Non-commutative Field Theory with Twistor-like Coordinates

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Problem

Quantum field theories are singular at short distances ? presence of ultra-violet divergences

These are handled by renormalization (whenever it is possible), often leaving some unpleasant "naturalness" problems

Non-renormalizable theories like quantum gravity are even worse: infinite number of input parameters (UV counterterms) ? no predictive power

So it is a good thing to construct UV softer, or even finite theories - c.f. superstrings, N=8 SUGRA (???) etc.

One way to change the short-distance behavior is to change the "particle" content. SUSY does it by pairing scalars with fermions, superstring theory by upgrading point-like particles to extended objects like strings. What happens in N=8 SUGRA is still unclear…

A more radical and profound idea is to change spacetime, replacing space-time continuum by some discrete or fuzzy "medium"

Very important (and in some way generic) examples of fuzzy spacetimes are those with non-commuting position coordinates

Questions

- *Does spacetime non-commutativity improve short-distance behavior of QFT?*
- *Twistor spacetime is fuzzy – can one think of fuzzy twistors in terms of some non-commutative geometry?*
- *If yes, how does it affect QFT at short (and long) distances?*

Outline

I. Non-commuting coordinates
II. Non-commutative Field Theory on Moyal Plane
III. Twistor Theory Revisited
IV. Quantum Fields with Twistor–like Coordinates

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Non-commuting Coordinates

Simplest and tractable example is the Groenewold-Moyal plane R with

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$

A radical step – the algebra of functions (fields) on R is modified
– the product is deformed to a star (Moyal) product:

$$(\phi_1 \star \phi_2)(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial^y_\mu \partial^z_\nu} \phi_1(y)\phi_2(z)|_{y=z=x}$$

Interesting mathematics. Appears in open string theory in the presence of a constant B-field B=T (Seiberg-Witten, '99). But is it physically sensible?

Non-commutative Field Theory

Is it physically sensible?

Non-commutative Lagrangians involve non-local interactions with star products

$$\int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{4!}\lambda\,\phi^4\right]$$

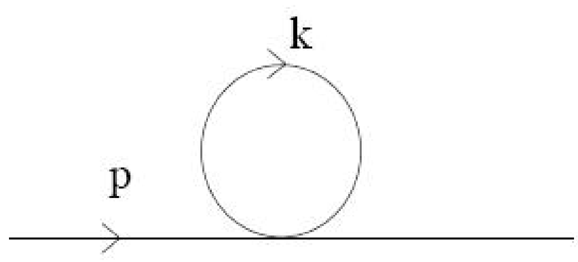
$$\rightarrow \int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{4!}\lambda\,\phi\star\phi\star\phi\star\phi\right]$$

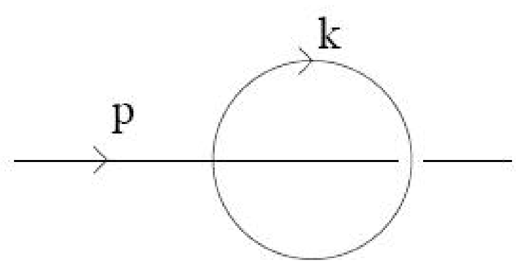
Free Feynman propagators are not affected, but the perturbative interaction vertices are modified by the factors

$$e^{i\sum_{i<j} C_{ij}\, k_{i\mu} k_{j\nu}\,\theta^{\mu\nu}}$$

(Minwalla et al, '99)

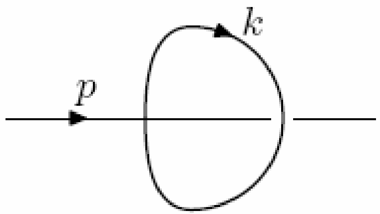
They affect UV and IR behavior of Feynman diagrams





Non-commutative Field Theory

Is it physically sensible and useful?



$$= \frac{\lambda}{12}\int \frac{d^4k}{(2\pi)^4}\frac{e^{ip_\mu k_\nu \Theta^{\mu\nu}}}{k^2+m^2}$$

from Rivasseau, '07

$$= \frac{\lambda}{48\pi^2}\sqrt{\frac{m^2}{(\Theta p)^2}}\,K_1\!\left(\sqrt{m^2(\Theta p)^2}\right) \underset{p\to 0}{\simeq} p^{-2}.$$

