Quantum Phases of k-Strings

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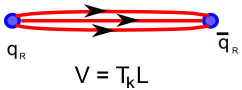
Based on 0810.3201 and 0908.4278 with Roberto Auzzi

Outline

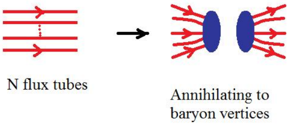
1. Confinement and QCD flux tubes or "k-strings"
2. Mass deformed $\mathcal{N} = 4$ theory and Olive-Montonen duality.
3. k-string solitons from gauge theory.
4. World-sheet theory
5. k-strings at large N

• Confinement in pure gauge theories (or with adjoint matter) is associated with the formation of colour flux tubes



• Flux tubes carry a discrete quantum number residing in the center of the gauge group. For $SU(N)$ theories, this is an integer $k \pmod N$ - "N-ality"

• k-strings interact with each other and in particular, annihilate in groups of N.



• The tension $T_k = \Lambda^2 f(k, N, \ldots)$ with $T_k = T_{N-k}$ by charge conjugation. The string tension and world-sheet dynamics of this soliton-like object is interesting in its own right. These are fundamental properties of the confining theory.

• Results from SUSY models in the "universality class" of $\mathcal{N} = 1$ SYM:

$$\frac{T_k}{T'_k} = \frac{\sin \frac{\pi k}{N}}{\sin \frac{\pi k'}{N}}$$

- MQCD (Hanany-Strassler-Zaffaroni 1997)
- Softly broken $\mathcal{N} = 2$ SYM (Douglas-Shenker 1995)
- Large N Gravity Duals: Maldacena-Nuñez and Klebanov-Strassler backgrounds (Herzog-Klebanov 2002)

• Lattice data for pure $SU(N)$ Yang-Mills are consistent with both Sine Law and Casimir Scaling $T_k \propto k(N-k)$. (Lucini-Teper-Wenger '02 , del Debbio-Panagopoulous-Vicari '03)

• It is interesting to explore possible behaviours in confining gauge theories where string tensions are calculable, and perhaps the world-sheet dynamics is tractable.

• A natural question is whether the k-string can be realized as a classical soliton in a confining theory.

• Generally speaking this would be possible, if there were some dual weakly coupled description of the confined phase, such as a Generalized Dual Meissner effect, containing solitonic magnetic k-strings.

Mass deformed $\mathcal{N} = 4$ SYM

- Olive-Montonen $SL(2, \mathbb{Z})$ duality of $\mathcal{N} = 4$ theory provides a precise setting for the above picture.

- Deform the $SU(N)$, $\mathcal{N} = 4$ theory with $\mathcal{N} = 1$ supersymmetric masses (m_1, m_2, m_3) for the adjoint matter fields $\Phi_{1,2,3}$ ($\mathcal{N} = 1$ multiplets). [Also known as $\mathcal{N} = 1^*$ SYM.]

- Classical vacuum equations coincide with $su(2)$ algebra

$$[\Phi_i, \Phi_j] = i\epsilon_{ijk}\Phi_k\, m_k$$

- Large number of ground states $\sim e^{\sqrt{N}}$ for large N. These include Higgs (**H**), confined (**C**), partially Higgsed/confined phases, etc.

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- S-duality:
  - $\frac{g_{YM}^2}{4\pi} \leftrightarrow \frac{4\pi}{g_{YM}^2} \implies$ **H** $\leftrightarrow$ **C** $\qquad (\theta = 0)$
  - Confining k-strings at $g_{YM} \gg 1 \rightarrow$ solitonic magnetic flux tubes in **H** at $g_{YM} \ll 1$.
  - Solitonic strings have quantum number $\pi_1 \left(SU(N)/\mathbb{Z}_N\right) = \mathbb{Z}_N$.

- To find these solitonic strings, we will take $m_1 = m_2 = m_3 = m$ $\implies$ global $O(3)$ flavor symmetry.

- In Higgs phase, VEVs $\langle\Phi_i\rangle = mJ_i$ $(J_i \in su(2))$, break $O(3)_f$, but a combination of global colour and flavour $O(3)_{c+f}$ is preserved.

