Matrix factorisations and D-branes

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mainly based on work with

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related work by Enger, Recknagel, Roggenkamp

1. Introduction

Phenomenologically interesting string compactifications usually involve Calabi-Yau manifolds (or orientifolds and orbifolds thereof). It is therefore important to understand D-branes on such spaces.

Many interesting Calabi-Yau manifolds can be described as hypersurfaces in complex projective space.

For example the quintic is the Calabi-Yau manifold

$$W = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0$$

in $\mathbb{CP}^4$

$\uparrow$

complex projective space

$$(x_1, \ldots, x_5) \sim (\lambda x_1, \ldots, \lambda x_5)$$

$\lambda \neq 0$.

For 'large' Calabi-Yaus can think of the D-branes geometrically as hypersurfaces that wrap non-trivial cycles. For example, for the quintic X

D0-brane $\leftrightarrow$ point on quintic

D6-brane $\leftrightarrow$ wraps entire space

D2-brane $\leftrightarrow$ wraps non-trivial element in $H_2(X)$

D4-brane $\leftrightarrow$ wraps non-trivial element in $H_4(X)$

generate D-brane (RR) charges

As is familiar from T-duality string theory does not always see this classical geometry.

Try to understand how the 'string sees the Calabi-Yau'

"quantum geometry"

From a microscopic (conformal field theoretic) point of view, Calabi-Yau compactifications are best understood at the so-called Gepner point, where they are described in terms of tensor products of $N=2$ minimal models.

$$\mathcal{A} = \bigotimes_i (N=2)_{k_i}$$

$$\uparrow$$

$$c_i = \frac{3k_i}{k_i+2} \qquad \sum_i c_i = 9$$

A certain class of D-branes (or boundary states) at the Gepner point were constructed some time ago by Recknagel & Schomerus. These branes are characterised by the property that they preserve the full chiral symmetry $\mathcal{A}$.

However, it has been known for some time that these RS branes do not account for all the D-brane (RR) charges.

Brunner, et al

For example for the case of the quintic

$$W = x_1^5 + .. + x_5^5 \quad \text{(LG superpot.)}$$

$$\updownarrow$$

$$\mathcal{A} = \bigotimes_{i=1}^{5} (N=2)_3$$

the RS branes only generate a sublattice of index 25 of the full charge lattice.

In particular, a single D0-brane on the quintic is not described by any of the RS boundary states. (Scheidegger)

In order to make progress with the problem of constructing the fundamental D-branes we can make use of recent results of Kontsevich and Diaconescu et al.

Matrix factorisations

in

Landau-Ginzburg models

Kontsevich

Kapustin, Li

Brunner et.al

Hori, Walcher

Maxim Kontsevich has proposed

that supersymmetric B-type

D-branes in Landau-Ginzburg

models can be characterised in

terms of matrix factorisations

$$Q^2 = W \cdot 1$$

LG superpotential

From a physics point of view,

this can be understood as follows:

Kapustin, Li
Brunner et al.

consider $\mathcal{N} = (2,2)$ supersymmetric

field theory in two dimensions.

$\mathcal{N} = (2,2)$ superspace

$$x^{\pm} = t \pm x \qquad \theta^+, \theta^- \qquad \bar{\theta}^+, \bar{\theta}^-$$

Chiral superfield Φ : $\bar{D}_{\pm} \Phi = 0$

$$D_{\pm} = \frac{\partial}{\partial \theta^{\pm}} - i \bar{\theta}^{\pm} \partial_{\pm}$$

$$\bar{D}_{\pm} = - \frac{\partial}{\partial \bar{\theta}^{\pm}} + i \theta^{\pm} \partial_{\pm} .$$

Action contains Landau-Ginzburg superpotential F-term

$$\int_{\Sigma} d^2x \; d\theta^+ d\theta^- \; W(\Phi) \; + \text{c.c.}$$

LG superpotential

D-branes $\cong$ worldsheet Σ with boundary

$$x^+ = x^- = t \qquad (x=0)$$

B-type boundary condition: preserve

$$Q = Q_+ + Q_-$$

$$\bar{Q} = \bar{Q}_+ + \bar{Q}_-$$

where $Q_\pm = \frac{\partial}{\partial \theta^\pm} + i\bar{\theta}^\pm \partial_\pm$

$$\bar{Q}_\pm = -\frac{\partial}{\partial \bar{\theta}^\pm} - i\theta^\pm \partial_\pm$$

Now consider susy variation of F-term

$$\int_\Sigma d^2x \, d\theta^+ d\theta^- \; \bar{\varepsilon}(\bar{Q}_+ + \bar{Q}_-) W$$

$$= \int_\Sigma d^2x \, d\theta^+ d\theta^- \; (-2i\bar{\varepsilon}\theta^+\partial_+ - 2i\bar{\varepsilon}\theta^-\partial_-) W$$

$\Big\ulcorner$ $\bar{D}_\pm W = \left(-\frac{\partial}{\partial\bar{\theta}^\pm} + i\theta^\pm\partial_\pm\right) W = 0$

since $W \equiv W(\Phi)$ and Φ chiral $\Big\lrcorner$

$$= \int_{\partial\Sigma} dt \, d\theta \; (-2i\bar{\varepsilon} W)$$

$\ulcorner$ on worldsheets with boundary.

