Bayesian structural learning and estimation in Gaussian graphical models and hierarchical log-linear models

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Table 1: The models with the highest posterior probabilities identified by MOSS for the Rochdale data (Whittaker, 1990). We report the models whose normalized posterior probabilities are greater than 0.05. We also give the 2.5%, 50% and 97.5% quantiles of the number of models visited by MOSS before completion across the five search replicates.

Search	Top models		*Models evaluated*
Dec.	$efg\|beg\|bdh\|bdg\|adg\|acg$	0.436	1123|5608|6240
	$efg\|ceg\|bdh\|adg\|acg$	0.369	
	$efg\|ceg\|beg\|bdh\|bdg\|acg$	0.069	
	$efg\|bh\|beg\|bdg\|adg\|acg$	0.068	
	$efg\|ceg\|bh\|bd\|adg\|acg$	0.058	
	$efg\|beg\|bdh\|bdg\|adg\|acg$	med.	
Graph./PM	$fg\|ef\|be\|bdh\|bdg\|adg\|acg\|ace$	0.462	240|369|608
	$fg\|ef\|bh\|be\|bd\|adg\|acg\|ace$	0.337	
	$fg\|ef\|bh\|be\|bdg\|adg\|acg\|ace$	0.072	
	$fg\|ef\|ce\|be\|bdh\|bdg\|adg\|acg$	0.067	
	$fg\|ef\|ce\|bh\|be\|bd\|adg\|acg$	0.061	
	$fg\|ef\|be\|bdh\|bdg\|adg\|acg\|ace$	med.	
Graph./Lapl	$fg\|ef\|be\|bdh\|adg\|acg\|ace$	0.507	29|515|926
	$fg\|ef\|ce\|be\|bdh\|adg\|acg$	0.184	
	$efg\|ceg\|be\|bdh\|adg\|acg$	0.112	
	$fh\|fg\|ef\|be\|bdh\|adg\|acg\|ace$	0.087	
	$fg\|ef\|bg\|be\|bdh\|ad\|acg\|ace$	0.056	
	$fg\|ef\|be\|bdh\|bdg\|adg\|acg\|ace$	0.055	
	$fg\|ef\|be\|bdh\|adg\|acg\|ace$	med.	
Hierar.	$fg\|ef\|dg\|cg\|cf\|ce\|be\|bdh\|ag\|ae\|ad\|ac$	0.076	1391|1417|1617
	$fg\|ef\|dg\|cg\|ce\|be\|bdh\|ag\|ae\|ad\|ac$	0.069	
	$fg\|ef\|dg\|cf\|ce\|be\|bdh\|ae\|ad\|acg$	0.057	
	$fg\|ef\|dg\|ce\|be\|bdh\|ae\|ad\|acg$	0.052	
	$fg\|ef\|dg\|cg\|cf\|ce\|be\|bdh\|ag\|ae\|ad\|ac$	med.	

Table 2: Results for the eight simulated models of Yuan and Lin (2007). We simulate 25 samples of dimension $p = 5$ and $p = 10$ from eight different models: AR(1), AR(2), AR(3), AR(4), a full graph, a star graph with every vertex conected to the first vertex and a circle graph. For each model, sample size and search type, we estimate the precison matrix K using the highest posterior probability graph (B), the median graph (M) and Bayesian model averaging(A). We employed the posterior mean estimator and assessed its performance using the average Kullback-Leibler (KL) loss across the replicates. We also report the number of incorrectly identified edges (false positives, FP) and the number of incorrectly missed edges (false negatives, FN) with respect to the median graph. In addition, we give the number of graphs in $\mathcal{S}$ ($|\mathcal{S}|$) and the number of graphs whose marginal likelihood was evaluated until the completion of MOSS (E).The standard errors across the 100 replicates are shown in parantheses.