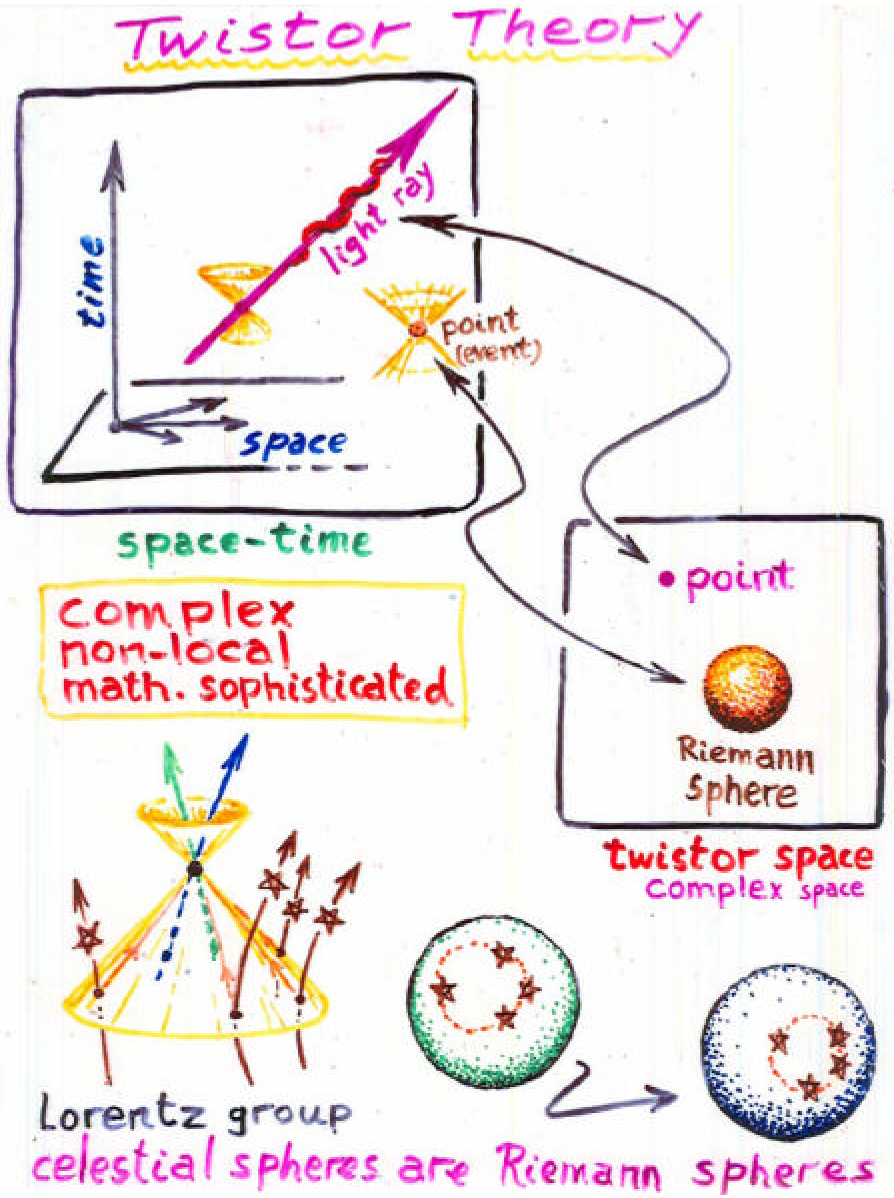
UV-IR mixing: ? 8 , p 0 limits' order matters

*Renormalizable F **4, without UV/IR mixing, can be constructed by modifying quadratic terms* *(Grosse, Wulkenhaar '04)*

In general, no significant improvement in UV – Feynman diagrams still have the same degree of divergences as commutative QFT

Moyal Plane non-commutativity isn't too useful for improving short-distance behavior *from Penrose, 2005 Oxford LMS Workshop*

Twistor Theory (Penrose '67)



*Twistors: Spinors representing null geodesics (light rays, world lines) in M***?**