At boundary $x^+ = x^- = t$ & $\theta^+ = \theta^- = \theta$ $\lrcorner$

Need to cancel this term – do so

by adding boundary F-term

$$\int_{\partial\Sigma} dt\, d\theta \;\; \underset{\substack{\uparrow \\ \text{fermionic} \\ \text{superfield}}}{\Gamma(t,\theta)} \;\; \underset{\substack{\uparrow \\ \text{polynomial} \\ \text{in chiral } \Phi.}}{E(\Phi)}$$

Choose $\Gamma(t,\theta)$ not to be chiral, but

$$\bar{D}\Gamma = (\bar{D}_+ + \bar{D}_-)\Gamma = J(\Phi)$$

This then cancels above variation

$$\left[\,\bar{Q} = \bar{Q}_+ + \bar{Q}_- = -\frac{\partial}{\partial\bar{\theta}} - i\,\theta\,\partial_t \text{ on } \partial\Sigma\,\right]$$

provided that

$$E(\Phi)\;J(\Phi) = 2i\;W(\Phi)$$

factorisation of superpotential !

Fermions act on some Clifford space:

in general $E(\Phi)$ and $J(\Phi)$ are

matrices: dropping the $(2i)$-factor have

$$E(\Phi) \cdot J(\Phi) = W(\Phi) \cdot \mathbb{1}$$

$d \times d$ matrices

⌈String theory: can think of E & J

as describing tachyon profile on

D-$\bar{D}$ system ⇝ K-theory, der. categories, ...

tachyon ↔ worldsheet fermion

'wrong' GSO-projection ⌋

Combine E and J into BRST matrix

$$Q = \begin{pmatrix} 0 & J \\ E & 0 \end{pmatrix} \qquad (2d) \times (2d) \text{ matrix}$$

Then Q satisfies

$$Q \cdot Q = W \cdot \mathbb{1}_{2d}$$

Its cohomology describes topological open string spectrum on the D-brane described by Q (see later).

Landau-Ginzburg models are closely related to N=2 minimal models (i.e. building blocks in Gepner model description).

Thus there must be correspondence

LG model		N=2 minimal model
Matrix factorisation	$\longleftrightarrow$	N=2 boundary states
$W = \sum_i x_i^{\ell_i}$		$\mathcal{A} = \bigotimes_i (N=2)_{\ell_i - 2}$

For the case of a single minimal model this correspondence has been understood, but in general very little is known.

Kapustin & Li
Brunner et al.

Here we want to study the next simple case

$$W = x_1^d + x_2^d$$

which will turn out to be already quite interesting.

In particular this is sufficient to make contact with the recent work of Diaconescu et.al. They showed that the D0-brane on the quintic corresponds to a matrix factorisation that involves a non-trivial factorisation of $W = x_1^5 + x_2^5$ (as well as simple factorisations of x_3^5, x_4^5 and x_5^5).

Calabi-Yau hypersurface

$W = x_1^5 + .. + x_5^5 = 0$

in $\mathbb{CP}^4$

'geometric' D-branes

(coherent sheafs)

$x_1 - \eta x_2 = x_3 = x_4 = x_5 = 0$

location of D0

linear σ-model

Landau-Ginzburg orbifold $\cong$ Gepner model in CFT

$(W = x_1^5 + \ldots + x_5^5)/\mathbb{Z}_5$ $\qquad$ $\left((N=2)_{k=3}^{\otimes 5}\right)/\mathbb{Z}_5$

Matrix factorisations $\qquad$ N=2 D-branes

$W = Q^2 \quad Q = \begin{pmatrix} 0 & J \\ E & 0 \end{pmatrix}$

D0-brane $\longleftrightarrow$ Permutation brane

$J: x_1 - \eta x_2, x_3, x_4, x_5$

[Recknagel]

By identifying the D0-brane on the quintic with a particular permutation brane we are also able to prove its stability.

The above picture generalises to other Gepner models and certain other classes of D-branes.

In particular, we have also managed to construct the boundary states of certain D2-branes for a number of models.

In this way we have found the D-brane boundary states whose charges generate the full charge lattice for a number of Gepner models (including the quintic).

2. The baby example

First we review the case of a single minimal model which corresponds to

$$W = x^d .$$

The corresponding $N=2$ minimal model has

$$c = \frac{3k}{k+2} \quad \text{with} \quad d = k+2 .$$

The bosonic subalgebra of the $N=2$ algebra can be described by the coset

$$(N=2)_{bos} = \frac{su(2)_k \oplus u(1)_4}{u(1)_{2k+4}} .$$

The spectrum of the theory (after GSO-projection) is then

$$\mathcal{H} = \bigoplus_{[l,m,s]} \left(\mathcal{H}_{[l,m,s]} \otimes \bar{\mathcal{H}}_{[l,m,-s]} \right)$$

rep of $(N=2)_{bos}$

We are interested in B-type gluing conditions

$$(L_n - \bar{L}_{-n}) \, \| B \rangle\rangle = 0$$

$$(J_n - \bar{J}_{-n}) \, \| B \rangle\rangle = 0$$

$$(G^{\pm}_{r} + i\eta \, \bar{G}^{\pm}_{-r}) \, \| B \rangle\rangle = 0$$

The corresponding Ishibashi states are supported in the sectors

$$[l,m,s] \otimes [l,-m,-s]$$

The B-type boundary states are then

MMS

$$\| L, S \rangle\!\rangle = \sqrt{2(2k+4)} \sum_{\substack{s,l \\ l+s \in 2\mathbb{Z}}} \frac{S_{L0S,\,l0s}}{\sqrt{S_{l0s,\,000}}} \, |[l,0,s]\rangle\!\rangle$$