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- Look for classical axially-symmetric solutions with

(Markov-Marshakov-Yung 2004; Auzzi-SPK 2009)

  - $\Phi_i(r \to \infty) \to mJ_i$
  - $\exp(i \oint A_\varphi(r \to \infty)) = e^{2\pi i k/N}$, $k = 1, 2 \ldots N-1$
  - Non-abelian flux $\oint A_\varphi \propto \mathrm{diag}(k, k, \ldots N-k, N-k)$ as $r \to \infty$
  - $O(3)_{c+f} \to U(1)_{c+f}$, and SUSY broken.

- Solutions exist, and are obtained numerically.

- The tensions evaluated numerically for $N = 4, 5, 6$

$$T_k \simeq \frac{2\pi m^2}{g_{YM}^2} k(N-k)$$

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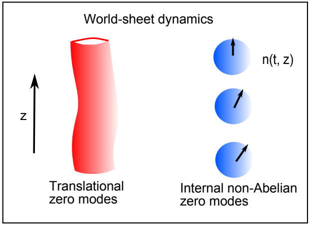
World-sheet theory

There is an $SO(3)_{c+f}/U(1)_{c+f} \simeq S^2$ moduli space of solutions.

- Bosonic $\mathbf{CP}^1$ sigma model with θ-term.

- $S_\sigma = \int dz\, dt \left(\frac{1}{g_\sigma^2} (\partial_s \vec{n})^2 - \frac{\theta_\sigma}{8\pi} \epsilon^{sr}\ \vec{n} \cdot \partial_s \vec{n} \times \partial_r \vec{n} \right)$

- $$\theta_\sigma = k(N-k)\,\theta_{YM}$$



- Asymp. free with a mass gap. Spectrum is an $O(3)$ triplet.

- $\theta_\sigma = \pi$. Flow to $c = 1$ CFT. Spectrum consists of deconfined doublets. (Zamolodchikov-Zamolodchikov)

[cf. Hanany,Tong 2003-4; Auzzi-Bolognesi-Evslin-Konishi-Yung 2003; Shifman-Yung 2004 ...]

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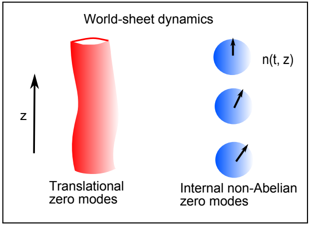

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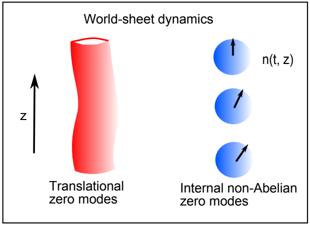

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Moving away from $O(3)$ symmetric point

- Take $m_1 = m_2 = m$ and $m_3 \neq m$. Preserves $U(1)_{c+f}$.
- Small enough deformation $\delta = m_3^2 - m^2$ induces a potential on the S^2 moduli space, $\boxed{\mathcal{L}_\sigma \to \mathcal{L}_\sigma - \delta\, n_3^2}$.

- $m_3 < m$ - Classically massive - 2 vacua.
- BPS kinks and "dyonic kinks"
- $m_3 > m$ - Classically massless O(2) model
- Vortex "merons" on the world-sheet

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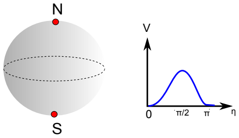

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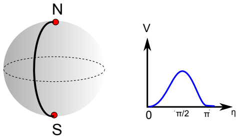

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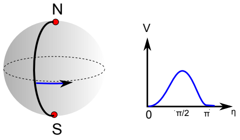

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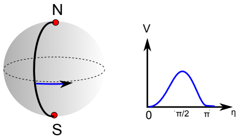

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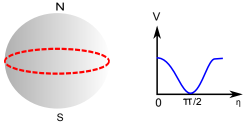

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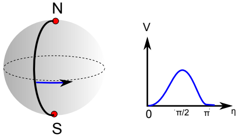

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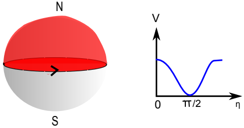

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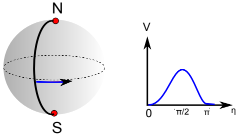

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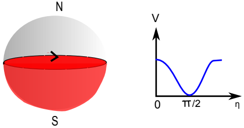