Intersections   *Points*

Notation   *(Penrose,Rindler '86)*

$$V^{AA'} = \begin{pmatrix} V^{00'} & V^{01'} \\ V^{10'} & V^{11'} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} V^0 + V^3 & V^1 + iV^2 \\ V^1 - iV^2 & V^0 - V^3 \end{pmatrix}$$

momentum $p_{AA'} = \pi_A \bar{\pi}_{A'}$

$$L^{AA'BB'} = i\,\omega^{(A}\bar{\pi}^{B)}\epsilon^{A'B'} - i\,\epsilon^{AB}\bar{\omega}^{(A'}\pi^{B')}$$

? angular momentum

Twistor Theory

Twistors $$Z^{\alpha} = (\omega^{A}, \pi_{A'}) \qquad \alpha = 1, 2, 3, 4$$

$$\omega^{A} = \check{\omega}^{A} - i x^{AA'} \check{\pi}_{A'} \quad ; \quad \pi_{A'} = \check{\pi}_{A'}$$

? reference point (? line) of angular momentum L

Dual Twistors $$\bar{Z}_{\alpha} = (\bar{\pi}_{A}, \bar{\omega}^{A'})$$

$$p^{0} = \frac{1}{\sqrt{2}}(Z^{3}\bar{Z}_{1} + Z^{2}\bar{Z}_{0}); \ldots$$

$$L^{01} = -L^{10} = \frac{i}{2}(Z^{0}\bar{Z}_{0} - Z^{1}\bar{Z}_{1} - Z^{2}\bar{Z}_{2} + Z^{3}\bar{Z}_{3}); \ldots$$

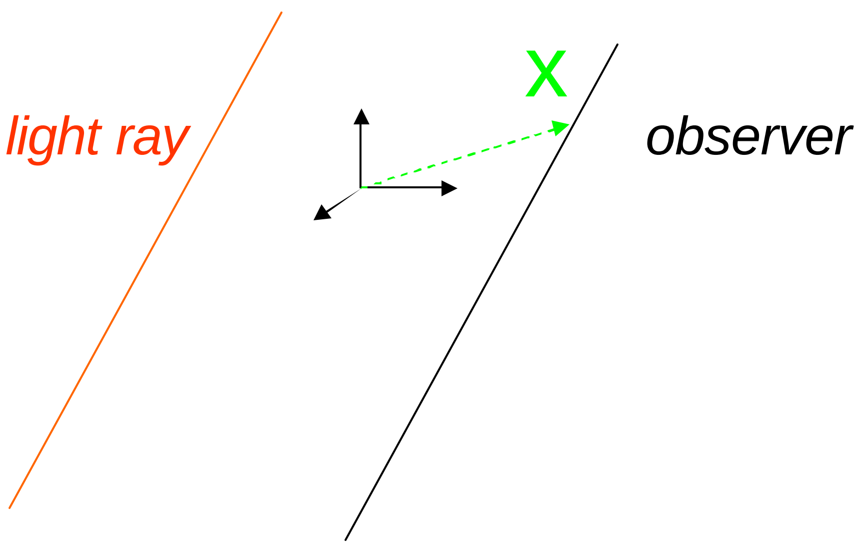
Interpretation

"Angular" twistor $\omega^A = \omega_0^A - i x^{AA'}\pi_{A'}$ *"incidence relation"*

Usually, one considers a fixed reference point, often setting $\omega_0^A = 0$
Then one thinks about x as running along the light ray

