Heterotic Twistor-Strings

David Skinner, Oxford & Perimeter

Based on arXiv:0807.2276 with Lionel Mason
also
Katz & Sharpe *hep-th/0406226*; Witten *hep-th/0504078*;
Adams, Distler & Ernebjerg *hep-th/0506263*
and standard twistor-string papers

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We'd like to understand these issues better, and also see how the Witten and Berkovits pictures are related.

Outline

- (0,2) Basics
  - Fields & action
  - Vertex operators
  - Anomalies
- Heterotic String Theory
  - Coupling to YM
  - Amplitudes
- Relation to other twistor-string models
  - Berkovits
  - Witten
- Summary

Twisted (0,2) models

A theory of smooth maps $\Phi : \Sigma \to X$ from a closed, compact Riemann surface Σ to a complex manifold X.

Fields are worldsheet scalars $(\phi^i, \phi^{\bar{\jmath}})$ and

$$\bar{\rho}^{\bar{\jmath}} \in \Gamma(\Sigma, \phi^* \overline{T}_X) \qquad \rho^i \in \Gamma(\Sigma, K_\Sigma \otimes \phi^* T_X)$$

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Susy transformations are

$$\{\overline{Q}, \phi^i\} = 0 \qquad \{\overline{Q}, \phi^{\bar{\jmath}}\} = \bar{\rho}^{\bar{\jmath}}$$
$$\{\overline{Q}, \rho^i\} = \overline{\partial}\phi^i \qquad \{\overline{Q}, \bar{\rho}^{\bar{\jmath}}\} = 0$$

and

$$\{\overline{Q}^\dagger, \phi^i\} = \rho^i \qquad \{\overline{Q}^\dagger, \phi^{\bar{\jmath}}\} = 0$$
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$\overline{Q}$ acts on functions of $\phi, \bar{\phi}$ as the $\bar{\partial}$ operator on $\mathrm{Maps}(\Sigma, X)$

Action

The basic action is

$$S_0 = \mathrm{t}\int_\Sigma g\left(\overline{\partial}\phi, \partial\bar{\phi}\right) - g(\rho, \nabla\bar{\rho}) + \int_\Sigma \phi^*\omega$$
$$= \mathrm{t}\left\{\overline{Q}, \int_\Sigma g(\rho, \partial\bar{\phi})\right\} + \int_\Sigma \phi^*\omega$$

for $\mathrm{t} \in \mathbb{R}^+$ and g a Hermitian (*not pseudo-Hermitian*) metric on X with $\omega(X, Y) = g(X, JY)$

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- Action is $\overline{Q}$-exact $\Rightarrow$ partition function independent of t, g
- $S_0 = -\mathrm{t}|\overline{\partial}\phi|^2 +$ fermions $\Rightarrow$ localize on holomorphic maps
- Manifestly invariant under $\overline{Q}$; also invariant under $\overline{Q}^\dagger$ if X is Kähler
- Can generalize by coupling to B-field: $\partial\overline{\partial}\omega = 0$ and ∇ has torsion determined by B

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Coupling to a bundle

We can also couple in a holomorphic bundle $\mathcal{V} \to X$ by introducing

$$\psi^a \in \Gamma(\Sigma, \phi^*\mathcal{V}) \qquad \bar{\psi}_a \in \Gamma(\Sigma, K_\Sigma \otimes \phi^*\mathcal{V}^\vee)$$
$$r^a \in \Gamma(\Sigma, \overline{K}_\Sigma \otimes \phi^*\mathcal{V}) \qquad \bar{r}_a \in \Gamma(\Sigma, K_\Sigma \otimes \phi^*\mathcal{V}^\vee)$$

with susy transformations

$$\{\overline{Q}, \psi^a\} = 0 \qquad \{\overline{Q}, \bar{\psi}_a\} = \bar{r}_a$$
$$\{\overline{Q}, r^a\} = \overline{D}\psi^a + F_{i\bar{\jmath}}{}^a{}_b \psi^b \rho^i \bar{\rho}^{\bar{\jmath}} \qquad \{\overline{Q}, \bar{r}_a\} = \bar{\partial}\bar{\psi}_a$$

and action

$$\begin{aligned} S_1 &= \left\{ \overline{Q}, \int_\Sigma \bar{\psi}_a r^a \right\} \\ &= \int_\Sigma \bar{\psi}_a \overline{D}\psi^a + F_{i\bar{\jmath}}{}^a{}_b \bar{\psi}_a \psi^b \rho^i \bar{\rho}^{\bar{\jmath}} + \bar{r}_a r^a \end{aligned}$$

