LMS DURHAM SYMPOSIUM

GENERALISED GEOMETRY & DUALITY

A GEOMETRY FOR NON-GEOMETRIC STRING BACKGROUNDS

hep-th/0406102

STRINGS AND GEOMETRY

GEOMETRIC BACKGROUND

MANIFOLD + TENSORS

g_{ij}, Φ, H_{ijk}, ...

STRINGS MOVE IN GEOMETRY

BUT

DUALITIES MIX

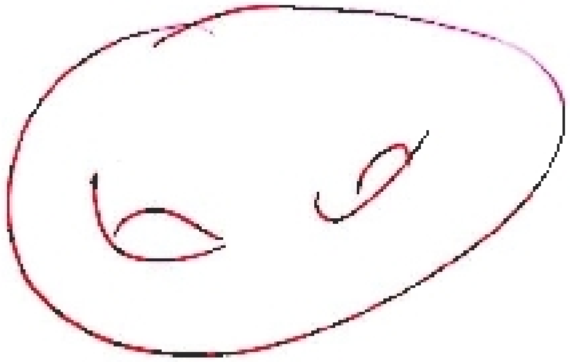
MOMENTA (GEOMETRIC) AND

STRING WINDING MODES

\+ BRANE WRAPPING MODES

NOT GEOMETRIC OR EXTRA "BRANEY" DIMENSIONS?

BACKGROUNDS WHERE MOMENTA "GRADUALLY" CHANGED TO BRANE MODES?



GEOMETRIC BACKGROUND

$M(g_{ij}, H_{ijk}, \Phi, \sim)$

$\Downarrow$ DUALITY

(T, U, MIRROR ~)

??

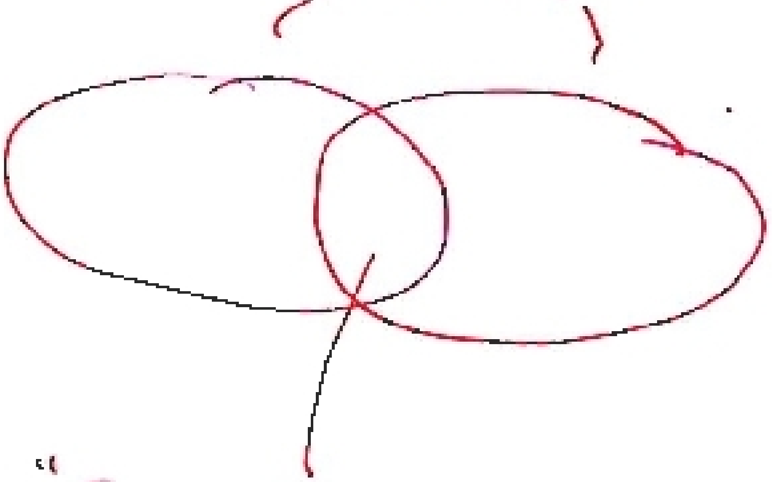
<u>USUALLY</u> TO ANOTHER GEOMETRIC BACKGROUND

BUT SOMETIMES NOT

- OBSTRUCTION TO DUALITY?
- NON-GEOMETRIC BACKGROUND?

GEOMETRIC BACKGROUND

PATCHES

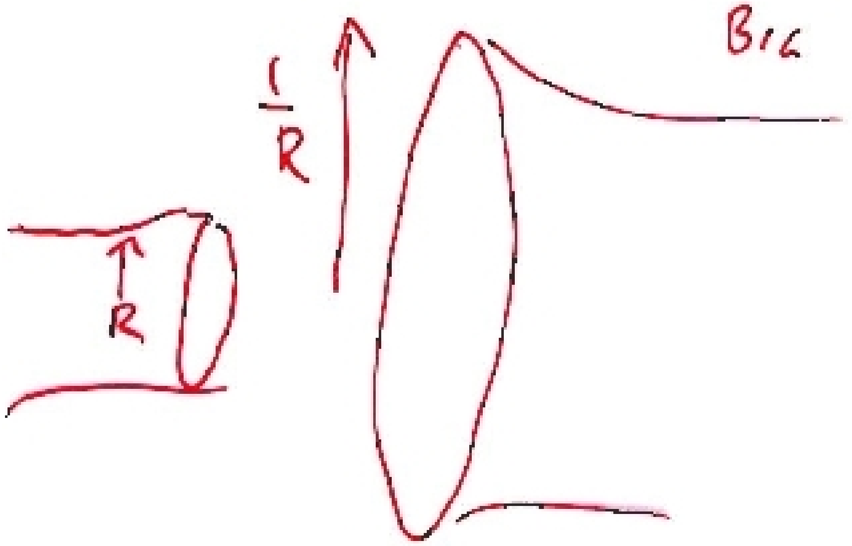


"GLUE" USING DIFFEOS,
GAUGE TRANSFORMATIONS —
ALL GAUGE SYMMS

T-FOLD

GLUE USING T-DUALITIES DISCRETE GAUGE SYMS

GLUE MOMENTUM MODES TO
WINDING MODES,
BIG S^1 TO SMALL S^1



GENERALISED GEOMETRY

(HITCHIN)