$$x^{AA'} \to x^{AA'} + k\pi^A\pi^{A'}$$

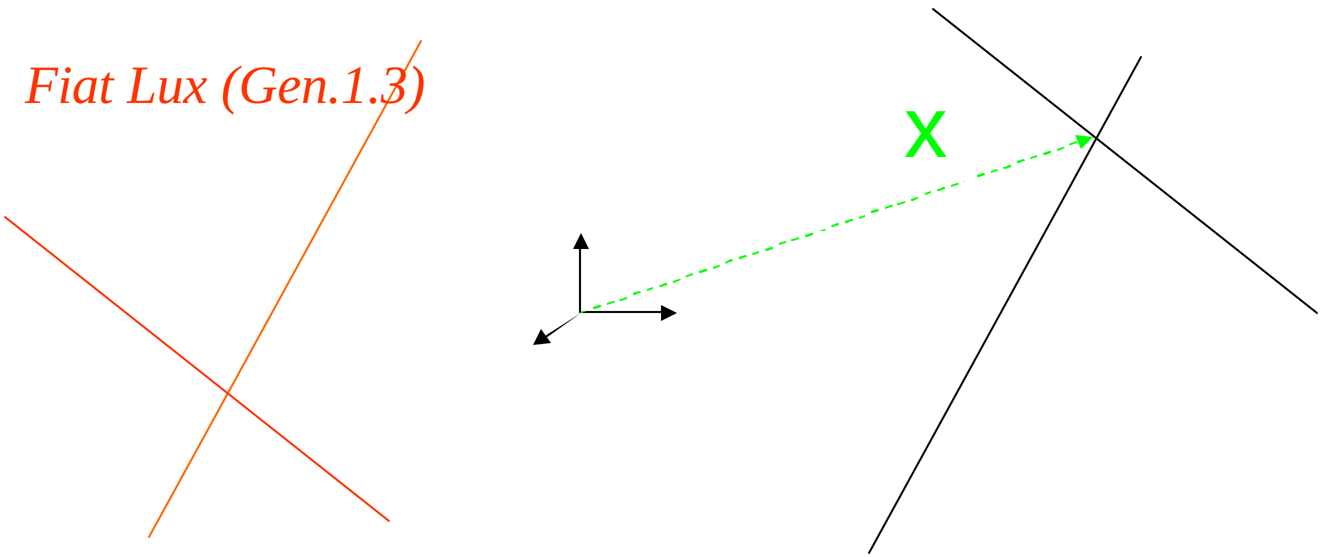
Another viewpoint: consider a fixed light ray :



By using 2 reference light rays, and measuring

$$Z_a^{\alpha} = (\omega_a^A ; \pi_{aA'}) \qquad a = 1;2$$

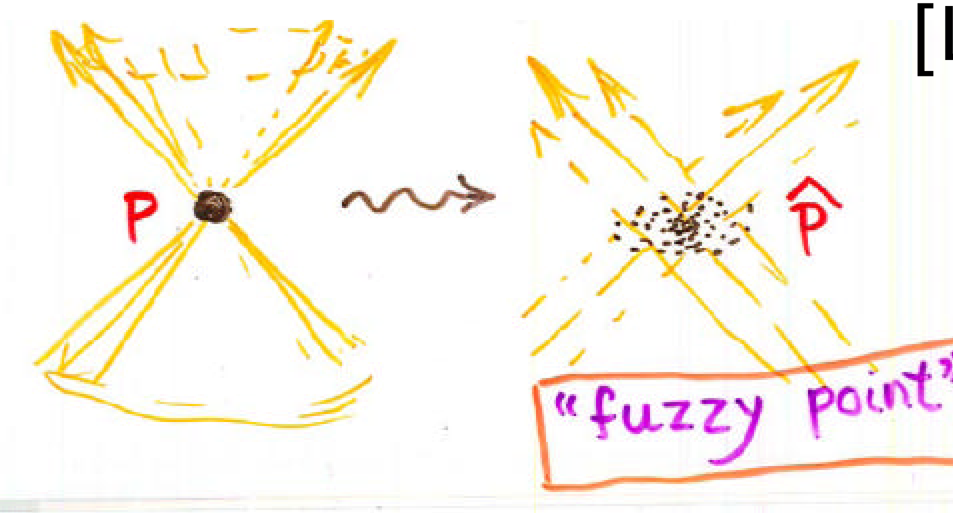
the observer can determine his/her position:



$$x^{AA'} = \frac{i}{\pi_{1B'}\pi_2^{B'}}\left[(\omega_1^A - \bar{\omega}_1^A)\pi_2^{A'} - (\omega_2^A - \bar{\omega}_2^A)\pi_1^{A'}\right]$$