		Decomposable							Unrestricted						
p	Model	B	M	A	FP	FN	$\lvert\mathcal{S}\rvert$	E	B	M	A	FP	FN	$\lvert\mathcal{S}\rvert$	E
	1	0.16	0.16	0.14	0.33	0	10	236	0.16	0.16	0.14	0.33	0	10	255
		(0.02)	(0.01)	(0.01)	(0.06)	(0)	(1)	(7)	(0.02)	(0.01)	(0.01)	(0.06)	(0)	(1)	(17)
	2	0.29	0.29	0.28	0.25	0.03	6	107	0.29	0.29	0.29	0.25	0.03	9	212
		(0.02)	(0.02)	(0.02)	(0.06)	(0.02)	(0)	(3)	(0.02)	(0.02)	(0.02)	(0.05)	(0.02)	(0)	(8)
	3	0.54	0.54	0.54	0.1	2.22	12	174	0.54	0.54	0.54	0.16	2.28	14	298
		(0.02)	(0.02)	(0.02)	(0.03)	(0.13)	(1)	(9)	(0.02)	(0.02)	(0.02)	(0.04)	(0.13)	(1)	(15)
	4	0.55	0.55	0.53	0.15	5.69	17	263	0.55	0.54	0.53	0.21	5.57	19	409
		(0.02)	(0.02)	(0.02)	(0.04)	(0.12)	(1)	(16)	(0.02)	(0.02)	(0.02)	(0.04)	(0.11)	(1)	(22)
5	5	0.58	0.59	0.57	0	7.17	16	268	0.58	0.58	0.57	0	7.07	17	368
		(0.02)	(0.02)	(0.02)	(0)	(0.11)	(1)	(14)	(0.02)	(0.02)	(0.02)	(0)	(0.12)	(1)	(18)
	6	0.84	0.87	0.83	0	4.58	17	274	0.84	0.87	0.83	0	4.53	18	348
		(0.04)	(0.03)	(0.03)	(0)	(0.31)	(1)	(22)	(0.04)	(0.03)	(0.03)	(0)	(0.31)	(1)	(26)
	7	0.35	0.35	0.32	0.21	2.92	16	306	0.35	0.35	0.32	0.21	2.9	16	389
		(0.01)	(0.01)	(0.01)	(0.05)	(0.09)	(1)	(14)	(0.01)	(0.01)	(0.01)	(0.05)	(0.09)	(1)	(17)
	8	0.48	0.48	0.47	1.56	0.12	9	128	0.37	0.37	0.36	0.32	0.04	8	191
		(0.02)	(0.03)	(0.02)	(0.08)	(0.03)	(0)	(4)	(0.02)	(0.02)	(0.02)	(0.06)	(0.02)	(0)	(8)
	1	0.16	0.16	0.13	1.04	0	45	5284	0.16	0.16	0.13	1.04	0	45	4158
		(0.01)	(0.01)	(0.01)	(0.11)	(0)	(4)	(253)	(0.01)	(0.01)	(0.01)	(0.11)	(0)	(4)	(280)
	2	0.27	0.28	0.26	0.79	0	24	2076	0.3	0.29	0.27	0.79	0	72	6464
		(0.01)	(0.01)	(0.01)	(0.12)	(0)	(2)	(63)	(0.01)	(0.01)	(0.01)	(0.11)	(0)	(5)	(354)
	3	0.49	0.5	0.51	0.46	1.28	20	1763	0.55	0.56	0.55	0.97	1.58	52	4541
		(0.02)	(0.02)	(0.02)	(0.08)	(0.1)	(2)	(56)	(0.02)	(0.02)	(0.02)	(0.11)	(0.12)	(5)	(360)
	4	0.86	0.88	0.84	0.47	12.91	77	4237	0.86	0.86	0.84	0.85	12.88	188	13047
		(0.02)	(0.02)	(0.02)	(0.07)	(0.27)	(10)	(345)	(0.02)	(0.02)	(0.02)	(0.11)	(0.22)	(19)	(1027)
10	5	0.77	0.78	0.76	0.17	20.59	72	4357	0.77	0.76	0.76	0.35	20.1	145	10342
		(0.02)	(0.02)	(0.02)	(0.04)	(0.21)	(6)	(241)	(0.02)	(0.02)	(0.02)	(0.06)	(0.2)	(14)	(752)
	6	1.21	1.21	1.18	0	18.04	173	10302	1.71	1.7	1.64	0	39.9	399	24572
		(0.05)	(0.05)	(0.04)	(0)	(1.99)	(26)	(1402)	(0.01)	(0.01)	(0.01)	(0)	(0.2)	(30)	(1543)
	7	0.53	0.53	0.52	1.08	4.23	154	10517	0.53	0.53	0.52	1.14	4.23	167	11822
		(0.02)	(0.02)	(0.02)	(0.1)	(0.15)	(17)	(761)	(0.02)	(0.02)	(0.02)	(0.1)	(0.15)	(18)	(957)
	8	0.52	0.48	0.48	5.75	0	531	16056	0.41	0.39	0.37	2.07	0	420	28062
		(0.02)	(0.02)	(0.01)	(0.24)	(0)	(47)	(1063)	(0.02)	(0.02)	(0.02)	(0.2)	(0)	(47)	(2422)

Table 3: Growth data example: posterior inclusion probabilities ($p(i)$), regression coefficients estimates ($\hat{\beta}_i$) and the coefficient of determination (R^2) by search type. Standard deviations of the estimates are given in parantheses.

Variable	Regression		Decomposable-GGMs		GGMs	
	$p(i)$	$\hat{\beta}_i$	$p(i)$	$\hat{\beta}_i$	$p(i)$	$\hat{\beta}_i$
Life	1	7e-04	1	7e-04	1	7e-04
		(2e-04)		(2e-04)		(1e-04)
GDPsh560	1	-0.0135	1	-0.0177	1	-0.018
		(0.0021)		(0.0021)		(0.002)
EquipInv	1	0.1161	1	0.2022	1	0.1969
		(0.0365)		(0.0366)		(0.0367)
SubSahara	1	-0.0188	1	-0.0217	1	-0.0195
		(0.0044)		(0.0033)		(0.0031)
Confuncious	1	0.0644	1	0.0348	1	0.0351
		(0.0112)		(0.0095)		(0.0094)
RuleofLaw	1	0.0112	1	0.0159	1	0.0164
		(0.0038)		(0.0037)		(0.0038)
Mining	0.981	0.0356	0	0	0	0
		(0.0122)		(0)		(0)
Protestants	0.968	-0.0127	0	0	0	0
		(0.0047)		(0)		(0)
Hindu	0.943	-0.0572	0	0	0	0
		(0.022)		(0)		(0)
EcoOrg	0.936	0.002	1	8e-04	0.587	0.0013
		(9e-04)		(7e-04)		(5e-04)
NEquipInv	0.91	0.0425	0	0	0.002	0
		(0.0212)		(0)		(0)
LabForce	0.898	0.1856	0	0	0	0
		(0.0878)		(0)		(0)
BlMktPm	0.894	-0.007	0	0	0	0
		(0.0035)		(0)		(0)
HighEnroll	0.873	-0.0748	0	0	0	0
		(0.0381)		(0)		(0)
LatAmerica	0.74	-0.0063	1	-0.0069	1	-0.0062
		(0.0049)		(0.0022)		(0.002)
EthnoLFrac	0.722	0.0069	0	0	0	0
		(0.0055)		(0)		(0)
PublEdu	0.651	0.143	0	0	0	0
		(0.1269)		(0)		(0)
R^2	0.9006		0.7869		0.7924	
	(0.017)		(0.0129)		(0.0106)	