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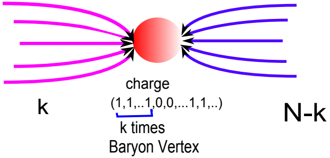
$m_3 < m$ and $\mathcal{N} = 2$ limit

- The semiclassical (anti)-kinks exhibit a 2D "Witten effect" when $\theta_\sigma \neq 0$ whereby they acquire a U(1) charge $(-)\frac{\theta_\sigma}{2\pi}$. (Dorey 1998, Abraham-Townsend 1991)

- Dyonic "rotating" kinks with $U(1)$ charge Q obey BPS mass formula $M_{\rm kink} \propto |Q + \frac{\theta_\sigma}{2\pi} + \frac{i}{g_\sigma^2}|$.
Kinks with charge Q and $-Q - 1$ become degenerate at $\theta_\sigma = \pi$.

- In 4D gauge theory, a kink interpolates between two flux orientations



• When $m_3 \ll m$ ($\mathcal{N} = 2$ limit), 4D theory has BPS monopoles with the above charges and
$M_{\rm mon} = \sqrt{2} m |Q + k(N-k)(\frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2_{YM}})|$

These undergo level crossing precisely when $k(N-k)\theta = \pi$.

(Auzzi-SPK, 2009)

• The identification of monopoles with σ-model kinks, explains why $\boxed{\theta_\sigma = k(N-k)\theta}$.

$m_3 > m$ and sigma-model merons:

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• Vacuum manifold is the equator: N pole and S pole vortices with topological charge $= \pm\frac{1}{2}$.

• Coulomb gas of vortex-merons $\rightarrow$ Sine-Gordon model

$\mathcal{L}_\sigma = g^2_\sigma (\partial\psi)^2 - 2\zeta \cos\frac{\theta_\sigma}{2} \cos\psi$

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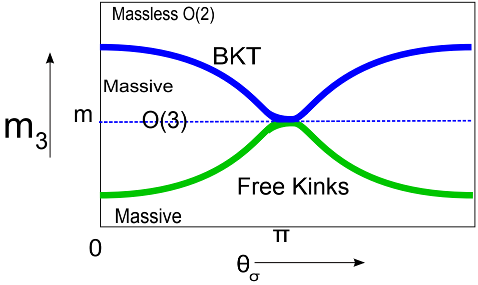
Phase diagram for the k-string



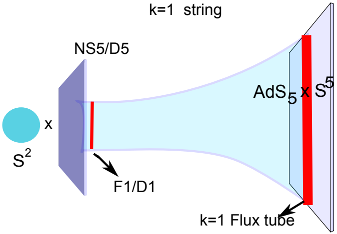
• The significance of the new massless internal mode on the world-sheet is unclear.

• However, it does have a physical effect, since it will contribute to the Luscher term for the k-string.

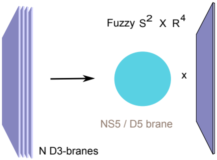
Large-N String Dual (**C**/**H** phases)

- At large N, flux tubes are F1/D1 strings in Polchinski-Strassler background - deformation of the $AdS_5 \times S^5$, type IIB geometry.

- Confinement/Higgs reflected by NS5/D5 with world-volume $\mathbb{R}^4 \times S^2$, cutting off IR.



- Motivated by weak-coupling picture of D3's blowing up into a transverse fuzzy S^2, $\Phi_1{}^2 + \Phi_2^2 + \Phi_3^2 = \mathbf{1}\,\frac{m^2(N^2-1)}{4}$



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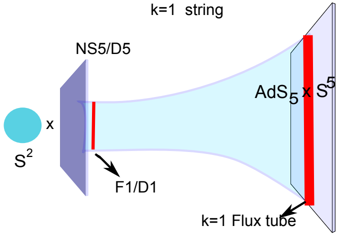

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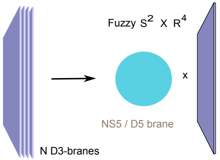

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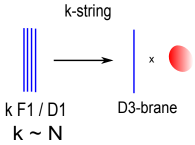
- $k = 1$ string tension -

Nambu-Goto for F1 in **C** phase ($\lambda = g_{YM}^2 N \gg 1$): $T_{\mathrm{F1}} = m^2 \frac{\lambda}{8\pi}$

DBI action for $D1$ in **H** phase $T_{\mathrm{D1}} = 2\pi m^2 \frac{N}{g_{YM}^2}$.