(k odd)

Here

$$S \text{ even / odd} \longleftrightarrow \eta = +1 \, / \, \eta = -1$$

Furthermore, we note that

$$\| L, S \rangle\!\rangle = \| k-L, S+2 \rangle\!\rangle$$

and

$$\| L, S \rangle\!\rangle = \overline{\| L, S+2 \rangle\!\rangle}$$

anti-brane

The corresponding open string spectrum can be determined from overlap

$$\langle\langle L\, S \,\|\, q^{L_0 + \bar{L}_0 - \frac{c}{12}} \,\|\, \hat{L}\, \hat{S} \rangle\rangle$$

$$= \sum_{[l m s]} \Big(\delta^{(4)}(\hat{S} - S + s)\, N_{\hat{L}\, l}{}^{L} + \delta^{(4)}(\hat{S} - S + 2 + s)\, N_{\hat{L}\, l}^{k-L} \Big)\, \chi_{[l m s]}(\tilde{q})$$

The topological chiral primaries are in the representations

NS $\quad (l, l, 0) \quad$ or $\quad (l, -l-2, 2)$

R $\quad (l, l+1, 1) \quad$ or $\quad (l, -l-1, -1)$

[Recall $(l, m, s) \sim (k-l,\, m+k+2,\, s+2)$]

Thus we can determine the

topological states that appear in the

open string spectrum between two such

branes:

$\langle\langle L, 0 \| \hat{L}, 0 \rangle\rangle$ has $[l, l, 0]$ in spectrum

for those l s.t. $N_{\hat{L}l}{}^{L} = 1$.

This can now be compared with the

results that come from

matrix factorisation.

The corresponding factorisations of

$W = x^d \quad (d = k+2)$ are

$$Q_r = \begin{pmatrix} 0 & x^r \\ x^{d-r} & 0 \end{pmatrix} \qquad J = x^r \qquad E = x^{d-r}$$

where $r = 1, 2, \ldots, d-1$. The dictionary is

$$Q_r \leftrightarrow \| r-1, 0 \gg$$

Kapustin, Li
Brunner et.al.

Since

$$\| r-1, 0 \gg \; = \; \| k+1-r, 2 \gg$$
$$= \overline{\| d-r-1, 0 \gg}$$

$$Q_r \leftrightarrow Q_{d-r}$$

antibranes of one another

This can be confirmed by computing topological spectrum from matrix factorisations:

$$
\begin{tikzcd}
Q_r & \mathbb{C}[x] \arrow[r, shift left, "J"] \arrow[d, "f_0"'] \arrow[dr, dashed, "t_1" near end] & \mathbb{C}[x] \arrow[l, shift left, "E"] \arrow[d, "f_1"] \arrow[dl, dashed, "t_0"' near end] & J = x^r \\
Q_{\hat{r}} & \mathbb{C}[x] \arrow[r, shift left, "\hat{J}"] & \mathbb{C}[x] \arrow[l, shift left, "\hat{E}"] & \hat{J} = x^{\hat{r}}
\end{tikzcd}
$$

Here f_0 and f_1 are polynomials in x that satisfy

$$\hat{J} f_0 = f_1 J$$
$$\hat{E} f_1 = f_0 E \qquad [\Leftrightarrow \text{"} Qf = 0 \text{"}]$$

These polynomials have to be determined up to Q-exact solutions, i.e. up to

$$f_1 = t_1 E - \hat{J} t_0$$

$$f_0 = -t_0 J + \hat{E} t_1$$

The corresponding Q-cohomology is then the space of topological open string states.

Brunner et.al
Kapustin, Li
Hori, Walcher

For example, for $Q_r = Q_{\hat{r}}$, the Q-closed condition is simply

$$f_0 = f_1 .$$

The Q-exact solutions are those for which $f_0 = f_1$ contains x^r (or x^{d-r}) as a factor.

For Q_r with $r \leq \frac{d-1}{2}$ (so that $r < d-r$) we therefore have

$$f_0 = f_1 = \underbrace{1, x, \ldots, x^{r-1}}_{r \text{ different states.}}$$

This agrees precisely with the topological spectrum of $\| r-1, 0 \gg$: there we have

$$[l, l, 0] \qquad l = 0, 2, \ldots, 2(r-1)$$

$[\, N_{r-1\, l}^{\;r-1} = 1$ for these values !]

One can also check that the

U(1) charges match!

