Non –Perturbative Field Theory

from 2d CFT to 4d QCD

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Book I

- Part I. Non-Perturbative Methods in Two Dimensional Field Theory:

- 1. From massless free scalar field to conformal field theories
- 2. Conformal field theory
- 3. Theories invariant under affine current algebras
- 4. Wess-Zumino-Witten model and Coset models
- 5. Solitons and two dimensional integrable models
- 6. Bosonization;
- 7. The large N limit of two dimensional models;

Book II

- Part II. Two Dimensional Non-Perturbative Gauge Dynamics:
- 8. Gauge theories in two dimensions - basics;
- 9. Bosonized gauge theories;
- 10. The t'Hooft solution of 2d QCD;
- 11. Mesonic spectrum from current algebra;
- 12. DLCQ and the spectra of QCD with fundamental and adjoint fermions;
- 13. The baryonic spectrum of multiflavour QCD2 in the strong coupling limit;
- 14. Confinement versus screening;
- 15. QCD2, Coset models and BRST quantization;
- 16. Generalized Yang Mills theory on Riemann surface;

Book III

- Part III. From Two to Four Dimensions:
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- 18. Integrability in four dimensional gauge dynamics;
- 19. Large N methods in QCD4;
- 20. From 2d bosonized baryons to 4d skyrmions;
- 21. From two dimensional solitons to four dimensional magnetic monopoles;
- 22. Instantons of QCD;
- 23. Summary, conclusions and outlook;

Two versus four dimensional field theories

- In two dimensions the underlying manifold is simpler.
- The number of degrees of freedom of each field is smaller.
- Certain symmetries associate with infinite dim algebras
- In one space dimension there is no rotation symmetry and no angular momentum.
- The light cone is disconnected and is composed of left moving and right moving branches. Therefore, massless particles are either on one branch or the other.
- On trivial topology pure gauge theory is empty.
- Also, the ultra-violet behavior is more convergent in two dimensions, making for instance QCD2 a super-convergent theory.

Outline

- Conformal symmetry
- Integrability
- Bosonization
- Weak strong duality
- Topological field configurations
- Confinement versus screening
- Hadronic spectra: mesons, baryons.
- Future outlook

Conformal invariance

- The conformal algebra is the infinite dimensional Virasoro algebra
- The conformal algebra is the finite dimensional SO(4,2)
- The collinear algebra is SL(2,R)
- The conformal transformations are holomorphic and anti-holomorphic
- The conformal transformations are global

Collinear algebra

- Use light-cone coordinates

$$A^\mu = A_- n_+^\mu + A_+ n_-^\mu + A_T^\mu$$

- The transformation of x_

$$x_- \to a x_- \qquad x_- \to x_- + a_- \qquad x_- \to x'_- = \frac{x_-}{1 + 2\tilde{a} x_-}$$

- The generators are

$$L_+ = -iP_+ \qquad L_- = \frac{i}{2} K_- \qquad L_0 = \frac{i}{2}(D + M_{-+})$$

- SL(2,R) algebra $[L_0, L_\pm] = \pm L_\pm \qquad [L_-, L_+] = -2L_0$

- For a field $\Phi(x)$ that depends $\Phi(\alpha) \equiv \Phi(\alpha n_+^\mu)$
- The collinear algebra

$$\alpha \to \alpha' = \frac{a\alpha + b}{c\alpha + d} \qquad \Phi(\alpha) \to (c\alpha + d)^{-2j} \Phi\left(\frac{a\alpha + b}{c\alpha + d}\right)$$

Primary field, highest weight

Virasoro in 2 dimensions

$$L_0[\phi(0)|0>] = h[|\phi(0)|0>]$$

$$L_n[\phi(0)|0>] = 0, \qquad n > 0$$

SL(2,R) Collinear 4 dimensions

$$L_0[\Phi(0)|0>] = j[|\Phi(0)|0>]$$

$$L_-[\Phi(0)|0>] = 0,$$

Conformal Operator Product Expansion

- 2d COPE

$$\emptyset_i(z,\bar{z})\emptyset_j(w,\bar{w}) \sim \sum_k C_{ijk}(z-w)^{h_k-h_i-h_j}(\bar{z}-\bar{w})^{\bar{h}_k-\bar{h}_i-\bar{h}_j}\emptyset_k(w,\bar{w})$$

