London Mathematical Society – EPSRC Durham Symposium

GEOMETRY AND ARITHMETIC OF LATTICES

Monday 4th July – Thursday 14th July 2011

All talks will take place in ROOM D110, Dawson Building

TUESDAY, 5TH

09:00-09:05	WELCOMING MESSAGE	
09:05-10:00	W.M. Goldman	*Geometric structures on manifolds, part 1*
10:00-11:00	M.S. Raghunathan	*The first Betti number of compact manifolds of constant curvature, part 1*
11:30-12:30	D. Toledo	*Periods of cubic surfaces*
16:00-17:00	T. Gelander	*On the growth of Betti numbers of arithmetic groups*
17:00-17:30	S. Baba	*Graftings and complex projective structures*
17:30-18:00	D. Panov	*Building complex manifolds using hyperbolic geometry*
19:45	*Wine reception at Grey College*	

WEDNESDAY, 6TH

09:00-10:00	W.M. Goldman	*Geometric structures on manifolds, part 2*
10:00-11:00	M.S. Raghunathan	*The first Betti number of compact manifolds of constant curvature, part 2*
11:30-12:30	F. Paulin	*Representation of integers by product of linear forms and diagonal flows*
16:00-17:00	N. Bergeron	*Some consequences of Arthur's work on the topology of hyperbolic manifolds*
17:00-17:30	A. Rahm	*Implications of hyperbolic geometry to operator K-theory of arithmetic groups*
17:30-18:00	M. Stover	*Arithmeticity and complex reflection groups*

THURSDAY, 7TH

09:00-10:00	W.M. Goldman	*Geometric structures on manifolds, part 3*
10:00-11:00	M.S. Raghunathan	*The first Betti number of compact manifolds of constant curvature, part 3*
11:30-12:30	J. Millson	*The geometric theta correspondence and totally geodesic cycles*

Afternoon Excursion to Durham Cathedral

FRIDAY, 8TH

09:00-10:00	V. Markovic	*Immersing nearly geodesic surfaces in hyperbolic manifolds and applications, part 1*
10:00-11:00	G. Prasad	*Volumes of arithmetic quotients of semi-simple groups, finiteness theorems, and fake projective spaces, part 1*
11:30-12:30	A. Rapinchuk	*Weakly commensurable arithmetic groups and length-commensurable locally symmetric spaces*
16:00-17:00	A. Salehi-Golsefidy	*Counting lattices in a simple Lie group*
17:00-17:30	V. Emery	*Covolumes of non-uniform arithmetic lattices in* $\mathrm{PU}(n,1)$
17:30-18:00	M.H. Sengun	*On the integral homology of Kleinian groups*
19:00	*Conference Dinner*	

SATURDAY, 9TH

09:00-10:00	V. Markovic	*Immersing nearly geodesic surfaces in hyperbolic manifolds and applications, part 2*
10:00-11:00	G. Prasad	*Volumes of arithmetic quotients of semi-simple groups, finiteness theorems, and fake projective spaces, part 2*
11:30-12:30	T.N. Venkataramana	*On Zariski dense discrete subgroups of* $\mathrm{SL}_n(\mathbb{R})$
16:00-17:00	E. Leuzinger	*A geometric characterization of arithmetic Fuchsian groups*
17:00-18:00	PROBLEM SESSION	

SUNDAY, 10TH

Day trip to Robin Hood's Bay and Whitby

MONDAY, 11TH

09:00-10:00	R. Kellerhals	*Hyperbolic orbifolds of small volume*
10:00-11:00	J. Ratcliffe	*Right-angled Coxeter polytopes, hyperbolic 6-manifolds, and a problem of Siegel*
11:30-12:30	B. Klingler	*Rigidity for complex hyperbolic lattices*
16:00-17:00	C.H. Grossi	*Constructing (complex) hyperbolic manifolds*
17:00-17:30	M. Deraux	*Non-arithmetic lattices in complex hyperbolic geometry, part 1*
17:30-18:00	J. Paupert	*Non-arithmetic lattices in complex hyperbolic geometry, part 2*