3. The product theory

Now we want to consider the product theory that corresponds to the superpotential

$$W = x_1^d + x_2^d .$$

The space of states of the corresponding CFT is (after GSO-projection)

$$\bigoplus_{\substack{[l_1\, m_1\, s_1] \\ [l_2\, m_2\, s_2] \\ s_1 - s_2 \in 2\mathbb{Z}}} \Big[\big([l_1\, m_1\, s_1] \otimes [l_2\, m_2\, s_2] \big) \otimes \big(\overline{[l_1\, m_1\, s_1]} \otimes \overline{[l_2\, m_2\, s_2]} \big)$$

$$\oplus \big([\quad] \otimes [\quad] \big) \otimes \big(\overline{[\quad s_1 + 2]} \otimes \overline{[\quad s_2 + 2]} \big) \Big]$$

This theory has the obvious tensor product branes that are labelled by

$$\| L_1 \, S_1 \, L_2 \, S_2 \gg$$

They correspond to the tensor products of the previous factorisations

Hori, Walcher

$$Q = \begin{pmatrix} 0 & J \\ E & 0 \end{pmatrix} \quad \text{with}$$

$$J = \begin{pmatrix} J_2 & J_1 \\ E_1 & -E_2 \end{pmatrix} \qquad E = \begin{pmatrix} E_2 & J_1 \\ E_1 & -J_2 \end{pmatrix}$$

and $\quad J_i = x_i^{L_i+1} \qquad E_i = x_i^{d-1-L_i}$.

In addition there are however also

rank 1 factorisations. To see how Diaconescu et al.

they arise we write

$$W = \prod_{\eta} (x_1 - \eta x_2)$$

all d'th roots of -1.

Label these roots as $\eta_m = e^{\pi i \frac{2m+1}{d}}$, and

call the label set $D = \{0, 1, \dots, d-1\}$. Then

we have the rank 1 factorisations

$$J = \prod_{m \in I} (x_1 - \eta_m x_2) \qquad E = \prod_{n \in D - I} (x_1 - \eta_n x_2)$$

for any subset $I \subset D$.

From the work of Orlov we can associate to any factorisation

$$P_1 \underset{E}{\overset{J}{\rightleftarrows}} P_0$$

the sheaf Coker J via the exact sequence

$$0 \to P_1 \xrightarrow{J} P_0 \to \operatorname{Coker} J \to 0 .$$

For the rank 1 factorisation $J = (x_1 - \eta x_2)$ we then find

$$\operatorname{Coker} J = \mathbb{C}[x_1, x_2] / (x_1 - \eta x_2)$$

i.e. the line with equation

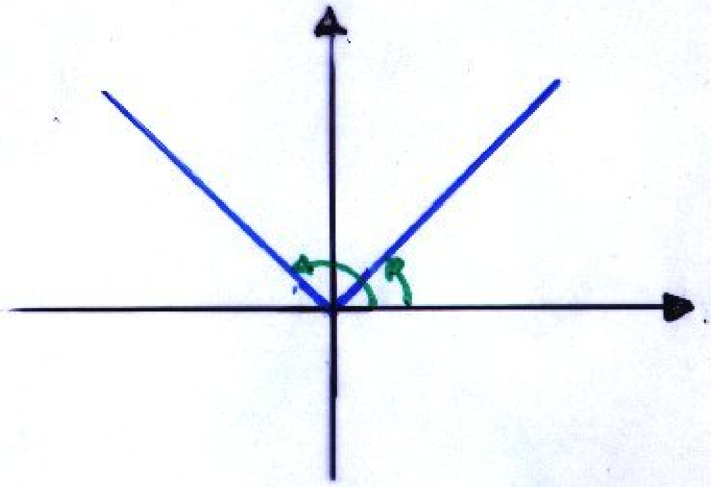
$$x_1 - \eta x_2 = 0 .$$

Similarly, the rank 1 factorisation

$J = \prod_{m \in I} (x_1 - \eta_m x_2)$ corresponds to the

union of lines,

e.g. for $|I| = 2$



This suggests that

the branes with

$|I| > 1$ can be obtained as bound states

from the fundamental branes with $|I|=1$.

Indeed one can show that if $I_1 \cap I_2 = \emptyset$

$$\left(Q_{I_1} \oplus Q_{I_2}\right)^{f} \cong Q_{I_1 \cup I_2}$$

switch on 'tachyon'

conjugation by polynomial matrix U

whose inverse is also polynomial

Herbst et al

One can also interpret topological open string spectrum between these branes from this point of view: the number of fermions between D_1 and D_2 (i.e chiral primaries between D_1 and $\bar{D}_2$) is just the intersection number between the corresponding union of lines. (In particular there is therefore always such a tachyon between Q_{I_1} and Q_{I_2} for $I_1 \cap I_2 = \emptyset$!)

We have also determined the topological spectrum between these rank 1 factorisations and the tensor product factorisations.

(cf. Diaconescu et al.)

- same number of bosons and fermions (no RR charge)
- at least for $|I|=1$ independent of choice of η_m.

This also fits together with the fact that one can obtain all tensor product factorisation from the rank 1 factorisations. In particular

$[L_1=0, L_2=0 \text{ tensor product}]$

$$\cong \left(\left[J = (x_1 - \eta_n x_2) \right] \oplus \left[J = \prod_{m \neq n} (x_1 - \eta_m x_2) \right] \right)^{\hat{f}}$$

antibranes of one another, i.e. $E \leftrightarrow J$.

So what is interpretation of these rank 1 factorisations in terms of the conformal field theory description?