"DOUBLE" TANGENT SPACE

$T \oplus T^*(M)$ $\quad$ $O(d,d)$ STRUCTURE GROUP

$$U^I = \begin{pmatrix} X^i \\ \xi_i \end{pmatrix}$$

$O(d,d)$ METRIC $\quad$ $L_{IJ}\, dU^I dU^J = 2\, dX^i d\xi_i$

$$L = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \qquad (\, , \,)$$

GENERALISED METRIC $\quad G_{ij}, B_{ij}$

$$T \oplus T^* \simeq V \oplus V^\perp$$

$L|_V$ +ve def $\quad$ $L|_{V^\perp}$ −ve def

$$\Rightarrow \mathcal{H} = L|_V - L|_{V^\perp} \quad \text{+ve def}$$

$$\mathcal{H}_{IJ} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

$$S^I{}_J = L^{IK}\mathcal{H}_{KJ} \qquad S^2 = \mathbb{1}, \quad S|_V = \mathbb{1}, \; S|_{V^\perp} = -\mathbb{1}$$

GENERALISED GEOMETRY (HITCHIN)

"DOUBLE" TANGENT SPACE

$T \oplus T^*(M)$ $\quad O(d,d)$ STRUCTURE GROUP

PRESERVES $O(d,d)$ METRIC $L_{IJ} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$G, B \longleftrightarrow \mathcal{H}_{IJ} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$

GENERALISED METRIC

$O(d,d) \longrightarrow O(d) \times O(d)$

$S = L^{-1}\mathcal{H} \qquad S^2 = \mathbb{1}$

(2,2) GEOMETRY

GENERALISED KAHLER

COMPLEX STRUCTURES $\longleftrightarrow$

J_+, J_- BIHERMITIAN G

$\nabla^{\pm} J_{\pm} = 0 \qquad \nabla^{\pm} = \nabla \pm \frac{1}{2} H$

$\mathbb{J}^I{}_J, \ \mathbb{J}^2 = -\mathbb{1}$

$[S, \mathbb{J}] = 0$

$O(d,d) \to U\left(\frac{d}{2}, \frac{d}{2}\right) \to U\left(\frac{d}{2}\right) \times U\left(\frac{d}{2}\right)$

(GATES HULL ROCEK)

[GUALTIERI]

GENERALISED GENERALISED GEOMETRY

T-FOLDS : DOUBLE MANIFOLD
($\Rightarrow$ DOUBLING OF TANGENT SPACE)

T^n FIBRATION

$$\begin{tikzcd} M_d \arrow[r] & T^n \arrow[d] \\ & N_{d-n} \end{tikzcd}$$

TRANSITION FUNCTIONS
$SL(n, \mathbb{Z}) \times \ldots$

DOUBLED SPACE

$$\begin{tikzcd} \mathcal{M}_{2d} \arrow[r] & T^{2n} \arrow[d] \\ & T^*N \end{tikzcd}$$

FIBRES
$T^n \times \tilde{T}^n$

TRANSITION FUNCTIONS $O(n,n;\mathbb{Z}) \times \ldots$

$$T\mathcal{M} \simeq T \oplus T^*(M)$$

H_{IJ} METRIC ON $\mathcal{M}$

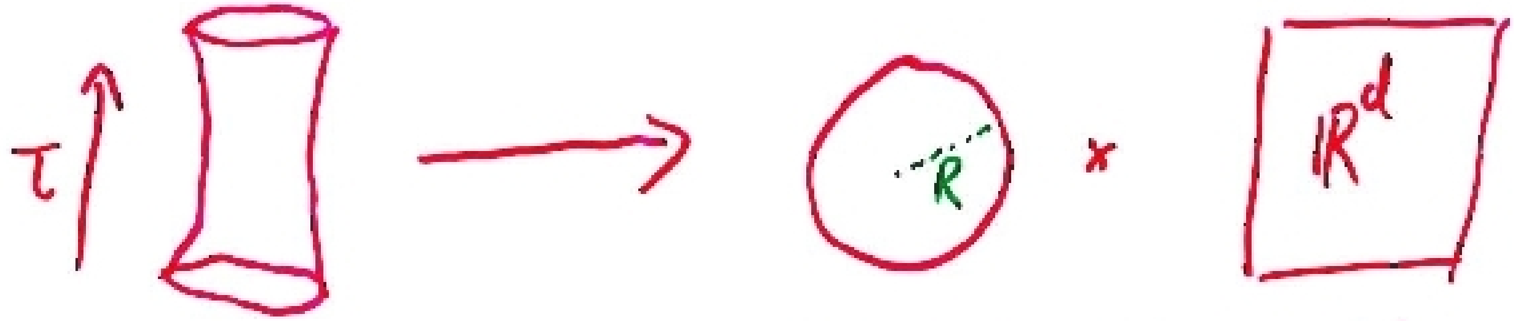
PROJECTS ONTO

$$\begin{tikzcd} \bar{\mathcal{M}}_{d+n} \arrow[r] & T^{2n} \arrow[d] \\ & N \end{tikzcd}$$

WITH ONLY FIBRES T^n
DOUBLED

POLARISATION: CHOOSE "PHYSICAL" $M_d \subset \mathcal{M}_{2d}$

T-Duality: Strings on $S^1 \times \mathbb{R}^d$



$\sigma \sim \sigma + 2\pi$ $\quad X \sim X + 2\pi R$ $\quad Y$

$$(\partial_\tau^2 - \partial_\sigma^2)\, X(\sigma,\tau) = 0$$

$$X = x + p\tau + w\sigma + \sum_n a_n e^{in(\sigma+\tau)} + \tilde{a}_n e^{in(\sigma-\tau)}$$