- 4d collinear COPE

$$\begin{aligned} A(x)B(0) &= \sum_{n=0}^{\infty} C_n \left(\frac{1}{x^2}\right)^{1/2(t_A+t_B-t_n)} \frac{x_-^{n+s_1+s_2-s_A-s_B}}{B(j_A-j_B+j_n, j_B-j_A+j_n)} \\ &\times \int_0^1 du\, u^{(j_A-j_B+j_n-1)}(1-u)^{(j_B-j_A+j_n-1)} \mathcal{O}_n^{j_1,j_2}(ux_-) \end{aligned}$$

Conformal invariance

- As an example let's compare the OPE of two currents. The expression in 2d reads

$$J^a(z)J^b(w) = \frac{k\delta^{ab}}{(z-w)^2} + i\frac{f_c^{ab}J^c(w)}{(z-w)} + finite\ terms$$

for any non-abelian group, and in particular for the abelian case the second term on the RHS is missing.

- For comparison the OPE of the transverse components of the electromagnetic currents takes the form

$$J^T(x)J^T(0) \sim$$

$$\sum_{n=0}^{\infty} C_n \left(\frac{1}{x^2}\right)^{(6-t_n)/2} (-ix_-)^{n+1} \frac{\Gamma(2j_n)}{\Gamma(j_n)\Gamma(j_n)} \int_0^1 du[u(1-u)]^{j_n-1} Q_n^{1,1}(ux_-)$$

- where

$$\mathcal{Q}_\mu(\alpha_1,\alpha_2) = \bar{\psi}(\alpha_1)\gamma_\mu P e^{ig\int_{\alpha_1}^{\alpha_2} dt A_+(t)}\psi(\alpha_2)$$

- and the corresponding local operators read

$$Q_n^{1,1}(\alpha) \quad = \quad (i\partial_+)^n \left[\bar{\psi}(\alpha)\gamma_+ C_n^{3/2}\left(\overleftrightarrow{D}_+/d_+\right)\psi(\alpha)\right],$$

- with

$$\overleftrightarrow{D}_+ = \overrightarrow{D}_+ - \overleftarrow{D}_+ \qquad d_+ = \overrightarrow{D}_+ + \overleftarrow{D}_+$$

- and where $C_n^{3/2}$ are the Gegenbaur polynomials.

Conformal invariance

- The conformal Ward identity associated with the dilatation operator in 4d

$$\sum_i^N (l_\phi + \gamma(g^*) + x_i \partial_i) < T\phi(x_1)...\phi(x_N) >= 0$$

where l_ϕ is the canonical dimension and $\gamma(g^*)$ is the anomalous dimension,

- This seems quite similar to the one in 2d

$$\sum_i (z_i \partial_i + h_i) < 0|\phi_1(z_1, \bar{z}_1)...\phi_n(z_n, \bar{z}_n)|0 >= 0$$

Conformal invariance

- In both cases one has to determine the full quantum conformal dimension of the various operators.
- However, in certain CFT models, like the unitary minimal models, there are powerful tools based on unitarity which enable us to determine exactly the dimensions h_i of all the primary operators and hence all the operators of the model.
- On the other hand, it is a non-trivial task to determine the anomalous dimensions in other models in 2d, and of course in 4d.
- In certain supersymmetric theories there are operators whose dimension is protected

Conformal invariance

- Using the Ward identity one can extract the form of the two point function of operators of spin s in 4d . It is given by

$$< \phi(x_1)\phi(x_2) >= N_2(g^*)(\mu^*)^{-2\gamma(g^*)} \left[\frac{1}{(x_1-x_2)^2}\right]^{l_\phi+\gamma(g^*)} \left(\frac{(x_1-x_2)_+}{(x_1-x_2)_-}\right)^s$$

- The corresponding two point function in 2d, which depends only on the conformal dimension of the operator h, reads

$$G_2(z_1,\bar{z}_1,z_2,\bar{z}_2) \equiv< 0|\phi_1(z_1,\bar{z}_1)\phi_1(z_2,\bar{z}_2)|0 >= \frac{c_2}{(z_1-z_2)^{2h_1}(\bar{z}_1-\bar{z}_2)^{2\bar{h}_1}}$$

Conformal invariance

- As for higher point functions, one can use in 2d the local Ward identities together with Virasoro null vectors to write down partial differential equations that determine the correlators. For instance , the four point function of the Ising model.
- Certain 2d conformal field theories are further invariant under affine Lie algebra transformations. Using combined null vectors one derives the so called Knizhnik-Zamolodchikov equations, which can be used to solve for instance the four-point function of the SU(N) WZW model .
- This type of differential equations that fully determine correlation functions are obviously absent in 4d interacting conformal field theories.