At least some of them correspond to

Permutation branes

Recknagel
(MRG, Schäfer-Nameki)

$$\left(L_n^{(1)} - \bar{L}_{-n}^{(2)}\right) \| B \rangle\rangle = \left(L_n^{(2)} - \bar{L}_{-n}^{(1)}\right) \| B \rangle\rangle = 0$$

$$\left(J_n^{(1)} - \bar{J}_{-n}^{(2)}\right) \| B \rangle\rangle = \left(J_n^{(2)} - \bar{J}_{-n}^{(1)}\right) \| B \rangle\rangle = 0$$

$$\left(G_r^{\pm(1)} + i\eta\, \bar{G}_{-r}^{\pm(2)}\right) \| B \rangle\rangle = \left(G_r^{\pm(2)} + i\eta\, \bar{G}_{-r}^{\pm(1)}\right) \| B \rangle\rangle = 0$$

that also preserve diagonal $N=2$ algebra!

Such permutation branes can be explicitly constructed

$$\| L, M, S_1, S_2 \rangle\rangle = \frac{1}{2\sqrt{2}} \sum_{l, m, s_1, s_2} \frac{S_{Ll}}{S_{0l}} \times$$

$$\times\, e^{i\pi M m / k+2}\, e^{-i\pi (S_1 s_1 - S_2 s_2)/2}\, | [l, m, s_1] \otimes [l, -m, -s_2] \rangle\rangle$$

where the Ishibashi state

$$| [l, m, s_1] \otimes [l, -m, -s_2] \rangle\rangle$$

$$\in \left([l, m, s_1] \otimes [l, -m, -s_2] \right) \otimes \left(\overline{[l, m, s_2]} \otimes \overline{[l, -m, -s_1]} \right)$$

related by gluing condition!

Here $L+M$, $S_1 - S_2$ are even, and we have

$$\| L, M, S_1, S_2 \rangle\rangle = \| L, M, S_1 + 2, S_2 + 2 \rangle\rangle$$

$$= \| k - L, M + k + 2, S_1 + 2, S_2 \rangle\rangle .$$

In particular we have checked that

- this identification reproduces correctly the topological open string spectrum.

- the RR charges of these boundary states are compatible with the two classes of topological flows. (In CFT can also determine direction of flow, i.e. stability, etc.)

While not all rank 1 factorisations can be accounted for in terms of the permutation branes, this is at least possible for the fundamental rank 1 factorisations (with $|I|=1$).

Thus the permutation branes account for all the charges since all rank 1 factorisations can be obtained as bound states of the fundamental ones.

4. Applications to Gepner models

Recall that Diaconescu had identified

D0-brane on quintic with matrix

factorisation

$$E: \quad x_1 - \eta x_2, \; x_3, \; x_4, \; x_5$$

Above analysis now predicts that

corresponding Gepner brane is

$$\text{permutation}_{12} \otimes \text{tensor}_{3,4,5}$$

(Recknagel)

[Note: know CFT identification of

matrix factorisation with $|I|=1$.]

For the quintic, the RS-branes generate the vector space of charges, and their large volume geometrical interpretation is known.

Thus can deduce geometrical interpretation of above boundary state by calculating the relative Witten index (intersection matrix) between above boundary state and RS-branes:

confirms D0-brane !

Actually, the above D0-brane boundary states (together with their images under the 'Gepner monodromy') do not generate the full RR-lattice either (but only sublattice of index 5).

To generate full RR-lattice need also the D2-branes

$$
\begin{array}{lcr}
x_1 - \eta x_2 & & \text{permutation } (12) \\
x_3 - \eta x_4 & \longleftrightarrow & \text{permutation } (34) \\
x_5 & & \text{tensor } (5)
\end{array}
$$

Terminology:

- vector space of RR charges (B-typ)

$$\cong H^{\text{even}}(CY)$$

- RR-lattice is lattice of D-brane charges in above vector space.

The full RR-lattice is self-dual with respect to the intersection form.

Thus the quintic is an example of a Gepner model where

RS-branes generate vector space of RR charges

permutation branes generate full RR lattice.

In general, however, the RS-branes do not generate even the full vector space of RR-charges.