Momentum $p = \frac{m}{R}$ $\quad m \in \mathbb{Z}$ $\quad (e^{ipX}$ ok$)$

$w = nR$ $\quad n \in \mathbb{Z}$ Winding No.

$$(\text{Mass})^2 \sim \left(\frac{m}{R}\right)^2 + (nR)^2 + \sum a_n a_{-n} + \tilde{a}_n \tilde{a}_{-n}$$

T-Duality $\quad X \rightarrow \tilde{X}$ $\quad dX = * d\tilde{X}$

$\tilde{m} = n$ $\quad \tilde{n} = m$ $\quad \tilde{R} = 1/R$

Momentum $\longleftrightarrow$ Winding

$R \longleftrightarrow 1/R$

Dual Formulation on $\tilde{S}^1$, $\tilde{R} = 1/R$

T-DUALITY ON S^1-BUNDLES

IF SPACETIME ADMITS FREE S^1 ACTION (ISOMETRY)

$$\begin{array}{lccc} x & S^1 & \longrightarrow & M \\ & & \pi & \downarrow \\ y^i & & & N \end{array}$$

$M(g_{\mu\nu}, B_{\mu\nu}, \Phi)$

$H = dB \in H^3(M, \mathbb{Z})$

H_3 CLOSED 3-FORM, B_2 LOCAL (GERB)

CHERN CLASS $\quad c_1(M) \in H^2(N, \mathbb{Z})$

H-CLASS $\quad \pi_*(H) \in H^2(N, \mathbb{Z})$

$\frac{1}{2} dy^i \wedge dy^j H_{ijx}$

T-DUAL

$$\begin{array}{lccc} \tilde{x} & S^1 & \longrightarrow & \tilde{M} \\ & & \tilde{\pi} & \downarrow \\ y^i & & & N \end{array}$$

$\tilde{M}(\tilde{g}, \tilde{B}, \tilde{\Phi})$

$g_{xx}(y) = R^2(y) \qquad \tilde{g}_{\tilde{x}\tilde{x}} = \tilde{R}^2 = 1/R^2$

$c_1(M) = \tilde{\pi}_*(\tilde{H})$

$c_1(\tilde{M}) = \pi_*(H)$

CHERN No. $\updownarrow$ H-FLUX

STRINGS ON $T^n \times \mathbb{R}^d$

T^n : X^i $i = 1, \dots, n$

MODULI G_{ij} B_{ij} (CONSTANT)

$O(n,n;\mathbb{Z})$ T-DUALITY SYMMETRY

i) $GL(n;\mathbb{Z})$ ACTS GEOMETRICALLY
MAPPING CLASS GROUP

ii) $B \to B + \theta$ $\theta \in \mathbb{Z}$

iii) $R \to 1/R$ EACH S^1 $(\mathbb{Z}_2)$

$$E_{ij} = G_{ij} + B_{ij}$$

$$E \longrightarrow (aE+b)(cE+d)^{-1}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(n,n;\mathbb{Z}) \text{ ; PRES} \begin{pmatrix} 0 & \mathbb{1}_n \\ \mathbb{1}_n & 0 \end{pmatrix}$$

$$\begin{pmatrix} p_i \\ w^i \end{pmatrix} = \begin{pmatrix} \text{MOMENTUM} \\ \text{WINDING No} \end{pmatrix} \quad \underset{\sim}{2n} \text{ OF } O(n,n)$$

T^n FIBRATIONS

$$\begin{array}{ccc} T^n & \longrightarrow & M \\ & & \downarrow \\ & & N \end{array}$$

$$(\underset{T^n}{\overset{\uparrow}{X^i}}, \underset{N}{\overset{\uparrow}{Y^m}})$$

$U(1)^n$ ISOMETRY

T^n MODULI $\quad E_{ij}(Y) = G_{ij} + B_{ij}$

$O(n,n;\mathbb{Z})$ T-DUALITY

$$E \longrightarrow (aE+b)(cE+d)^{-1}$$

$\left.\begin{array}{l} n \quad \text{CHERN Nos} \\ n \quad \text{H-FLUXES} \end{array}\right\} \in$ LATTICE $\Gamma^{n,n}$

$O(n,n;\mathbb{Z})$ ACTS ON LATTICE

$\left.\begin{array}{ll} n \text{ MOMENTA} & p_i \\ n \text{ WINDING Nos} & w^i \end{array}\right]$ NARAIN LATTICE $\tilde{\Gamma}^{n,n}$

T-DUALITY

GIVEN $M(g, B, \Phi)$ WITH T^n FIBRATION, ACTING WITH $O(n, n; \mathbb{Z})$ GIVES "NEW" STRING BACKGROUND GEOMETRY WITH <u>EQUIVALENT</u> <u>PHYSICS</u>

$GL(n, \mathbb{Z}) \subset O(n, n; \mathbb{Z})$ DIFFEOS

CHANGES GEOMETRY & TOPOLOGY

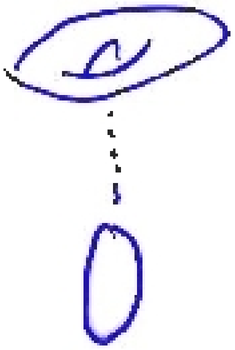
<u>PROBLEM</u>: SOMETIMES, APPLYING LOCAL RULES GIVE $(\tilde{g}, \tilde{B}, \hat{\phi})$ WHICH ARE <u>NOT</u> TENSOR FIELDS ON APPROPRIATE MANIFOLD $\widetilde{M}$

OBSTRUCTION?