Integrability

- Systems with a finite number of degrees of freedom, like spin chain models are integrable if there is an equal number of conserved charges.
- Integrable field theories admit an infinite countable number of conserved charges.
- The scattering processes of those models always involve a conservation of the number of “particles”.
- In two dimensions there are continuous integrable models like the sine-Gordon model as well as discretized ones like the XXX spin chain model.

- In two dimensions the spin chain models follow from a discretization of the space coordinate, by placing a spin variable on each site that can take several values, and by imposing periodicity.

Integrability

- The integrable sectors of 4d gauge dynamical systems are based on identifying an exact map between certain properties of the systems and a spin chain structure.
- In the four dimensional N =4 super YM theory the spin chain corresponds to a trace of field operators
- In high-energy scattering of QCD it is a ``chain'' of reggeized gluons exchanged in the t-channel of a scattering process.
- A summary of the comparison among the basic two dimensional spin chain, the ``spin chains'' associated with the planar N= 4 SYM, and the high energy scattering in QCD, is given in table

Integrability

Spin chain	Planar $\mathcal{N}=4$ SYM	High energy scattering in QCD
Cyclic spin chain	Single trace operator	Reggeized guons in t-channel
Spin at a site	Field operator	$SL(2)$ spin
Number of sites	Number of operators	Number of gluons
Hamiltonian	Anomalous dilatation operator	H_{BFKL}
Energy eigenvalue	Anomalous dimension $g^{-2}\delta\mathcal{D}$	$\sim \frac{1}{\lambda}\frac{log\mathcal{A}}{logs}$
evolution time	dilatation variable	the total rapidity $logs$
Zero momentum $U=1$	Cyclicity constraint	

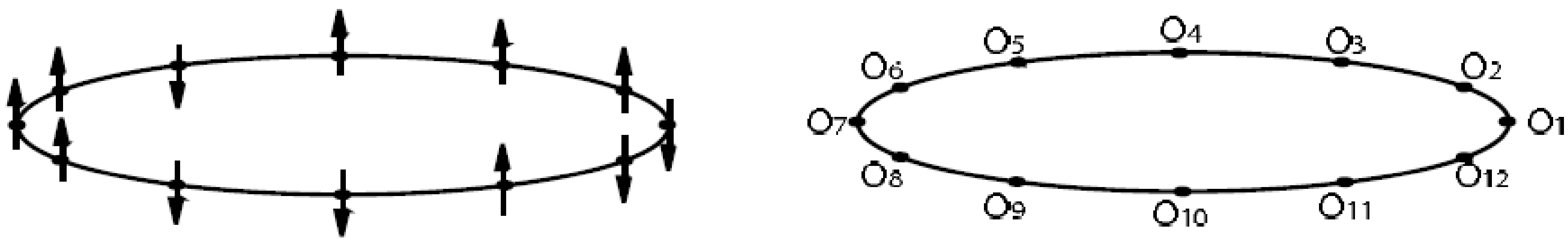

Fig. 18.1. A single trace operator as a spin chain.