Caviezel, Fredenhagen, MRG

1 Models that have RR ground states that couple to tensor product branes or permutation branes

1.1 Lattices generated by RS branes

#	CY - hypersurface	Gepner model	RR	maximal rank	submatrix with Det= 1
6	$\mathbb{P}_{(1,1,1,2,5)}[10]$	(8, 8, 8, 3, 0)	4	I_{RS}	I_{RS}
18	$\mathbb{P}_{(1,1,1,6,9)}[18]$	(16, 16, 16, 1, 0)	6	I_{RS}	I_{RS}
27	$\mathbb{P}_{(1,1,2,8,12)}[24]$	(22, 22, 10, 1, 0)	8	I_{RS}	I_{RS}
48	$\mathbb{P}_{(1,2,3,12,18)}[36]$	(34, 16, 10, 1, 0)	12	I_{RS}	I_{RS}
85	$\mathbb{P}_{(2,3,6,22,33)}[66]$	(31, 20, 9, 1, 0)	20	I_{RS}	I_{RS}
92	$\mathbb{P}_{(1,1,12,28,42)}[84]$	(82, 82, 5, 1, 0)	24	I_{RS}	I_{RS}
113	$\mathbb{P}_{(1,2,18,42,63)}[126]$	(124, 61, 5, 1, 0)	36	I_{RS}	I_{RS}
117	$\mathbb{P}_{(1,12,13,52,78)}[156]$	(154, 11, 10, 1, 0)	48	I_{RS}	I_{RS}
120	$\mathbb{P}_{(1,3,24,56,84)}[168]$	(166, 54, 5, 1, 0)	48	I_{RS}	I_{RS}
126	$\mathbb{P}_{(2,3,30,70,105)}[210]$	(103, 68, 5, 1, 0)	60	I_{RS}	I_{RS}
135	$\mathbb{P}_{(1,7,48,112,168)}[336]$	(334, 46, 5, 1, 0)	96	I_{RS}	I_{RS}
137	$\mathbb{P}_{(2,7,54,126,189)}[378]$	(187, 52, 5, 1, 0)	108	I_{RS}	I_{RS}
139	$\mathbb{P}_{(3,7,60,140,210)}[420]$	(138, 58, 5, 1, 0)	120	I_{RS}	I_{RS}
147	$\mathbb{P}_{(1,42,258,602,903)}[1806]$	(1804, 41, 5, 1, 0)	504	I_{RS}	I_{RS}

1.2 Lattices generated by RS- and permutation-branes

#	CY - hypersurface	Gepner model	RR	maximal rank	submatrix with Det= 1
1	$\mathbb{P}_{(1,1,1,1,1)}[5]$	(3, 3, 3, 3, 3)	4	I_{RS}	$I_{(12)(34)}$
2	$\mathbb{P}_{(1,1,1,1,2)}[6]$	(4, 4, 4, 4, 1)	4	I_{RS}	$I_{(12)(34)}$
3	$\mathbb{P}_{(1,1,1,1,4)}[8]$	(6, 6, 6, 6, 0)	4	I_{RS}	$I_{(12)(34)}$
4	$\mathbb{P}_{(1,1,2,2,2)}[8]$	(6, 6, 2, 2, 2)	6	I_{RS}	$I_{(12)(34)}$
7	$\mathbb{P}_{(1,1,1,3,6)}[12]$	(10, 10, 10, 2, 0)	8	$I_{(45)}$	$I_{(12)(45)}$
8	$\mathbb{P}_{(1,1,2,2,6)}[12]$	(10, 10, 4, 4, 0)	6	I_{RS}	$I_{(12)(34)}$
11	$\mathbb{P}_{(1,2,2,3,4)}[12]$	(10, 4, 4, 2, 1)	6	I_{RS}	$I_{(23)}$
12	$\mathbb{P}_{(1,2,3,3,3)}[12]$	(10, 4, 2, 2, 2)	8	I_{RS}	$I_{(\hat{1}2)(34)}$
14	$\mathbb{P}_{(1,2,2,2,7)}[14]$	(12, 5, 5, 5, 0)	6	I_{RS}	$I_{(23)}$
16	$\mathbb{P}_{(1,3,3,3,5)}[15]$	(13, 3, 3, 3, 1)	8	I_{RS}	$I_{(23)}$
17	$\mathbb{P}_{(1,1,2,4,8)}[16]$	(14, 14, 6, 2, 0)	10	$I_{(45)}$	$I_{(12)(45)}$
19	$\mathbb{P}_{(1,2,3,3,9)}[18]$	(16, 7, 4, 4, 0)	8	I_{RS}	$I_{(34)}$
21	$\mathbb{P}_{(2,2,2,3,9)}[18]$	(7, 7, 7, 4, 0)	10	$I_{(45)}$	$I_{(12)(45)}$
23	$\mathbb{P}_{(1,2,2,5,10)}[20]$	(18, 8, 8, 2, 0)	14	$I_{(45)}$	$I_{(23)(45)}$
24	$\mathbb{P}_{(1,4,5,5,5)}[20]$	(18, 3, 2, 2, 2)	12	I_{RS}	$I_{(34)}$
30	$\mathbb{P}_{(1,2,3,6,12)}[24]$	(22, 10, 6, 2, 0)	14	$I_{(45)}$	$I_{(\hat{1}2)(45)}$
31	$\mathbb{P}_{(1,3,4,4,12)}[24]$	(22, 6, 4, 4, 0)	14	$I_{(25)}$	$I_{(25)(34)}$
34	$\mathbb{P}_{(2,3,3,4,12)}[24]$	(10, 6, 6, 4, 0)	14	$I_{(45)}$	$I_{(23)(45)}$
37	$\mathbb{P}_{(1,2,4,7,14)}[28]$	(26, 12, 5, 2, 0)	18	$I_{(45)}$	$I_{(\hat{1}2)(45)}$
38	$\mathbb{P}_{(1,1,3,10,15)}[30]$	(28, 28, 8, 1, 0)	12	$I_{(35)}$	$I_{(12)(35)}$
39	$\mathbb{P}_{(1,2,2,10,15)}[30]$	(28, 13, 13, 1, 0)	10	I_{RS}	$I_{(23)}$
41	$\mathbb{P}_{(1,3,5,6,15)}[30]$	(28, 8, 4, 3, 0)	16	$I_{(35)}$	$I_{(\hat{1}2)(35)}$
43	$\mathbb{P}_{(2,2,5,6,15)}[30]$	(13, 13, 4, 3, 0)	16	$I_{(35)}$	$I_{(12)(35)}$
44	$\mathbb{P}_{(2,3,5,5,15)}[30]$	(13, 8, 4, 4, 0)	16	$I_{(25)}$	$I_{(34)(25)}$
52	$\mathbb{P}_{(2,3,4,9,18)}[36]$	(16, 10, 7, 2, 0)	22	$I_{(45)}$	$I_{(\hat{1}3)(45)}$

