Strongly coupled gauge theories on anti-de Sitter space

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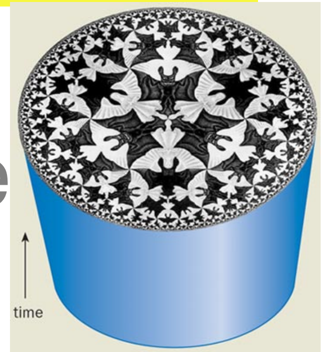

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Based on two projects in progress, in collaboration with Don Marolf+Mukund Rangamani, and with Micha Berkooz+David Tong+Shimon Yankielowicz

Outline

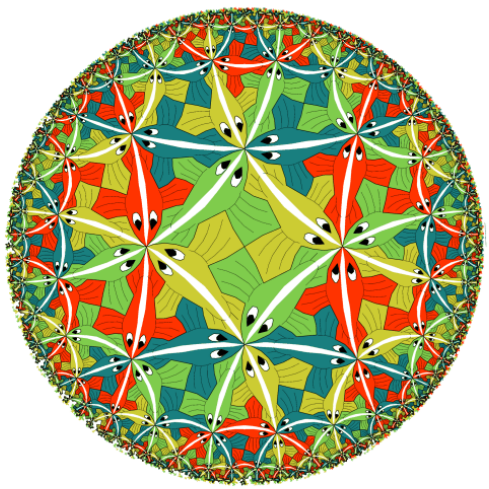
1) Motivations
2) Gauge theories in anti-de Sitter space
3) Weakly coupled theories and large N phase transitions
4) Strongly coupled $\mathcal{N}$=4 supersymmetric Yang-Mills (SYM) on AdS_4 :
   (a) S-duality, (b) AdS_5 dual
5) Confining gauge theories in anti-de Sitter space

Motivations

- We will consider theories on global anti-de Sitter space in d space-time dimensions, to avoid having horizons and associated boundary conditions. This space, with metric

$$ds^2 = L^2(-\cosh^2(\rho)dt^2 + d\rho^2 + \sinh^2(\rho)d\Omega_{d-2}^2)$$

behaves like a box in many ways. In particular, classical fields on this space all have energies #/L with positive coefficients. So, anti-de Sitter space provides an IR cutoff in a maximally symmetric fashion (SO(d-1,2) isometry).



- Such theories are also interesting in the context of the AdS/CFT correspondence, in two ways :

a) Field theories on AdS_d which are coupled to gravity are believed to have dual descriptions as conformal field theories in d-1 dimensions. When the coupling to gravity is weak it should not affect the properties of the field theory on AdS_d. What does strong coupling mean in dual CFT ?

b) Some conformal field theories on AdS_d are expected to have gravitational duals involving AdS_{d+1}, which could be weakly curved when the field theory is strongly coupled (e.g. we expect this for $d=4$ $\mathcal{N}=4$ SYM). What are they ?

Gauge theories in AdS space

- AdS space has a boundary, $S^{d-2} x R$. To define field theories on this space we need to impose boundary conditions. Recall that for scalar fields of mass m, the behavior near the boundary is : (with $m^2L^2=\Delta(\Delta-(d-1))$)

$$\Phi(\rho,x) \xrightarrow{\rho\to\infty} \phi_1(x)e^{(\Delta-d-1)\rho} + \phi_2(x)e^{-\Delta\rho} + \cdots$$

and we need to impose some boundary condition of the form $\phi_1(x)=f(x)$ to make the theory well-defined. In AdS/CFT we identify the bulk field ϕ with an operator $O(x)$ in the dual CFT_{d-1}, of dimension Δ. The boundary condition is associated with a deformation of the action by $\int dx f(x) O(x)$, and $\langle O(x)\rangle \cong \phi_2(x)$.

