2 so K has weight zero

e.g.

$$w = \frac{1}{\delta^{A'B'} \pi_{A'} \pi_{B'}}$$

and dk=0

OK for real Z, but poles if continued to complex Z

Choice of w doesn't affect results

1) **N=4 SYM + SUGRA**, any gauge group

N=4 PT has dimension 3/4 fibred over $CP^{1|N}$

$$K^i = (K, K^a)$$

scaled by suitable functions to be weight zero

Conclusions

- New twistor strings: no world-sheet anomalies, spectra of interesting sugras
- N=4, N=8 have correct MHV 3-graviton interactions
- Chiral supergravity or full supergravity?
- Loops? Fixing no. of vector multiplets?
- Extract YM loop amplitudes for N=4?