In some of these examples,

the permutation branes generate

the full vector space of RR charges

(and sometimes they also generate

the full RR lattice)

but sometimes even they

do not ...

2 Models that have RR ground states that do not couple to tensor product branes or permutation branes

2.1 Lattices generated by constructions with different values of M

#	CY - hypersurface	Gepner model	RR	maximal rank	submatrix with Det= 1
36	$\mathbb{P}_{(3,3,4,6,8)}[24]$	$(6,6,4,2,1)$	16	$I_{(\widetilde{35});0,2}$	$I_{(12)(\widetilde{35});0,2}$
42	$\mathbb{P}_{(1,3,6,10,10)}[30]$	$(28,8,3,1,1)$	40	$I_{(\widetilde{23})(45);0,2,4,6;0,2,4}$	$I_{(\widetilde{23})(45);0,2,4,6;0,2,4}$
46	$\mathbb{P}_{(3,5,6,6,10)}[30]$	$(8,4,3,3,1)$	32	$I_{(34)(\widetilde{25});0,2,4,6;0,2,4}$	$I_{(34)(\widetilde{25});0,2,4,6;0,2,4}$
47	$\mathbb{P}_{(1,1,4,12,18)}[36]$	$(34,34,7,1,0)$	16	$I_{(\widetilde{34});0,2}$	$I_{(12)(\widetilde{34});0,2}$
53	$\mathbb{P}_{(2,4,9,9,12)}[36]$	$(16,7,2,2,1)$	40	$I_{(\widetilde{25})(34);0,2,4}$	$I_{(\widetilde{25})(34);0,2,4}$
66	$\mathbb{P}_{(1,3,12,16,16)}[48]$	$(46,14,2,1,1)$	54	$I_{(\widetilde{23})(45);0,2,4}$	$I_{(\widetilde{23})(45);0,2,4}$
68	$\mathbb{P}_{(1,2,6,18,27)}[54]$	$(52,25,7,1,0)$	22	$I_{(\widetilde{34});0,2}$	$I_{(\widetilde{34});0,2}$
71	$\mathbb{P}_{(1,4,5,20,30)}[60]$	$(58,13,10,1,0)$	24	$I_{(\widetilde{24});0,2}$	$I_{(\widetilde{13})(\widetilde{24});0,2}$
78	$\mathbb{P}_{(2,3,15,20,20)}[60]$	$(28,18,2,1,1)$	64	$I_{(\widetilde{23})(45);0,2,4}$	$I_{(\widetilde{23})(45);0,2,4}$
79	$\mathbb{P}_{(3,3,4,20,30)}[60]$	$(18,18,13,1,0)$	24	$I_{(\widetilde{34});0,2}$	$I_{(12)(\widetilde{34});0,2}$
81	$\mathbb{P}_{(3,5,12,20,20)}[60]$	$(18,10,3,1,1)$	64	$I_{(\widetilde{13})(45);0,2,4,6;0,2,4}$	$I_{(\widetilde{13})(45);0,2,4,6;0,2,4}$
82	$\mathbb{P}_{(3,10,12,15,20)}[60]$	$(18,4,3,2,1)$	32	$I_{(\widetilde{25});0,2}$	$I_{(\widetilde{25});0,2}$
84	$\mathbb{P}_{(4,6,15,15,20)}[60]$	$(13,8,2,2,1)$	56	$I_{(\widetilde{15})(34);0,2,4}$	$I_{(\widetilde{15})(34);0,2,4}$
89	$\mathbb{P}_{(1,3,8,24,36)}[72]$	$(70,22,7,1,0)$	32	$I_{(\widetilde{34});0,2}$	$I_{(\widetilde{12})(\widetilde{34});0,2}$
98	$\mathbb{P}_{(3,4,7,28,42)}[84]$	$(26,19,10,1,0)$	34	$I_{(15)(\widetilde{24});0,2}$	$I_{(15)(\widetilde{24});0,2}$
100	$\mathbb{P}_{(3,4,21,28,28)}[84]$	$(26,19,2,1,1)$	84	$I_{(\widetilde{13})(45);0,2,4}$	$I_{(\widetilde{13})(45);0,2,4}$
102	$\mathbb{P}_{(2,3,10,30,45)}[90]$	$(43,28,7,1,0)$	38	$I_{(\widetilde{34});0,2}$	$I_{(\widetilde{34});0,2}$
105	$\mathbb{P}_{(1,4,20,25,50)}[100]$	$(98,23,3,2,0)$	68	$I_{(\widetilde{23})(45);0,2,4,6;0,2}$	$I_{(\widetilde{23})(45);0,2,4,6;0,2}$
123	$\mathbb{P}_{(1,9,20,60,90)}[180]$	$(178,18,7,1,0)$	86	$I_{(25)(\widetilde{34});0,2}$	$I_{(25)(\widetilde{34});0,2}$
124	$\mathbb{P}_{(4,5,36,45,90)}[180]$	$(43,34,3,2,0)$	112	$I_{(\widetilde{13})(45);0,2,4,6;0,2}$	$I_{(\widetilde{13})(45);0,2,4,6;0,2}$
125	$\mathbb{P}_{(2,9,22,66,99)}[198]$	$(97,20,7,1,0)$	92	$I_{(25)(\widetilde{34});0,2}$	$I_{(25)(\widetilde{34});0,2}$
127	$\mathbb{P}_{(6,14,15,70,105)}[210]$	$(33,13,12,1,0)$	88	$I_{(\widetilde{24})(35);0,2}$	$I_{(\widetilde{24})(35);0,2}$
128	$\mathbb{P}_{(1,8,27,72,108)}[216]$	$(214,25,6,1,0)$	98	$I_{(\widetilde{24})(35);0,2}$	$I_{(\widetilde{24})(35);0,2}$
131	$\mathbb{P}_{(3,8,33,88,132)}[264]$	$(86,31,6,1,0)$	116	$I_{(\widetilde{24})(35);0,2}$	$I_{(\widetilde{24})(35);0,2}$
132	$\mathbb{P}_{(1,6,42,98,147)}[294]$	$(292,47,5,1,0)$	96	$I_{(\widetilde{23});0,\ldots,10}$	$I_{(\widetilde{23});0,\ldots,10}$
134	$\mathbb{P}_{(3,22,30,110,165)}[330]$	$(108,13,9,1,0)$	120	$I_{(\widetilde{24});0,2}$	$I_{(\widetilde{24});0,2}$
136	$\mathbb{P}_{(1,18,38,114,171)}[342]$	$(340,17,7,1,0)$	144	$I_{(\widetilde{34});0,2}$	$I_{(\widetilde{34});0,2}$
141	$\mathbb{P}_{(6,7,78,182,273)}[546]$	$(89,76,5,1,0)$	168	$I_{(\widetilde{13});0,\ldots,10}$	$I_{(\widetilde{13});0,\ldots,10}$
143	$\mathbb{P}_{(1,14,90,210,315)}[630]$	$(628,43,5,1,0)$	192	$I_{(\widetilde{24});0,2}$	$I_{(\widetilde{24});0,2}$
144	$\mathbb{P}_{(3,14,102,238,357)}[714]$	$(236,49,5,1,0)$	216	$I_{(\widetilde{24});0,2}$	$I_{(\widetilde{24});0,2}$