- In the case that $(d-3)/2 \leq \Delta \leq (d+1)/2$, there are two possible choices for the boundary conditions, since both modes of the scalar field are normalizable. These have different dual field theories, with operators of different dimensions.
- For other fields (fermions, massive p-form fields) there is a similar story. For massless gauge fields this was worked out by Ishibashi+Wald, Marolf+Ross. I will focus on the case d=4 for simplicity. One can phrase the boundary conditions in a gauge-invariant way, or fix a convenient gauge such as $A_\rho=0$. In this gauge, the behavior of the gauge field near the boundary is as :

$$A_i(\rho, x) \xrightarrow{\rho\to\infty} a_i(x) + b_i(x)e^{-\rho} + \cdots$$

- The "standard boundary condition" that is used in the context of AdS_4/CFT_3 is to fix $a_i(x)$ to some value. With this boundary condition the gauge field corresponds to a conserved current $J_i(x)$ in the CFT_3, of dimension $\Delta=2$ (more generally $\Delta=d-2$). The value of the boundary condition corresponds to deforming the CFT_3 action by $\int dx a_i(x) J^i(x)$.

  (We do not really need the CFT_3 for this discussion).
- However, just like for some scalar fields, there is also another allowed boundary condition, where we fix $b_i(x)$ and allow $a_i(x)$ to fluctuate. We can think of this as gauging the global symmetry; we now have a "boundary gauge field" $a_i(x)$ (but with no kinetic term). ("modified boundary condition")

- With the “standard boundary condition” there is no Gauss’ law – charged states are allowed (=globally charged states in AdS/CFT). On the other hand, the “modified boundary conditions” allow gauge transformations that do not vanish on the boundary, so there is a Gauss’ law and no charged states are allowed.
- Witten (2003) noted that for a free U(1) theory on AdS_4, the two boundary conditions are related by S-duality. So, in a gauge theory on AdS_4, can have either electric charges, or magnetic charges, but not both. (Actually have full SL(2,Z).) In the context of AdS_4/CFT_3, the bulk S-duality gives an equivalence between different CFT_3’s (related as above).

- Consider now an SU(N) $\mathcal{N}=4$ SYM theory on AdS_4 at large N, with the standard boundary conditions. At weak coupling and low energies, the spectrum includes arbitrary gluon states, so the low-energy density of states is $O(N^2)$. This raises the following two confusions :
- This theory has an S-duality symmetry; naively at very strong coupling, $g_{YM} >> 1$, we get the same theory at weak coupling with the opposite boundary conditions, which allow only singlets, but then the density of states at low energies is $O(1)$. How does the transition between these configurations ("free electric gluons" and "confined magnetic gluons") happen as we raise the coupling ?

- In the large N limit with fixed 't Hooft coupling $\lambda=g_{YM}^2N$, expect to have a holographic dual of this theory, locally given by type IIB string theory on $AdS_5 x S^5$. But how can this have a density of states of order $O(N^2)$ at very low temperatures, since this requires a horizon ?
- We will see below how to resolve these two confusions, related to the behavior of the theory at strong coupling. But before that, let us briefly consider another (related) aspect of weakly coupled $SU(N)$ gauge theories on AdS_d in the large N limit.

Weakly coupled large N phase transitions

- Weakly coupled large N gauge theories (possibly with adjoint matter) on compact manifolds were shown to have a phase transition as a function of the temperature (hep-th/0310285). This arose because they have a Gauss' law, forcing all states to be singlets of the gauge group. At low temperatures the dominant configurations are single-trace combinations of gluons (and other adjoint fields), with a density of states of $O(1)$. At high temperatures the entropy dominates the free energy, and we have approximately a gas of free fields with $F \sim O(N^2)$.

- In the large N limit there is a sharp transition between these two possibilities, which can be identified as a deconfinement transition by the usual order parameters. This transition is first order at zero coupling, and becomes either first order or continuous at weak ‘t Hooft coupling.
- With the “modified boundary conditions”, gauge theories on AdS_d space also have a discrete spectrum with no zero modes, and a Gauss’ law. Thus, by the same arguments, they also exhibit a large N phase transition, at a temperature T~#/L. The coefficient may be easily determined from the particle content. For $\mathcal{N}$=4 d=4 SYM the zero coupling transition happens at T=1/(2 ln(3) L).
- No transition for “standard boundary conditions”.