from Penrose, 2005 Oxford LMS Workshop

Twistor Quantization



$$[L_{10}, P_\rho] = \eta_{1\rho}P_0 - \eta_{0\rho}P_1$$

canonical quantization of Poincare algebra

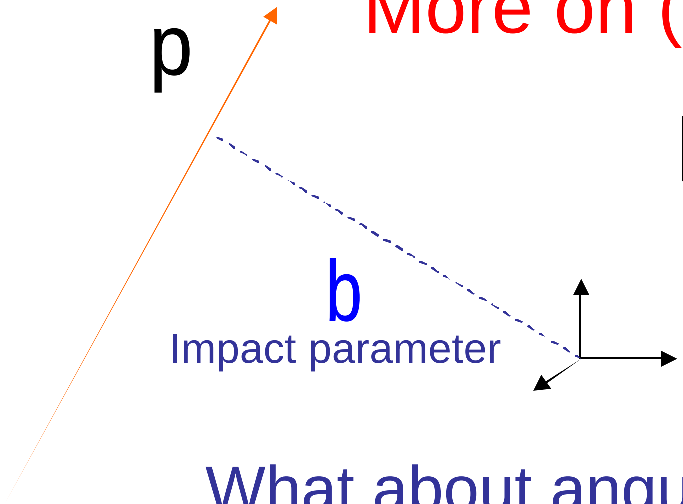
$$[Z^\alpha, \bar{Z}_\beta] = \hbar\delta^\alpha_\beta$$

$$[\omega^A, \bar{\pi}_B] = \hbar\delta^A_B \qquad [\pi_{A'}, \bar{\omega}^{B'}] = \hbar\delta^{B'}_{A'}$$

n.b. units: $\omega = x\sqrt{p}$; $\pi = \sqrt{p}$

Fuzzy space-time needs no quantum gravity?
What are the consequences
of such non-commutativity?

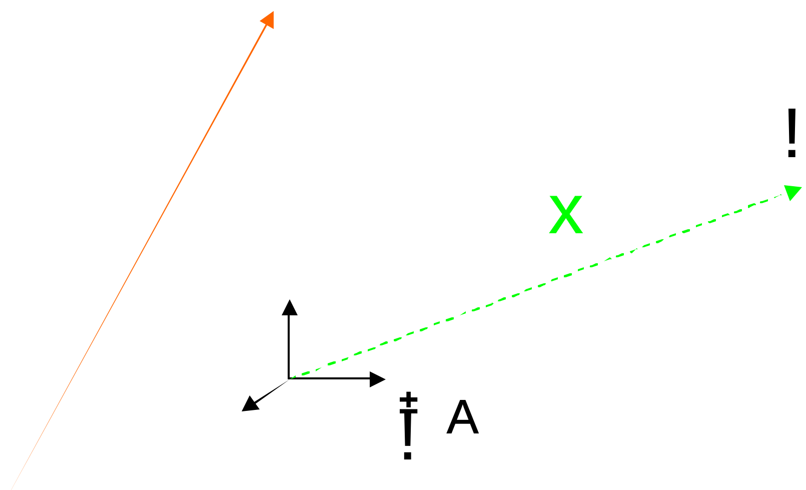
More on (obvious) interpretation



$[\,!^{A};\ ¼_{B}] = \sim\pm^{A}_{B}$ $\qquad$ $[¼_{A'};\ !^{B'}] = \sim\pm^{B'}_{A'}$

$¢x_{?}\,¢p_{?} \;¸\; \frac{\sim}{2}$

What about angular momenta w.r.t. different points?



$!^{A} = ‡!^{A} \;¡\; ix^{AA'}¼_{A'}$

$¢‡! \;!\; ¢¼ \;!\; ¢!$

$[\,!^{A}(x_1);\ !^{A}(x_2)] = i\sim(x_2^{AA'} \;¡\; x_1^{AA'})$

Spacetime parameterized by non-commuting twistor-like position coordinates

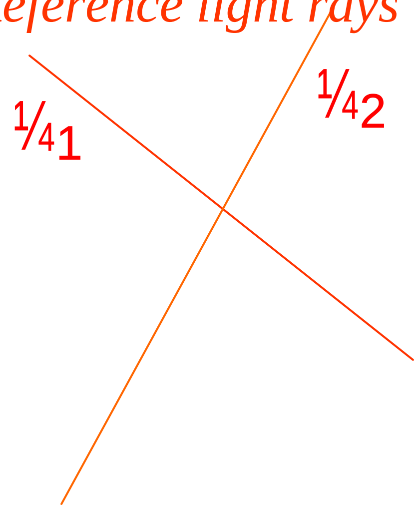
$$[\omega^{A}_{a}(x_1), \bar{\pi}^{A'}_{b}(x_2)] = i\hbar(x_2^{AA'} - x_1^{AA'})\delta_{ab}$$

LOCALLY COMMUTING BUT

NON-LOCALLY NON-COMMUTING

UNCERTAINTY GROWS WITH SEPARATION (LIKE IN A FOG…)