2.2 Lattices generated by constructions with different factorisations

#	CY - hypersurface	Gepner model	RR	maximal rank	submatrix with Det= 1
114	$\mathbb{P}_{(1,6,14,42,63)}[126]$	$(124,19,7,1,0)$	58	$I_{(\widetilde{34});0,2-(\widetilde{24});0,2}$	$I_{(\widetilde{34});0,2-(\widetilde{24});0,2}$

For A-type Gepner models

$$W = \sum_i x_i^{d_i}$$

at least, we have now identified the matrix factorisations that generate the full RR-lattice in all cases.

Caviezel, Fredenhagen, MRG

These involve only one small generalisation of the above constructions:

This arises when we have

a pair of factors

$$x_1^{d_1} + x_2^{d_2} \qquad (d_1 \neq d_2)$$

for which d_1 and d_2 are not coprime,

$(d_1, d_2) = d > 1$. Then we get

generalised permutation factorisations

from

$$x_1^{d_1} + x_2^{d_2} = \prod_{\eta} (x_1^{n_1} - \eta\, x_2^{n_2})$$

$d_i = n_i \cdot d$

η: d'th roots of -1.

We are currently trying to construct the corresponding conformal field theory D-branes.

Note that situation is analogeous to WZW models where, in general, the 'symmetric' constructions do not account for all the K-theory charges.

(MRG, Gannon, Roggenkamp)

Also, for product groups, e.g.

$$su(2)_{k_1} \otimes su(2)_{k_2}$$

Fredenhagen & Quella have found that the K-theory charges are not accounted for in terms of the usual tensor & permutation branes whenever $k_1 \neq k_2$

$$(k_1, k_2) = k > 1$$

Need generalised permutation construction!

5. Conclusions

- studied relation between matrix factorisations and the CFT description of B-type branes for product of two minimal models (with same k)

- thus identified certain boundary states with D0/D2-branes for a number of models (including quintic).

- for all A-type Gepner models have found factorisations that generate full RR-lattice.

- these factorisations correspond to

  tensor product (RS)

  and

  permutation (transposition) branes

  but some involve also

  'generalised permutations'

Open problems

- CFT construction for 'generalised permutations' and 'non-consecutive' rank 1 factorisations.
- structure of factorisations vs. gluing conditions
- direct geometric interpretation of factorisations [Orlov]
- repeat analysis for D-model [Brunner, MRG]
- superpotentials, A_∞-structure, ...