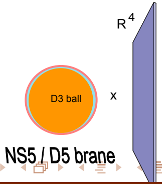
- What is a k-string when $k \sim \mathcal{O}(N)$? (e.g. Klebanov-Herzog)



- Flux tubes made of same material as baryon-vertex (with attached strings). $\mathcal{N} = 4$ SYM baryon-vertex - D5-brane wrapping S^5. Flux tube is a D5-brane wrapping $S^4 \subset S^5$. (Callan-Guijosa-Savvidy)

- Baryon vertex of $\mathcal{N} = 1^*$ is a D3-ball.

- k-string $\sim$ slice of D3-ball i.e. D3 disk.



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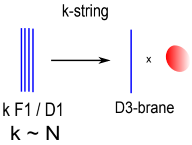

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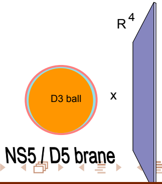
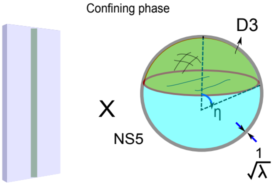


D3-brane as a k-string $k \sim N$ and $\sqrt{\lambda} \gg 1$

- Confining vacuum, k-string is the expanded D3 "cap + disk".
- Dilaton diverges in small region $\sim \frac{1}{\sqrt{\lambda}} \times$ sphere radius.



- Large dilaton forces cap to settle near NS5, $T_{\rm cap} \to 0$.
- Near NS5 $C_2^{RR} \neq 0$ and D3-cap acquires k-string charge:

$S_{WZ} \sim \int_{\rm cap} {}^*C_2 \wedge F_{tz}$ and $\boxed{(1-\cos\eta) = \frac{2k}{N}}$.

- D3-disk sees flat space.

k-string tension $=$ Disk area $= \frac{m^2 \lambda N}{32\pi} \sin^2 \eta$.

$\boxed{T_k = \frac{m^2 \lambda}{8\pi} k(1 - \frac{k}{N})}$ + unknown corrections from edge ...

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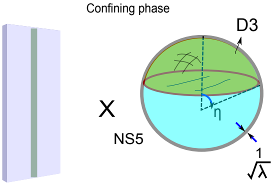

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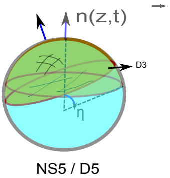
Moduli space dynamics

- The moduli space dynamics is obtained by allowing the D3-cap orientation to depend on string coordinates.



- Theta-dependence is easily included in the Higgs vacuum via $SL(2,\mathbb{R})$ transformation, $C_2^{RR} \to C_2^{RR} + \frac{\theta}{2\pi} B_2^{NS}$.

- From the WZ terms of the D3-cap $S_{WZ} =$

$$k(N-k)\,\frac{\theta}{8\pi}\int dtdz\,\epsilon^{sr}\,\vec{n}\cdot\partial_s\vec{n}\times\partial_r\vec{n}.$$

- The expanded D3-cap and the $k(1-\frac{k}{N})$ dependence is reminiscent of Abelian-Higgs vortices on the sphere in 2D which behave as hard-core discs (Manton-Nasir). The volume of the moduli space of k-coincident vortices $= k$ (1- $k\times$Area excluded by disc). (In our case, the NC string instantons have a core area $\sim \frac{1}{N}$).

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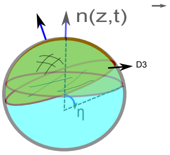

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Summary and questions

- Magnetic k-string world-sheet, at weak coupling, can peer into aspects of 4D gauge theory physics.
- The interpretation of phase transitions on the k-string and appearance new massless modes is puzzling – unclear what it says about the 4D gauge theory.
- Connection to instantons in $\mathbb{R}^4 \times S^2_{NC}$ and large-N.
- Casimir scaling, if correct for $k \sim \mathcal{O}(1)$ implies $\mathcal{O}(\frac{1}{N})$ corrections. This is a potential conflict with expectations in a theory with a $\frac{1}{N^2}$ expansion. (e.g. Armoni-Shifman, 2003).
- Exploration of k-string tensions at large N, in other massive phases involving multiple 5-branes. Moving away from the $O(3)$ symmetric theory in gravity dual to look for world-sheet kinks/monopoles.