Total action $S_0 + S_1$ is twisted version of heterotic string on general background

Twistor theory

We could choose $X = \mathbb{P}^{3|4}$, but

- Difficult to interpret bosonic worldsheet superpartners of fermionic target coordinates
- Not clear how to promote to string theory
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Instead, we'll choose $X = \mathbb{P}^3$ and include the bundle $\mathcal{V} = \mathcal{O}(1)^{\oplus 4}$

The advantages are

- ψ is a worldsheet scalar, as it would be with $\mathbb{P}^{3|4}$ target, but $\overline{\psi}$ is a 1-form – naturally on different footing
- First-order action for worldsheet fermions
- Worldsheet superpartners are auxiliary

Sheaves of chiral algebras

The antiholomorphic stress tensor $T_{\bar{z}\bar{z}} = \{\overline{Q}, \overline{G}_{\bar{z}\bar{z}}\}$, so all the antiholomorphic Virasoro generators $\overline{L}_n$ are $\overline{Q}$-exact.

$[\overline{L}_0, \mathcal{O}] = \bar{h}\mathcal{O}$, but since $\overline{L}_0 = \{\overline{Q}, \overline{G}_0\}$ we find

$$\bar{h}\mathcal{O} = \left[\{\overline{Q}, \overline{G}_0\}, \mathcal{O}\right] = \underbrace{\{\overline{Q}, [\overline{G}_0, \mathcal{O}]\}}_{\overline{Q}\text{-exact}} + \underbrace{\{[\overline{Q}, \mathcal{O}], \bar{G}_0\}}_{=0}$$

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In the A- or B-model, we'd similarly find $h = 0$, but in a (0,2) model there is no holomorphic susy and all $h \geq 0$ are allowed. Vertex operators form "sheaf of chiral algebras" over target.

(0,2) model is holomorphic (*not topological*) field theory.

(0,2) moduli

Focus on operators with $(h, \bar{h}) = (1, 0)$ and ghost number $+1$ (*related to deformations of the (0,2) action via descent*).

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- $b \to b + \partial\chi$ changes vertex operator by total derivative (*upto* ρ *eom*) $\Rightarrow \mathcal{H} = \partial b$ nontrivial in $H^{0,1}(\mathbb{PT}', \Omega^2_{\rm cl})$, plus super extension

(0,2) moduli correspond to states of $\mathcal{N} = 4$ conformal supergravity under the Penrose transform

Anomalies

Sigma model anomaly unless

$$\mathrm{ch}_2(T_X) - \mathrm{ch}_2(\mathcal{V}) = 0 \qquad \mathrm{c}_1(T_\Sigma)\,(\mathrm{c}_1(T_X) - \mathrm{c}_1(\mathcal{V})) = 0$$

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Anomalies in global symmetries

$$\mathrm{ind}(\overline{\partial}_{\phi^* T_{\mathbb{P}^3}}) = 4d + 3(1-g)$$
$$\mathrm{ind}(\overline{\partial}_{\phi^* \mathcal{O}(1)^{\oplus 4}}) = 4(d+1-g)$$

for a map of degree d, genus g.

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Amplitudes with n_h external SYM states of helicity h supported on maps of degree

$$d = g - 1 + \sum_{h=-1}^{+1} \frac{h+1}{2} n_h$$

Coefficient of $(\psi)^{\mathrm{top}}$ *is a section of canonical bundle of instanton moduli space*

Perturbative corrections

There are also perturbative corrections to the theory. (0,2) susy ensures that $\Delta\overline{T}_{\bar{z}\bar{z}}$ and $\Delta T_{z\bar{z}}$ are $\overline{Q}$-exact, but there is no such statement for T_{zz}.
At one loop, correction to worldsheet action is

$$\Delta S^{1-\text{loop}} = \left\{\overline{Q}, \int_\Sigma R_{i\bar{\jmath}}\rho^i\partial\phi^{\bar{\jmath}} + g^{i\bar{\jmath}}F_{i\bar{\jmath}}{}^a{}_b\bar{\psi}_a r^b\right\}$$

- On $\mathbb{P}^{3|4}$ we have $R = 0$ and no bundle
- For $\mathbb{P}^3$ and bundle $\mathcal{O}(1)^{\oplus 4}$ we have $R_{i\bar{\jmath}} = 4g_{i\bar{\jmath}}$ and $F_{i\bar{\jmath}}{}^a{}_b = \delta^a{}_b g_{i\bar{\jmath}}$ so the 1-loop correction is $\propto$ classical action.