T^3 H-FLUX $H = N \star \mathrm{Vol}$

$\Downarrow$ T-DUAL ON S^1

$T^2 \to S^1$



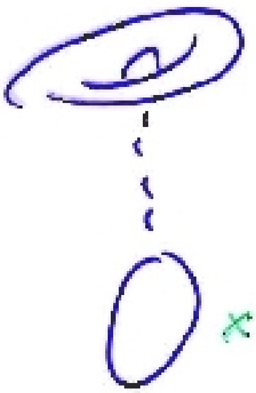
MONODROMY $\begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in SL(2,\mathbb{Z})$

$H = 0$

$\Downarrow$ T-DUAL ON S^1 FIBRE

T^2 y, z

S^1 x



$$ds^2 = \frac{1}{1+N^2x^2}\left(dy^2 + dz^2\right) + dx^2$$

$$B_{yz} = \frac{Nx}{1+N^2x^2}$$

BUT x PERIODIC ?

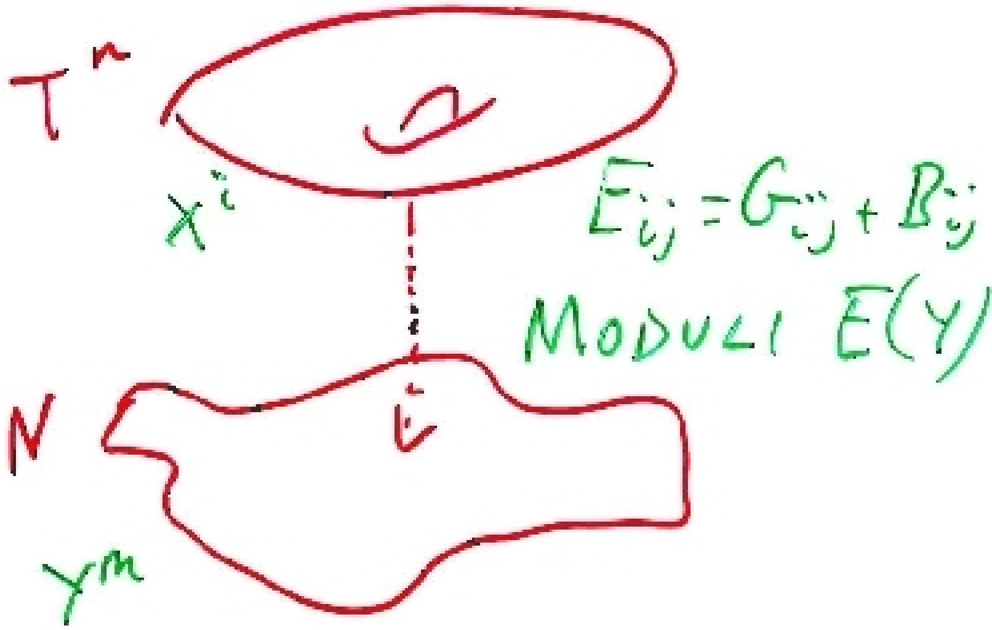
$$E(x+2\pi) = \frac{aE+b}{cE+d}$$

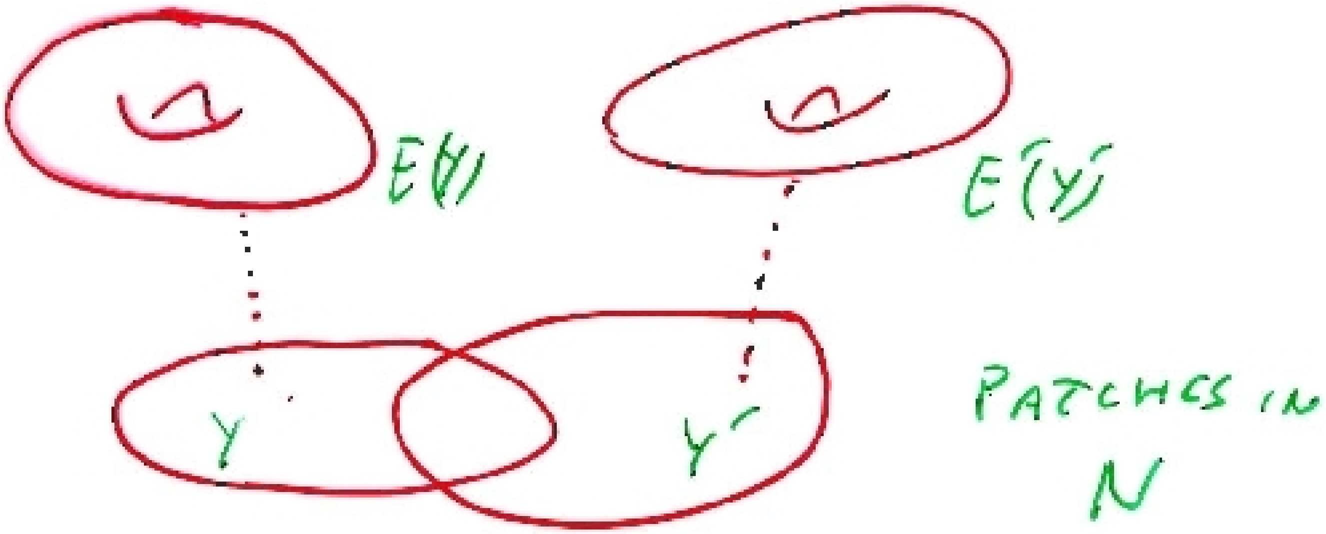
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(2,2;\mathbb{Z})$$