Topological field configurations

- Topological charges in any dimensions are conserved regardless of the equations of motion of the corresponding systems.
- In two dimensions it is very easy to write down a topological current

$$J_\mu = \epsilon_{\mu\nu}\partial^\nu \phi \qquad J_\mu = \epsilon_{\mu\nu} g^{-1}\partial^\nu g$$

- The corresponding topological charge (abelian)

$$Q_{top} = \int dx \phi' = [\phi(t, +\infty) - \phi(t, -\infty)] \equiv \phi_+ - \phi_-$$

- For the compact S^1 this is the winding number

Topological field configurations

- Obviously one cannot have such topologically conserved currents and charges in 4d.
- However, for theories that are invariant under a non-abelian group, one can construct also in four dimensions a topological current and charge, like for the cases of Skyrmions, magnetic monopoles and instantons. For the Skyrmions the topological current is given by

$$J^{\mu}_{skyre} = \frac{i\epsilon^{\mu\nu\rho\sigma}}{24\pi^2} Tr[L_\nu L_\rho L_\sigma]$$

where $L_\mu = U^\dagger \partial_\mu U$ with $U \in SU(N_f)$

Topological field configurations

- The topological charges, for compact spaces, are the winding numbers of the corresponding topological configurations. For a compact one space dimension, we have the map of $S^1 \to S^1$ related to the homotopy group $\pi_1(S^1)$.
- In two space dimensions, the windings are associated with the map $S_2 \to S_2^{G/H}$ as for the magnetic monopoles.
- For three space dimensions, it is $S^3 \to S^3$ for the Skyrmions at N_f=2, and the non-abelian instantons for the gauge group SU(2) . The topological data of the various models is summarized in the table .

Topological field configurations

Table 2: Topological classical field configurations in two and four dimensions

classical field	dim.	map	topological current
soliton	two		$\epsilon^{\mu\nu}\partial_\nu\phi$
baryon	two	$S^1 \to S^1$	$\epsilon^{\mu\nu}Tr[g^{-1}\partial_\nu g];\ \ g \in U(N_f)$
Skyrmion	four	$S^3 \to S^3$	$\frac{i\epsilon^{\mu\nu\rho\sigma}}{24\pi^2}Tr[L_\nu L_\rho L_\sigma]$
monopole	four	$S^2_{space} \to S^2_{G/H}$	$\frac{1}{8\pi}\epsilon_{\mu\nu\rho\sigma}\epsilon^{abc}\partial^\nu\hat{\Phi}^a\partial^\rho\hat{\Phi}^b\partial^\sigma\hat{\Phi}^c$
instanton	four	$S^3_s \to S^3_g$	$\frac{i\epsilon^{\mu\nu\rho\sigma}}{16\pi^2}Tr[A_\nu\partial_\rho A_\sigma + \frac{2}{3}A_\nu A_\rho A_\sigma]$

Bosonization

Bosonization is the formulation of fermionic systems in terms of bosonic variables. (No spin in 2d)

It has several advantages:

- It is usually easier to deal with commuting fields rather than anti-commuting ones.

- In certain examples, like the Thirring model, the fermionic strong coupling regime turns into the weak coupling one in its bosonic version, the Sine-Gordon model.

- One loop fermionic computations involving the currents turn into tree level consideration in the bosonized version. The best known example of the latter are the chiral or axial anomalies.

operator	fermionic	bosonic
$J_+(x^+)$	$:\psi_L^\dagger \psi_L:$	$\partial_+\phi$
$J_-(x^-)$	$:\psi_R^\dagger \psi_R:$	$\partial_-\phi$
$T_{++}(x^+)$	$-\frac{1}{2}:[\psi_L{}^\dagger \partial\psi_L - \partial\psi_L{}^\dagger \psi_L]:$	$-\frac{1}{2}:\partial_+\phi\partial_+\phi(x^+):$
$T_{--}(x^-)$	$-\frac{1}{2}:[\psi_R{}^\dagger \partial\psi_R - \partial\psi_R{}^\dagger \psi_R]:$	$-\frac{1}{2}:\partial_-\phi\partial_-\phi(x^+):$
fermion$_L$	$\psi_L(x^+)$	$\sqrt{\frac{c\mu}{2\pi}}:\exp\left(-i\sqrt{\pi}\left(\int_{-\infty}^{x} d\xi\pi(\xi)+\phi(x)\right)\right):$
fermion$_R$	$\psi_R(x^-)$	$\sqrt{\frac{c\mu}{2\pi}}:\exp\left(-i\sqrt{\pi}\left(\int_{-\infty}^{x} d\xi\pi(\xi)-\phi(x)\right)\right):$
mass term	$\psi_L^\dagger(x^+)\psi_R(x^-)+\psi_R^\dagger(x^+)\psi_L(x^-)$	$\mu: cos\hat{\phi}(x^+,x^-):$