The twistor model is a holomorphic CFT provided we study correlators of $\overline{Q}$-closed operators.

Holomorphic bc-system

Supercurrent $\overline{G}_{\bar{z}\bar{z}}$ plays role of $\bar{b}$-antighost

No left-moving susy, so need to include holomorphic bc-ghost system

$$S = \int_\Sigma b\overline{\partial}c \qquad b \in \Gamma(\Sigma, K_\Sigma \otimes K_\Sigma) \ ; \ c \in \Gamma(\Sigma, T_\Sigma)$$

- Provides holomorphic BRST operator Q
- $Q + \overline{Q}$ has complete descent chain

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- Provides holomorphic BRST operator Q
- $Q + \overline{Q}$ has complete descent chain
- Fixed vertex operators $\Rightarrow$ sigma-model vertex operators of $(h, \bar{h}) = (1, 0)$, contracted with c

Physical string states $\Leftrightarrow$ (0,2) moduli $\Leftrightarrow$ $\mathcal{N} = 4$ conformal supergravity

Yang-Mills current algebra

In order for $Q^2 = 0$ we need to include a holomorphic current algebra contributing central charge $c = 28$ $(= 26 + 2 \times (4 - 3))$, as in both Berkovits' and Witten's models (*see later* ...)

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e.g. Could include further fermions

$$\lambda^\alpha \in \Gamma(\Sigma, \sqrt{K_\Sigma} \otimes \phi^* E) \qquad \bar{\lambda}_\alpha \in \Gamma(\Sigma, \sqrt{K_\Sigma} \otimes \phi^* E^\vee)$$

for some holomorphic bundle $E \to X$ (together with auxiliary superpartners).

- Conformal invariance requires $\mathrm{c}_1(E) = 0$
- Freedom from sigma model anomalies requires $\mathrm{ch}_2(E) = 0$

$\Rightarrow$ E corresponds to a zero-instanton spacetime bundle
Vertex operators $c\mathcal{A}_{\bar{\jmath}}{}^{\alpha}{}_{\beta}\bar{\lambda}_\alpha \lambda^\beta \Leftrightarrow$ External states in $\mathcal{N} = 4$ SYM

Yang-Mills instantons

Heterotic strings contain NS branes which couple magnetically to the NS B-field.

- Physical heterotic strings (10-manifold) $\longrightarrow$ 5-branes
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Modified Green-Schwarz condition

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$\Rightarrow$ instanton backgrounds allowed

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e.g. 't Hooft $SU(2)$ k-instanton

$$A(x) = \mathrm{i}\,\mathrm{d}x^\mu \sigma_{\mu\nu} \partial^\nu \log \Phi\ , \qquad \Phi(x) = \sum_{i=0}^{k} \frac{\lambda_i}{(x - x_i)^2}$$

wrap NS branes on the $k+1$ lines in twistor space corresponding to the x_is.

A puzzle

	Physical heterotic	Twistor-string
c	16	28
Field theory	$SO(32)$, $E_8 \times E_8$, $E_8 \times U(1)^{248}$, $U(1)^{496}$	$SU(2) \times U(1)$, $U(1)^4$
Modular invariance	$SO(32)$, $E_8 \times E_8$	??

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Modular invariance	$SO(32)$, $E_8 \times E_8$	??

- Change level of current algebra?
- Include additional fields contributing to c?
- Promote to string theory by some other means than bc-system?

Clear that modular invariance is key test.

Amplitudes and contours

Choose basis of Beltrami differentials μ and compute

$$\left\langle \prod_{i=1}^{3g-3+n} (\mu^{(i)}, b)(\overline{\mu}^{(i)}, \overline{G}) \prod_{j=1}^{n} \mathcal{O}_j \right\rangle$$

where $\mathcal{O}_j$ are fixed vertex operators.

- bc-ghost number anomaly absorbed by (μ, b) and vertex operators
- $U(1)_R$ anomaly is $3(1-g)+4d$. Remaining anomaly of $4d = \mathrm{vdim}_{\mathbb{C}}\overline{\mathcal{M}}_{g,0}(\mathbb{P}^3, d)$

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Integrand is effectively a $(4d, 0)$ form on moduli space of stable maps $\Rightarrow$ contour integral.