MONODROMY MIXES MOMENTUM + WINDING

TORUS BUNDLES

$$\mathcal{M} = T^n \downarrow N$$





GLUE T^n FIBRES WITH $h \in GL(n, \mathbb{Z})$

$$E' = h E h^t$$

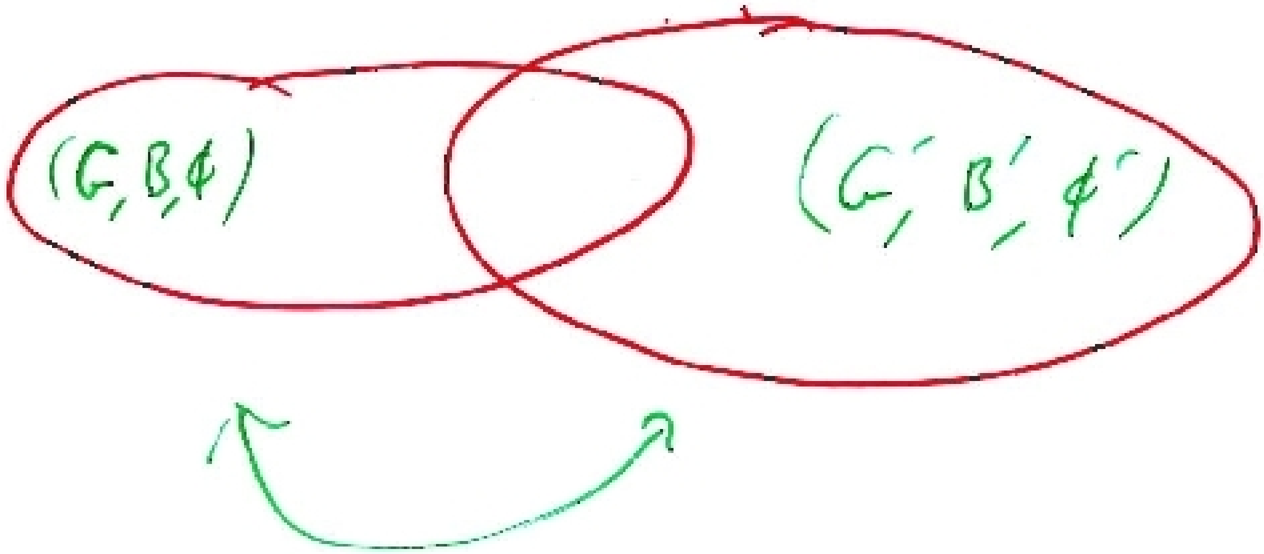
$\Downarrow$ T-DUALITY $g \in O(n,n;\mathbb{Z})$

$$h \to k = g h g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(n,n;\mathbb{Z})$$

$$E' = \frac{aE + b}{cE + d}$$

NOT TENSOR!

MANIFOLD



TRANSITION FNS:

DIFFEO + GAUGE TRANS

$$\Downarrow$$

"T-FOLD"

TRANS FUNCTIONS ARE T-DUALITIES

$$E' = \frac{aE + b}{cE + d} \qquad E_{ij} = G_{ij} + B_{ij}$$

MANY CONSISTENT NON-GEOMETRIC STRING BACKGROUNDS

NOT MANIFOLDS, BUT

T-FOLDS, U-FOLDS, ...

WITH TRANSITION FUNCTIONS IN

T-DUALITY, U-DUALITY, MIRROR MAPS...

CMH + DABOLKHAR

HELLERMAN MCGREEVY WILLIAMS

KACHRU SCHULZ TRIPATHY TRIVEDI

LOWE NASTASE RAMGOOLAM

FLOURNOY WECHT WILLIAMS

STRINGS ON T^n

$\Box \mathbb{X} = 0$

$$\mathbb{X} = X_L(\sigma+\tau) + X_R(\sigma-\tau)$$

$$X = X_L + X_R \qquad \tilde{X} = X_L - X_R$$

CONJ TO WINDING NO., T-DUAL OF X

$$\partial_\alpha X = \epsilon_\alpha{}^\beta \partial_\beta \tilde{X} \qquad \boxed{dX = * d\tilde{X}}$$

NEED "AUXILIARY" $\tilde{X}$:

1) VERTEX OPS $\quad e^{ik_L \cdot X_L}, \; e^{ik_R \cdot X_R}$

2) STRING FIELD THEORY

$$\Psi[X, \tilde{X}, a_n, \tilde{a}_n]$$

T-DUALITY : $O(n,n)$ ACTS ON $\begin{pmatrix} X^i \\ \tilde{X}_i \end{pmatrix}$

T-FOLDS $\begin{pmatrix} X' \\ \tilde{X}' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} X \\ \tilde{X} \end{pmatrix}$