Bosonization

operator	fermionic	bosonic
$J(z)$	$:\psi^\dagger\psi(z):$	$i\partial\phi(z)$
$\bar{J}(\bar{z})$	$:\tilde{\psi}^\dagger\tilde{\psi}(\bar{z}):$	$-i\bar{\partial}\phi(\bar{z})$
$T(z)$	$-\frac{1}{2}:[\psi^\dagger\partial\psi-\partial\psi^\dagger\psi]:$	$-\frac{1}{2}:\partial\phi\partial\phi(z):$
$\bar{T}(\bar{z})$	$-\frac{1}{2}:[\tilde{\psi}^\dagger\partial\tilde{\psi}-\partial\tilde{\psi}^\dagger\tilde{\psi}]:$	$-\frac{1}{2}:\bar{\partial}\phi\bar{\partial}\phi(\bar{z}):$
fermion$_L$	$\psi(z)$	$:e^{i\phi(z)}:$
fermion$_R$	$\tilde{\psi}(\bar{z})$	$:e^{i\phi(\bar{z})}:$
mass term	$\tilde{\psi}^\dagger(\bar{z})\psi(z)+\psi^\dagger(z)\tilde{\psi}(\bar{z})$	$\mu:cos\hat{\phi}(z,\bar{z}):$

Bozonization

- The non-abelian bosonization, especially in the product scheme, offers a separation between colored and flavored degrees of freedom, which is very convenient for analyzing the low lying spectrum.

- Baryons composed of N_C quarks are a many-body problem in the fermion language, while simple solitons in the boson language.

Bozonization

- In four dimensions, spin is obviously non-trivial and one cannot constitute generically a bosonization equivalence. However, in certain circumstances a systems can be described approximately by fields that depend only on the time and the radial direction.

- Examples are monopole induced proton decay, and fractional charges induced on monopoles by light fermions. In these cases the relevant degrees of freedom are in an s-wave .

- There is a slight difference with two dimensions, as the radial coordinate goes from zero to infinity, so "half" a line. Appropriate boundary conditions enable us to use a reflection, so to extend to a full line.

Strong –weak duality

- A very important phenomenon that occurs in both two and four dimensions is the strong-weak duality, and the duality between a soliton and an elementary field.
- In two dimensions it is the relation between the Thirring model and the sine-Gordon model.

$$\frac{\beta^2}{4\pi} = \frac{1}{1+\frac{g}{\pi}}$$

- This also relates the elementary fermion field of the Thirring model with the soliton of the sine-Gordon model. In particular for g=0 corresponding to $\beta^2=4\pi$ the Thirring model describes a free Dirac fermion, while the soliton of the corresponding sine-Gordon theory is the same fermion in its bosonization disguise.

Strong- weak duality

- An analog in four dimensions is the Olive-Montonen duality , which relates

  electric charge e $\longleftrightarrow$ magnetic charge $e_m = 4\pi/e$

  elementary states $\longleftrightarrow$ magnetic monopoles

- On top of the self-duality of the spectrum, there is a similar duality also in the low energy scattering.
- There is no net force between (BPS) magnetic monopoles This follows up from an exact cancelation between the magnetic repulsion and the attraction due to an exchange of a Higgs scalar.

The N= 4 SYM admits a complete invariance under the Olive Montonen duality

Confinement versus screening

- In 2d the string tension is proportional to

$$T_s \sim m_d\, g$$

- This implies that massless dynamical quarks always screen. In particular that massless dynamical adjoint fermions can screen fundamental fermions.
- This follows from the computation of

$$T_s = \langle H \rangle - \langle H_o \rangle$$

- With this definition of the string tension, the screening behavior in 2d and in 4d are very diffferent.

