A disintegration theorem for infinite variance superprocesses

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joint work with

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Superprocesses as diffusion approximation of spatial population models:

- Dawson-Watanabe (DW) superprocess
  Branching Brownian motion
  Population size evolves randomly
- Fleming-Viot (FV) superprocess
  "Spatial" Moran model
  Population size is fixed

Feller rescaling:
Mass of each individual $\sim 1/n$
Branching/Sampling rate speeded by n

Perkins' Disintegration Theorem.

If we normalize a DW superprocess X_t by its total mass $|X_t|$ (stopped before extinction) we recover a FV superprocess with time varying sampling rate $\sim 1/|X_t|$.

"Skew-Product" representation for DW superprocess :

$$X_t = |X_t| Z_t$$

$|X_t|$: Total mass process, given by a Feller diffusion.

Z_t: Time-changed FV superprocess.

$\Rightarrow$ Genealogy of the DW superprocess can be linked via a time change to that of the FV superprocess.

Donnelly-Kurtz construction.

Simultaneous representation of measure-valued population models and their genealogy.

Countable representation.

Relabelling according to the longest line of descent.

Infinite variance setting:

[Etheridge, Williams]

[Birkner, Möhle, Wakolbinger]

Offspring generating function:

$$\Phi(s) = \frac{1}{1+\beta}(1-s)^{1+\beta} + s, \quad 0 < \beta \leq 1$$

Tails $\sim n^{-1-\beta}$.

$\Rightarrow$ Stable branching mechanism of index $1+\beta$.

Population size described by a CSBP with unbounded variation.

Exchangeable Λ-coalescents.

[Pitman97], [Sagitov99]

Multiple mergers of ancestral lineages occur

If there are b lineages, then a k-tuple of lineages merges with rate :

$$\lambda_{b,k} = \int_0^1 x^{k-2}(1-x)^{b-k}\Lambda(dx)$$

where Λ is a finite measure on $[0,1]$.

- No *simultaneous* multiple mergers are allowed.
- $\Lambda = \delta_0 \rightarrow$ Kingman coalescent.

Generalized Fleming-Viot processes.

[Bertoin, LeGall]

$(\rho_t, t \geq 0) \in \mathcal{M}_1(\mathbb{R}^d)$

At a branching event:

$$\rho_{t-} \to (1-X)\rho_{t-} + X\delta_Y$$

$X \sim \nu(\cdot)/\nu(]0,1])$, ν finite measure on $]0,1]$.

$Y \sim \rho_{t-}$.

Generalized Fleming-Viot (GFV) processes can be interpreted as diffusion approximation of Cannings' models.

Generator for ρ_t:

$$\mathcal{L}G(\rho_t) = \int \nu(dx) \int \rho_t(da) G((1-x)\rho_t + x\delta_a) - G(\rho_t)$$

The GFV process appears as *measure-valued dual* of the Λ-coalescents.

- $\nu(dx) = x^{-2}\Lambda(dx)$

X_t : Infinite variance superprocess

We study the normalized process (stopped before extinction):

$Y_t = X_t/|X_t| \in \mathcal{M}_1(\mathbb{R}^d)$

Y_t is a *time-changed GFV process*, with coalescent measure $\Lambda = Beta(1-\beta, 1+\beta)$.

Generator for Y_t (ignoring spatial component):

$$\mathcal{G}f(Y_t) = \frac{C}{|X_t|^\beta}\mathcal{L}f(Y_t)$$

The jumping rate is function of the total mass.

A Disintegration Theorem for infinite variance superprocesses.

"Skew-product" representation :

$$X_t = |X_t| Y_t$$

$|X_t|$: Total mass process, given by a CSBP.

Y_t: Time-changed GFV process.

$\Rightarrow$ Genealogy of a sample from an infinite variance superprocess can be represented in terms of a Λ-coalescent, with rates rescaled by its total mass.

For more general branching mechanisms, the jump measure ν will depend on $|X_t|$ in a complicated way.

$\Rightarrow$ "Skew-product" representation *only in the stable case.*