MIXES $X, \tilde{X}$; NO GLOBAL WAY OF SEPARATING OUT "REAL SPACE" X

DOUBLED FORMALISM

- USE DOUBLED VARIABLES

$$\mathbb{X} = \begin{pmatrix} X \\ \tilde{X} \end{pmatrix} \in T^{2n}$$

- SELF DUALITY CONSTRAINT

$$dX \sim * d\tilde{X} + \cdots$$

½ D.O.FREEDOM

- $O(n,n)$ COVARIANT

CREMMER, JULIA, LIU, POPE

TSEYTLIN, CMH DUFF MAHARANA + SCHWARZ

T^{2n} BUNDLE:

$2n$ CHERN NOS c_1^I $O(n,n,\mathbb{Z})$ ACTS

$\longrightarrow$ T^n BUNDLE WITH

n CHERN NOS

n H- FLUXES

G_{ij}, B_{ij} MODULI PARAMETRISE

$$O(n,n)/O(n)\times O(n)$$

$\mathcal{V}(y) \in O(n,n)$ VIELBEIN

$$\mathcal{V} \longrightarrow k(y)\,\mathcal{V}\,g$$

$g \in O(n,n)$ $k \in O(n)\times O(n)$

CHOOSE $\mathcal{V} = \begin{pmatrix} e^t & 0 \\ -e^{-1}B & e^{-1} \end{pmatrix}$

$O(n,n)$ METRIC $L_{IJ} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$

+ve DEF METRIC $\mathcal{H} = \mathcal{V}^t\mathcal{V}$

$$\mathcal{H}_{IJ} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

$$\mathcal{H} \longrightarrow g^t\,\mathcal{H}\,g$$

$S^I{}_J \equiv L^{IK}\mathcal{H}_{KJ}$ $\underline{S^2 = \mathbb{1}}$

$$\mathbb{X}^I = \begin{pmatrix} X \\ \tilde{X} \end{pmatrix} \qquad \mathbb{P}^I = d\mathbb{X}^I$$

$$\mathcal{L} = \frac{1}{2} \mathcal{H}_{IJ}(Y)\, \mathbb{P}^I \wedge * \mathbb{P}^J + \mathbb{P}^I \wedge * J_I + \mathcal{L}(Y)$$

$$J_I = \mathcal{H}_{IJ}(A^J + * B^J)$$

$$A^I = A^I{}_m(Y)\, dY^m$$

$A^I{}_m$ CONNECTION ON T^{2n} BUNDLE

$$B = -SA$$

$$(\mathbb{P} + A) = S * (\mathbb{P} + A)$$

SELF-DUALITY CONSTRAINT

$O(n,n)$: $\mathcal{H} \to g^t \mathcal{H} g \qquad \mathbb{X} \to g^{-1} \mathbb{X}$

$J \to g^t J \qquad A \to g^{-1} A$

DOUBLED FORMALISM

DOUBLED FIBRE

$$\mathbb{X}^I = \begin{pmatrix} X \\ \tilde{X} \end{pmatrix}$$

y^m

$\bar{M}$

$O(n,n;\mathbb{Z}) \subset GL(2n;\mathbb{Z})$

"NON-GEOMETRIC" TRANSITION FNS

$\rightarrow$ T^{2n} BUNDLE OVER N

– STRING THEORY ON $\bar{M}$

$O(n,n)$ DUALITY COVARIANT

– CONSTRAINT : $\frac{1}{2}$'s $\mathbb{X}$ d.o.f.

$$d\mathbb{X} = S * d\mathbb{X} + \cdots$$

$$S^2 = \mathbb{1}, \quad S(Y) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + O(Y) \ldots$$

$$\rightarrow \quad dX \sim * d\tilde{X}$$

CHOOSE POLARIZATION

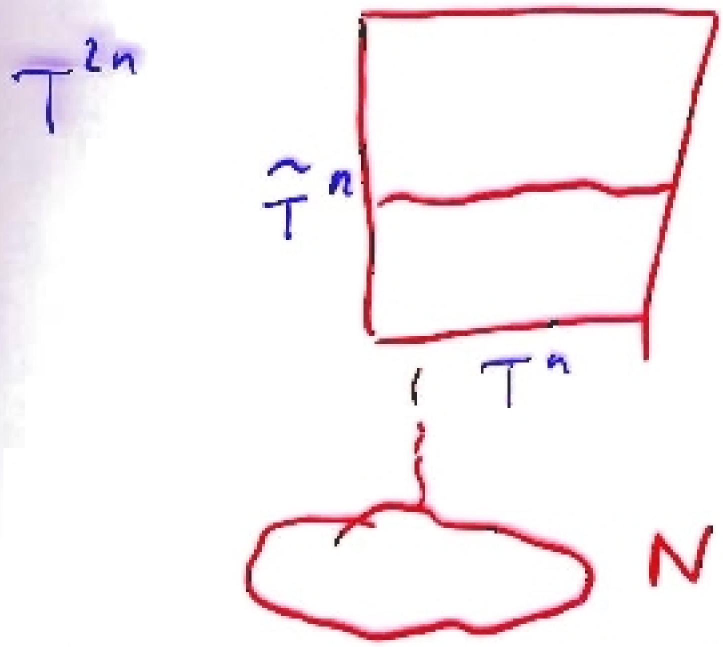
$$\mathbb{X}^I \longrightarrow \begin{cases} X^i & \text{— 'FUNDAMENTAL' SPACETIME} \\ \tilde{X}_i & \text{— AUXILIARY} \end{cases}$$

i.e. PICK $T^n \subset T^{2n}$ SPACETIME

$O(n,n)$ METRIC $\quad ds^2 = 2\,dX^i\,d\tilde{X}_i$

$\rightarrow$ T^n MAXIMAL NULL (ISOTROPIC)