Reference light rays



LORENTZ SYMMETRY BROKEN BY TIME-LIKE

$$I_{AA'} = \bar{\pi}_{1A}\pi_{1A'} + \bar{\pi}_{2A}\pi_{2A'}$$

SO(1,3) ? SO(3)

$$I = (\mu\ ;\ \vec{0})$$

NON-COMMUTATIVITY SCALE

SO IT'S A…


FOGGY ÆTHER
©

Free Fields propagating in *FOGGY ÆTHER*

$$\phi(\omega,\theta) = \int \frac{d^3\vec{p}}{\sqrt{(2\pi)^3\, 2|\vec{p}|}} \left(a_p e^{\omega^A_a \theta^a_A/\hbar} + a_p^\dagger e^{\theta^{-1\,a}_{A'}\omega^{A'}_a/\hbar} \right)$$

SECOND QUANTIZATION:

$$p_{AA'} = \theta^a_A \pi_{aA'} \qquad [a_p, a^\dagger_{p'}] = \delta^3(\vec{p} - \vec{p}\,')$$

FEYNMAN PROPAGATOR:

$$iD(x' - x) = \langle 0|T(\phi(\omega',\theta')\phi(\omega,\theta))|0\rangle$$

NON-COMMUTATIVE (BAKER-HAUSDORFF)

$$e^A e^B = e^{(A+B+\frac{1}{2}[A,B])}$$

$$e^{\omega^A_a(x')\theta^a_A(p')/\hbar}\, e^{\theta^{-1\,b}_{A'}(p)\,\omega^{A'}_b(x)/\hbar}$$

$$= e^{[\omega^A_a(x')\theta^a_A(p') + \theta^{-1\,a}_{A'}(p)\,\omega^{A'}_a(x) + \frac{i}{2}(x - x')^{AA'}\theta^a_A(p')\theta^{-1\,a}_{A'}(p)]/\hbar}$$

Feynman Propagator

$$iD(x'-x) = \int \frac{d^3\vec{p}}{(2\pi)^3 2|\vec{p}|}\left[e^{-ip\cdot(x'-x)(p+2l)^2/(4\mu^2\hbar)}\theta(t'-t)\right.$$

$$\left. + \, e^{ip\cdot(x'-x)(p+2l)^2/(4\mu^2\hbar)}\theta(t-t')\right]$$

$$= i\int \frac{d^4k}{(2\pi)^4} e^{-ik(x'-x)}\tilde{D}(k)$$

$$\tilde{D}(k) = \frac{1}{k^2} \times \frac{2}{\sqrt{|\vec{k}|/\mu+1}\left(\sqrt{|\vec{k}|/\mu+1}+1\right)}$$

$\mathrm{UV}: \ |\vec{k}| \gg \mu \quad \tilde{D}(k) \sim \frac{1}{k^3}$

ABOVE μ NON-COMMUTATIVITY SCALE

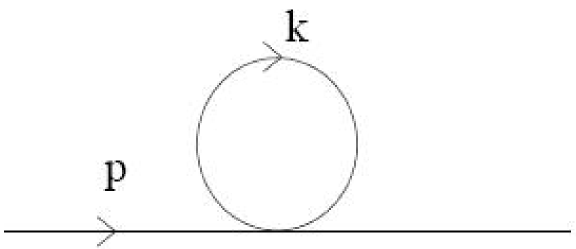
$\mathrm{IR}: \ |\vec{k}| \ll \mu \quad \tilde{D}(k) \sim \frac{1}{k^2}$

STANDARD PROPAGATOR

Interacting Fields (very preliminary)

Coordinates are locally commuting ? local interactions unchanged

UV behavior of Feynman diagrams determined by propagators



$$\gg \; Z \; \frac{d^4k}{k^3}$$

Linear (instead of quadratic) UV divergence

No UV/IR mixing

gauge theories in FOGGY ÆTHER : perturbatively finite ?

Conclusions

- Non-local (foggy) non-commutativity in twistor space
- Determined by 2 constants: *h*, *μ*
- Lorentz symmetry broken: preferred time direction, fundamental time unit $\tau = \hbar/(\mu c^2)$ and rotational invariance in the Æther frame
- Assuming Æther = CMB ? *μ > 10 TeV*
- Foggy spacetime tames UV divergences of QFT, no UV/IR mixing
- Many open questions: interacting QFT formalism, divergences, gravity,…

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