Determination of the string tension

- The bosonized action of massive QCD2 with fermions in the fundamental representation

$$S_{fundamental} = \frac{1}{8\pi}\int_{\Sigma} d^2x\ tr(\partial_\mu g \partial^\mu g^\dagger) \tag{14.42}$$
$$+\frac{1}{12\pi}\int_{B} d^3y \epsilon^{ijk}\ tr(g^\dagger\partial_i g)(g^\dagger\partial_j g)(g^\dagger\partial_k g)$$
$$+\frac{1}{2}m\mu_{fund}\int d^2x\ tr(g+g^\dagger) - \int d^2x \frac{1}{4e^2}F^a_{\mu\nu}F^{a\mu\nu}$$
$$-\frac{1}{2\pi}\int d^2x\ tr(ig^\dagger\partial_+ gA_- + ig\partial_- g^\dagger A_+ + A_+ gA_- g^\dagger - A_+A_-)$$

where g is NxN unitary matrix and $\mu = e\frac{\exp(\gamma)}{(2\pi)^{\frac{3}{2}}}$

- In the gauge A_=0 the action reads

$$S = S_0 + \frac{1}{2}m\mu_R\int d^2x\ tr(g+g^\dagger)$$
$$-\frac{ik_{dyn}}{4\pi}\int d^2x\ (g\partial_- g^\dagger)^a A^a_+,$$

Determination of the string tension

- An external source is addes as

$$-\frac{ik_{ext}}{4\pi}\int d^2x\ (u\partial_- u^\dagger)^a A_+^a$$

where
$$u = [\exp -i4\pi\left(\theta(x^- + L) - \theta(x^- - L)\right)]T^3_{ext},$$

- The combined action reads

$$S = S_0 + \frac{1}{2}m\mu_R\int d^2x\Big\{\ tr(g+g^\dagger)+ \\ [-\frac{ik_{dyn}}{4\pi}(g\partial_- g^\dagger)^a + k_{ext}\delta^{a3}(\delta(x^- + L) - \delta(x^- - L))]A_+^a\Big\}$$

Determination of the string tension

- The external sources can be eliminated bv transforming

$$-\frac{ik_{dyn}}{4\pi}(\tilde{g}\partial_-\tilde{g}^\dagger)^a = -\frac{ik_{dyn}}{4\pi}(g\partial_- g^\dagger)^a + k_{ext}\delta^{a3}\left(\delta(x^- + L) - \delta(x^- - L)\right)$$

- This is solved by

$$\begin{aligned} \tilde{g}^\dagger &= P\exp\left\{\int dx^- \left(g\partial_- g^\dagger + i4\pi\frac{k_{ext}}{k_{dyn}}(\delta(x^- + L) - \delta(x^- - L))T^3_{dyn}\right)\right\} \\ &= e^{i4\pi\frac{k_{ext}}{k_{dyn}}\theta(x^- + L)T^3_{dyn}} g^\dagger e^{-i4\pi\frac{k_{ext}}{k_{dyn}}\theta(x^- - L)T^3_{dyn}} \end{aligned} \quad (14.47)$$

- The resulting action

$$S = S_{WZW}(\tilde{g}) + S_{kinetic}(A_\mu) - \frac{ik_{dyn}}{4\pi}\int d^2x\ (\tilde{g}\partial_-\tilde{g}^\dagger)^a A^a_+ + \frac{1}{2}m\mu_R\int d^2x\ tr(\tilde{g}e^{i4\pi\frac{k_{ext}}{k_{dyn}}T^3_{dyn}} + e^{-i4\pi\frac{k_{ext}}{k_{dyn}}T^3_{dyn}}\tilde{g}^\dagger)$$

Determination of the string tension

- The expectation value of the Hamiltonian is

$$< H >= -\frac{1}{2} m\mu_R \, tr(e^{i4\pi \frac{k_{ext}}{k_{dyn}} T^3_{dyn}} + e^{-i4\pi \frac{k_{ext}}{k_{dyn}} T^3_{dyn}}) = -m\mu_R \sum_i \cos(4\pi\lambda_i \frac{k_{ext}}{k_{dyn}})$$

- The string tension thus is given by

$$\sigma = m\mu_R \sum_i \left(1 - \cos(4\pi\lambda_i \frac{k_{ext}}{k_{dyn}}) \right)$$

K-string

- The Wilson line that associates with the potential of a quark anti-quark pair in an N_c anality=k is the k-string. Various methods including large Nc , lattice and holography were used to determine it.
- The string tension of such a configuration is believed to follow either a Casimir or sinusoidal rules

$$\sigma_k^{cas} \sim \frac{k(N-k)}{N} \qquad \sigma_k^{sin} \sim \sin(\frac{\pi k}{N})$$

- The 2d analog of the 4d YM (or N=1 SYM) is QCD with adjoint fermions. The 2d k-string tension is

$$\sigma_k^{2d} \sim \sin^2(\frac{\pi k}{N})$$

Hadronic spectra

- In 2d the mesonic spectrum of QCD can be worked out in:
- 't Hooft seminal large N_c limit in the fermionic picture.
- The currentization method for massless quarks.
- The DLCQ approach for fundamental and adjoint fermions.
- The 2d baryonic spectrum can be extracted using bosonization and the strong coupling limit.