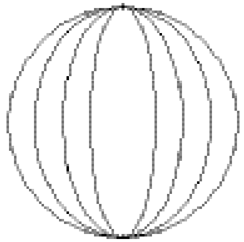
Strongly coupled d=4 $\mathcal{N}$=4 SYM on AdS$_4$

- To resolve the confusions discussed above we need to understand the strong coupling behavior of d=4 $\mathcal{N}$=4 SYM on AdS$_4$, using the usual tools of S-duality and the AdS/CFT correspondence. Let us start with very strong coupling, $g_{YM}>>1$, where S-duality is useful. We naively assumed that as in the U(1) case, the S-dual of the "standard boundary conditions" for the SU(N) theory gives the "modified boundary conditions" for this theory. Actually, we should be more precise now, since we also have 6 scalar fields whose boundary conditions we need to specify (they can have Δ=1,2).

- Generally, we have control over the strong coupling behavior only for supersymmetric theories. So, we consider boundary conditions which preserve the maximal possible amount of supersymmetry. In this case the maximal choice is 16 supercharges (out of 32). This requires giving a boundary condition of one type to 3 of the scalars, and of the opposite type to the other 3 scalars. (Breitenlohner+Freedman,1982)
- In fact, Gaiotto and Witten (2008) classified all boundary conditions of d=4 $\mathcal{N}$=4 SYM on a half-line which preserve half of the supersymmetry. This configuration is related by a conformal transformation to AdS_4, so it classifies also half-SUSY boundary conditions on AdS_4.

- They found that the S-dual of Neumann ("modified") b.c.s in this non-Abelian case is *not* given by Dirichlet ("standard") b.c.s. This can be seen from a brane picture. Neumann b.c. arise for D3-branes ending on a NS5-brane. The S-dual of this is D3-branes ending on a D5-brane, but this gives boundary conditions which explicitly break the SU(N) gauge symmetry (3 scalar fields behave near the boundary like the N-dimensional representation of SU(2)). So, we do not get $O(N^2)$ degrees of freedom at low energies. Similarly, the S-dual of Dirichlet boundary conditions (D3-branes ending on N D5-branes) involve a complicated CFT at the boundary (D3-branes ending on N NS5-branes). This evades the confusion…

- For a different range of couplings – very large N, large but finite 't Hooft coupling λ – we may expect to have a weakly curved gravitational dual for this SYM theory on AdS_4. Locally the dual should be type IIB string theory on $AdS_5 x S^5$. In fact, AdS_5 has an AdS_4 slicing (used e.g. for Randall-Sundrum). However, in this slicing the boundary of AdS_5 is mapped to <u>two</u> copies of AdS_4, connected at their boundary, so it is not quite what we want.



- The simplest way to get a holographic dual of the SYM theory on a single AdS_4 is to perform an orbifold / orientifold that identifies the two copies of AdS_4. This can preserve half of the SUSY for an orbifold / orientifold 5-plane wrapping $AdS_4 x S^2$.

- Gaiotto and Witten also analyzed the boundary conditions for D3-branes ending on orbifolds / orientifolds, and how they transform under S-duality. These boundary conditions are highly non-trivial. For example, for D3-branes ending on an $O5^0$ orientifold, the boundary condition is "modified" for a USp(N) subgroup of SU(N), and "standard" for the other generators, giving a theory with USp(N) gauge symmetry (and Gauss' law), and there is also a fundamental hypermultiplet living on the boundary. The S-dual of this has "modified b.c." for an SU(N/2)xSU(N/2) subgroup of SU(N), and "standard" for the rest. In all cases a large gauge group is preserved; O(1) degrees of freedom at low energies.