- Absorb anomaly by inserting Poincaré dual into path integral, soaking up remaining $\bar{\rho}$ zero-modes (*Dolbeault picture*).
- Choice of contour $\Leftrightarrow$ choice of spacetime signature
- Leading-trace SYM amplitudes agree with Witten's & Berkovits' models. Sub-leading trace = cSUGRA (by unitarity)

Instanton corrections and twistor actions

At degree d, the heterotic generating function for amplitudes in $\mathcal{N} = 4$ csugra + SYM is

$$\int_{\mathcal{M}_{g,d}} \mathrm{d}\mu \;\exp\left(\frac{-A(C)}{2\pi} + \mathrm{i}\int_C B\right) \frac{\det \overline{\partial}_{E\otimes S_-}}{\det' \overline{\partial}_{N_{C|\mathbb{PT}_s}}} \qquad (\star)$$

- $\mathcal{M}_{g,d}$ is contour in space of genus g, degree d curves, measure $\mathrm{d}\mu$ ($= \mathrm{d}^{4|8}x$ at $g = 0$, $d = 1$)
- $A(C)$ = area of curve C (*from the restriction of the Kähler form*)
- $N_{C|\mathbb{PT}_s}$ is normal bundle to C in supertwistor space

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In compactifications on $CY \times \mathbb{R}^4$, $(\star)$ describes instanton corrections to $4d$ superpotential.
Here, the $d = 1$ contribution can be used together with the Chern-Simons ($d = 0$ term) as a twistor action.

Berkovits' model I

On contractible open patch $U \subset \mathbb{PT}$

- Action becomes free
- $H^p(U, \mathcal{S}) = 0$ for $p > 0 \Rightarrow$ Vertex ops independent of $\bar{\rho}$

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- Action becomes free
- $H^p(U, \mathcal{S}) = 0$ for $p > 0 \Rightarrow$ Vertex ops independent of $\bar{\rho}$

$\Rightarrow$ All correlation functions on U obtainable from

$$S_{\beta\gamma} = \int_\Sigma \beta_i \bar{\partial}\gamma^i + \bar{\psi}_a \bar{\partial}\psi^a$$

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Cover target with patches, each supporting free $\beta\gamma$ system

- Anomaly conditions arise from consistency in gluing
- Higher vertex operators described by Čech cohomology

Berkovits' model II

Equivalently, work on non-projective space

$$S = \int_\Sigma Y_I \overline{D} Z^I \qquad I = (\alpha|a) = (1,\ldots,4\,|\,1,\ldots,4)$$

with $\overline{D} = \overline{\partial} + A$ a $GL(1,\mathbb{C})$ connection. To recover previous description: integrate out $A \Rightarrow Y_I Z^I = 0$ and solve on patches $Z^\alpha \neq 0$.

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- Introduce holomorphic bc-system and current algebra as before
- Path integral only involves holomorphic Zs $\Rightarrow$ contour still needed

Given antiholomorphic involutions on Σ and $\mathbb{P}^3$, perform orientifold projection. $++--$ orientifolded theory $\Leftrightarrow$ "open string theory" on Σ' with action

$$S = \int_{\Sigma'} Y_I \overline{D} Z^I + \overline{Y}_{\bar{I}} D Z^{\bar{I}} + b\overline{\partial}c + \bar{b}\partial\bar{c} + S_{\mathrm{YM}}$$

where $Z(\partial\Sigma') \subset \mathbb{RP}^3$, and $Z^I|_{\partial\Sigma'} = \overline{Z}^{\bar{I}}|_{\partial\Sigma'}$ *etc.*

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$$\int_C \Phi_1 \overline{\partial} \Phi_0 \ ; \qquad \Phi_0 \in \Gamma(C, N_{C|\mathbb{PT}_s}) \ , \ \Phi_1 \in \Gamma(C, K_C \otimes N^\vee_{C|\mathbb{PT}_s}) \ .$$

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Combining these ingredients gives exactly the same contribution as the heterotic worldsheet instantons.

Conclusions & Outlook

We've given a construction of twistor-string theory as a heterotic string.

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  - *Should generalize to non-pert. top. str. on standard CY*

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Oustanding problems:

- Modular invariance & $c = 28$
- Penrose transform complete action
- Contour integrals
  - *Derivation of RSV? Connected/disconnected equivalence?*
- Replace $\mathcal{O}(1)^{\oplus 4}$ by another bundle?
- Poincaré supergravity? Pure SYM? Phenomenology?