- In 4d one can use lattice simulations and approximate methods like
- Large N_c
- Skyrme model for the baryons …

Mesons

- The spectrum of mesons is 2d is characterized by

$$M_{mes}\,(g, N_c\ , N_f\ , m_q\ ,n)$$

- The highly excited states behave like

$$M^2_{mes} \sim \pi\,(\,g^2 N_c\,)\,n$$

- This is the Regge behavior which mesons in nature admit. It is easily derived from the quantization of a string model but it is hardly ever the result of 4d field theory.

- The opposite limit of the ground state and low lying states

- In the limit $m_q \gg g \quad M^0_{mes} \sim (\,m_1 + m_2\,)$
- In the limit $m_q \ll g \quad (\,M^0)^2_{mes} \sim (g^2 N_c)^{1/2}(\,m_1 + m_2\,)$

Mesons

- For massless quarks this implies a massless Meson

- This is similar to the GOR relation for the pion mass

$$m_\pi^2 \sim \frac{<\bar{\psi}\psi>}{f_\pi^2}(m_1 + m_2)$$

- We cannot deduce form 't Hooft model the dependence on N_f . This can be done using the currentization method. We get

$$M^2_{mes} \sim N_f$$

Mesons

- Whereas the ‘t Hooft model presents a solution of the mesonic spectra in 2d, in 4d one does not know the corresponding mesonic spaectra in the planar limit. One can only determine the scaling with N_c of the mass, the size, scattering amplitudes etc.

Baryons

- In 2d the spectrum of the baryons can be determined in the strong coupling limit using bosonization. $\frac{m_q}{e_c} \to 0$
- After integrating the colored d.o.f one finds an exact expression for the action of the flavored d.o.f.
- The mass of the baryon takes the form

$$E = 4m\sqrt{\frac{2N_C}{\pi}} + m\sqrt{2}\sqrt{\left(\frac{\pi}{N_C}\right)^3}\left[C_2 - N_C^2\frac{(N_F-1)}{2N_F}\right]$$

where $m = [N_C c m_q (\frac{e_c\sqrt{N_F}}{\sqrt{2\pi}})^{\Delta_C}]^{\frac{1}{1+\Delta_C}}$ $\Delta_C = \frac{N_C^2-1}{N_C(N_C+N_F)}$

In 2d for N_f =3 the lowest state is the totally symmetric **10** and not the **8**

Large N scaling and flavor content

- The scaling with N_C in 2d and 4d are different

	two dimensions	four dimensions
Classical baryon mass	N_C	N_C
Quantum correction	N_C^0	N_C^{-1}

- Both in 2d and 4d the mass depends on N_f via the second Casimir operator.
- The flavor content of the baryons in 2d and 4d is

	two dimensions state	value	four dimensions state	value
$\langle \bar{u}u \rangle$	Δ^+	$\frac{1}{2}$	p	$\frac{2}{5}$
$\langle \bar{d}d \rangle$	Δ^+	$\frac{1}{3}$	p	$\frac{11}{30}$
$\langle \bar{s}s \rangle$	Δ^+	$\frac{1}{6}$	p	$\frac{7}{30}$
$\langle \bar{s}s \rangle$	Δ^{++}	$\frac{1}{6}$	Δ	$\frac{7}{24}$

Summary

- In general the non-perturbative techniques are more powerful in 2d than in 4d.
- There are certain similarities between the application of CFT in 2d and 4d.
- Integrability in 4d is based on mapping sectors of 4d CFT to 2d integrable models.
- There are methods that apply only in 2d like bosonization.
- Strong-weak and particle-soliton dualities occour in both 2d and 4d.
- Screening versus confinement seems to be different in 2d and 4d