- The full list Gaiotto+Witten found is given by :

Gauge group	Quotient	Dual gauge group	Nahm data	Boundary dof
$SO(N)$	$O5^+$	$SU(N)$	0	$2+1$ SCFT
$USp(N)$	$O5^-$	$SU\left(\frac{N}{2}\right)_d \subset SU(N)$	N-dim irrep of $SU(2)$	0
$SU(N/2) \times SU(N/2)$	$\mathbb{Z}_2(-1)^{F_L}$	$USp(N)$	0	D5-hypers
$SU(p) \times SU(q)$	$\mathbb{Z}_2(-1)^{F_L}$	$USp(2q)$	$N = (p-q) + q \times 1$	0

- In all cases O(1) low energy degrees of freedom, so consistent with absence of horizons. But, for "standard b.c." in full SU(N) should still have O(N^2) degrees of freedom at low energies. To find the dual of this we need to find the near-horizon limit of D3-branes ending on a N D5-branes, which is difficult (and probably not weakly coupled; SU(N) global symmetry). Similar challenge for most b.c.s.

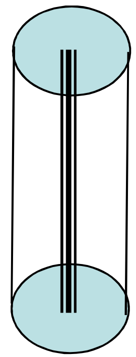
Confining gauge theories in anti-de Sitter space

- Next, let's discuss confining gauge theories in anti-de Sitter space. Don't necessarily need large N; we can take some asymptotically free SU(N) gauge theory like d=4 QCD on AdS_4 (but applies more generally).
- Consider the "standard boundary conditions", for which we have an SU(N) global symmetry and no Gauss' law. The gauge theory is characterized by a strong coupling scale Λ, and we can analyze two different limits. (For d=3, replace by g_{YM}^2.)
- For $\Lambda L \ll 1$, the gauge theory is weakly coupled at the scale $1/L$ of the mass gap. The spectrum consists of weakly coupled gluons and quarks in AdS.

- On the other hand, for $\Lambda L \gg 1$, we expect confinement to take place at a much higher scale than the IR cutoff $1/L$. So, we expect to have no charged states under the $SU(N)$ global symmetry. This is clearly true in the bulk, and states near the boundary should also be very heavy (as we take the boundary to infinity). Where did all the charged states disappear ? Is there some sharp phase transition as a function of ΛL ?
- May think states continuously become massive, but this cannot happen since state created by J^a must have energy exactly $2/L$ (conserved current) as long as it exists (for small ΛL it is a single-gluon state, with an energy fixed by gauge-invariance).

- Thus, for (and only for) "standard boundary conditions", expect a sharp phase transition as we increase ΛL, around $\Lambda L \sim 1$. As we pass through this phase transition, all charged states disappear from the spectrum, and the global symmetry disappears.
- From the dual CFT point of view (once we couple to gravity) this sounds very strange. Can imagine examples where M_p and L are fixed, and we continuously change gauge coupling (Λ) by an exactly marginal deformation. However, usually in AdS/CFT, ΛL is not a continuous parameter in the dual CFT, but discrete, so we do not have any concrete examples of this yet.

- What happens at finite temperature T ? For $\Lambda L >> 1$, may expect a deconfinement transition at $T \sim \Lambda$. However, recall that the effective temperature in global AdS is position-dependent (if we define it as usual by periodicity of Euclidean global time), diverging near the boundary and reaching a minimum at the "center" $\rho = 0$. So, it seems that the transition in this case proceeds by forming a deconfined bubble near the center, which grows with T. Sharp transition at $T \sim \Lambda$ (but not in gravity).



- If we have a dual description as a (d+1)-dimensional gravity theory, with the deconfined phase a black hole as usual, this implies a black hole localized in the IR region, near the bubble. Interesting to find such solutions.

- Why have we never encountered such confining gauge theories in the bulk in the AdS/CFT context ? Obtaining them once we couple the theory to bulk gravity requires some fine-tuning. Usually we discuss backgrounds in which all distance scales are the same ~L, and we get weakly coupled gauge theories at the scale 1/L, so $\Lambda L \ll 1$. To get $\Lambda L \gg 1$ we need to get a weakly coupled gauge theory at a scale exponentially larger than 1/L. The scale L is determined by the total cosmological constant, so this can be achieved by canceling several different contributions to the cosmological constant (as in KKLT), but requires fine-tuning. In SUSY case may be able to control this by canceling different contributions to the superpotential.

Summary

- After 13 years, still seem to be many surprises and unexplored corners in AdS/CFT, and even just in the basic phase structure of field theories on anti-de Sitter space.
- Work in progress, many more directions to explore - stay tuned…