'BREAK' $O(n,n;\mathbb{Z}) \rightarrow GL(n,\mathbb{Z})$



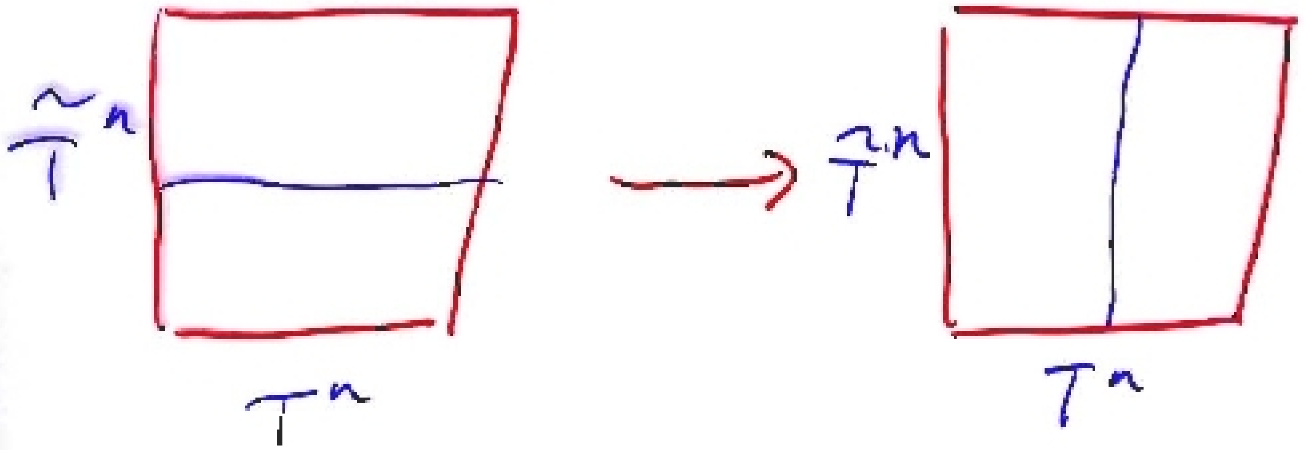
PICK "SECTION" TO GET SPACETIME

$$\begin{array}{c} T^n \\ \downarrow \\ N \end{array}$$

SOLVE $d\mathbb{X} = S \star d\mathbb{X} + \cdots$ IN TERMS OF $X \longrightarrow$ CONVENTIONAL FORMULATION

T-DUALITY CHANGES

POLARIZATION



T-DUALITY A SYMMETRY $\Rightarrow$

PHYSICS INDEPENDENT OF

POLARIZATION CHOICE

SPACETIME A 'BRANE'

IN DOUBLED SPACE

T-FOLD

BUNDLE

$$\begin{array}{c} T^{2n} \\ \downarrow \\ N \end{array}$$

TRANSITION FUNCTIONS

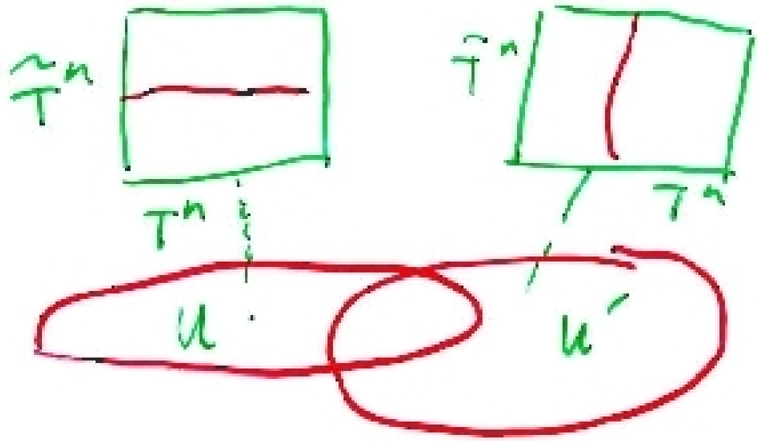
IN $O(n,n;\mathbb{Z}) \subset GL(2n,\mathbb{Z})$

LOCAL SPACETIME

$$\begin{array}{ccc} T^n & \subset & T^{2n} \\ \downarrow & & \downarrow \\ U & & U \end{array}$$

FOR EACH OPEN SET $U \subset N$:

PICK "SPACETIME" $T^n \times U$



T-DUALITY

TRANSITION

FNS

CHANGE POLARIZATION

FROM PATCH TO PATCH

GEOMETRIC BACKGROUND

$GL(n,\mathbb{Z})$

LOCAL SECTIONS PATCH TOGETHER TO GIVE SPACETIME SUBMANIFOLD

NON-GEOMETRIC

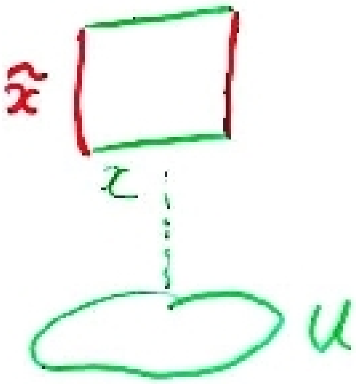
LOCAL SPACETIME PATCHES, BUT DON'T FIT TOGETHER TO MANIFOLD

UNIVERSAL BUNDLE

$$\begin{array}{cc} T^{2n} & X^I \\ \downarrow & \\ N & y \end{array}$$

CONTAINS ALL DUAL GEOMETRIES

LOCAL SECTION OVER PATCH $U \subseteq N$

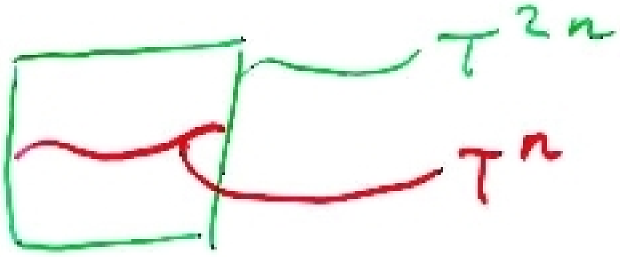


CHOOSE $T^n \subset T^{2n}$

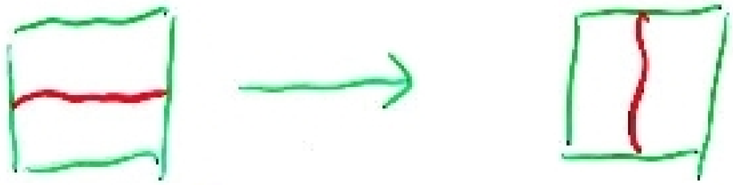
POLARIZATION $X^I \rightarrow \begin{pmatrix} x^i \\ \tilde{x}_i \end{pmatrix}$

PHYSICAL SPACE $\{x^i\}$

n- BRANE IN T^{2n} "BRANE-WORLD"



T-DUALITY CHANGES SECTION



TRANSITION FUNCTIONS : T- DUALITIES

IF THERE IS GLOBAL SECTION,

BRANE - WORLDS PATCH TO FORM SUBMANIFOLD

THEN THERE IS "GEOMETRIC" BACKGROUND

$$\begin{array}{c} T^n \\ \downarrow \\ N \end{array}$$

CHOOSE POLARIZATION

$$SL(n,\mathbb{Z}) \subset O(n,n\,\mathbb{Z})$$

$$\{x^i\} \subset \{X^I\}$$

$$T^n \subset T^{n,n}$$

MAXIMAL $\left\{\begin{array}{l}\text{NULL} \\ \text{ISOTROPIC}\end{array}\right.$ SUBSPACE

$O(n,n)$ METRIC $ds^2 = 2\,dx^i\,d\tilde{x}_i$

SUBSPACES ~ FORMS ~ SPINORS

MAXIMAL NULL SUBSPACE

$\simeq$ PURE SPINOR OF $O(n,n)$

OPEN STRINGS AND D-BRANES

IF X^i NEUMANN B.C.'S / DIR

T-DUAL $\tilde{X}_i$ DIRICHLET B.C.'S / NEU

$$2n \;\; X^I \longrightarrow \begin{cases} n & \text{DIRICHLET } X_D^i \\ n & \text{NEUMANN } X_N^i \end{cases}$$

$\{X_D\}$ MAXIMAL NULL SUBSPACE

$\{X_D\} \cap \{X^i\} \to$ D-BRANE

"PHYSICAL" POLARIZATION

EXAMPLE: $T^{3,3} \to N$, N HAS Dp-BRANE

$X_D^1 \quad X_D^2 \quad X_D^3 \quad X_N^1 \quad X_N^2 \quad X_N^3$ Dp-BRANE

$X_D^1 \quad X_D^2 \quad X_D^3 \quad X_N^1 \quad X_N^2 \quad X_N^3$ D(p+1)

$X_D^1 \quad X_D^2 \quad X_D^3 \quad X_N^1 \quad X_N^2 \quad X_N^3$ D(p+2)

$X_D^1 \quad X_D^2 \quad X_D^3 \quad X_N^1 \quad X_N^2 \quad X_N^3$ D(p+3)

- NO OBSTRUCTION TO T-DUALITY
- IN GENERAL GIVES T-FOLDS

LOCAL SPACETIME PATCHES
BUT NO GLOBAL SPACETIME MANIFOLD

- D-BRANES & SUSY INCORPORATED

- T-FOLDS MOMENTUM & WINDING MIX

U-FOLDS, U-DUALITY TRANSITIONS
MOMENTUM MODES & BRANE
WRAPPING MODES MIX

DOUBLED FORMALISM

T-DUALITY GEOMETRIC, T^{2n} FIBRATION

LOCAL SPACETIME PATCH FROM LOCAL
CHOICE OF $T^n \subset T^{2n}$

- GENERALIZATION TO SPACES THAT
ARE NOT TORUS FIBRATIONS ?