TUESDAY, 12TH

09:00-10:00	D. Cartwright	*Enumerating the fake projective planes, part 1*
10:00-11:00	A. Lubotzky	*Counting arithmetic subgroups, surfaces and manifolds, part 1*
11:30-12:30	B. Remy	*Quasi-isometry classes of twin building lattices*
16:00-17:00	N. Gusevskii	*A characterization of R-Fuchsian groups acting on the complex hyperbolic plane*
17:00-17:30	P. Will	*Teichmüller slices in the* $\mathrm{PU}(2,1)$ *representation variety of surface groups*
17:30-18:00	J. Thompson	*Deformed non-equilateral triangle groups*
19:45	*Wine reception at Grey College*	

WEDNESDAY, 13TH

09:00-10:00	D. Cartwright	*Enumerating the fake projective planes, part 2*
10:00-11:00	A. Lubotzky	*Counting arithmetic subgroups, surfaces and manifolds, part 2*
11:30-12:30	A. Gorodnik	*Large sieve for arithmetic groups*
16:00-17:00	N. Nikolov	*Local convergence of congruence subgroups in an arithmetic lattice*
17:00-18:00	H. Abels	*The coarse geometry of arithmetic quotients, a question of Siegel*

Titles and Abstracts

H. Abels (Universität Bielefeld)

The coarse geometry of arithmetic quotients, a question of Siegel

This is a report on joint work with G. A. Margulis. It started from the following question of Siegel (1959). Let Γ be an arithmetic subgroup of a reductive group G and let S be a Siegel domain for Γ in G, a certain fundamental set for Γ in G constructed in reduction theory. Is the natural map $S \to G/\Gamma$ an isometry up to a bounded error? We give a positive answer for every global field by making reduction theory more precise, determine the coarse isometry type of G/Γ, and discuss for which metrics on G the result holds.

S. Baba (Universität Bonn)

Graftings and complex projective structures

We consider complex projective structures with fixed holonomy representation from a closed surface group into $\mathrm{PSL}(2,\mathbb{C})$. (2π)-grafting is a certain surgery operation that transforms a projective structure to another, preserving its holonomy.

We show that, given two projective structures with fixed holonomy, there is a finite sequence of graftings and inverse-graftings that transforms one to the other, under the assumption that the holonomy representation is a generic one in the character variety.

N. Bergeron (Université Pierre et Marie Curie)

Some consequences of Arthur's work on the topology of hyperbolic manifolds

D. Cartwright (University of Sydney)

Enumerating the fake projective planes

1. A fake projective plane is a smooth compact complex surface P which is not biholomorphic to the complex projective plane $\mathbb{P}^2_{\mathbb{C}}$, but has the same Betti numbers as $\mathbb{P}^2_{\mathbb{C}}$, namely 1, 0, 1, 0, 1. A fake projective plane is determined by its fundamental group,

In their 2007 Inventiones paper, Gopal Prasad and Sai-Kee Yeung showed that these fundamental groups are the torsion-free subgroups Π, with finite abelianization, of index $3/\chi(\bar{\Gamma})$ in a maximal arithmetic subgroup $\bar{\Gamma}$ of $\mathrm{PU}(2,1)$. They show that only a small number of $\bar{\Gamma}$ can arise, list them explicitly, and found many of the possible subgroups Π.

Making heavy use of computers, Tim Steger and I have found all the possible groups Π, for all of these $\bar{\Gamma}$'s, by finding explicit generators and relations for each of these groups $\bar{\Gamma}$. We have therefore found all the fake projective planes. It turns out that there are, up to homeomorphism, exactly 50 of them (100 up to biholomorphism). The fundamental group of Mumford's original fake projective plane will be identified.

2. Continuing with the notation of the abstract of part 1, I plan to give some more details of how the fundamental groups Π of the fake projective planes were found. In particular, for some of the groups $\bar{\Gamma}$, Prasad and Yeung left open the question of whether there are any subgroups Π corresponding to fake projective planes. The groups $\bar{\Gamma}$ in question were all groups of unitary matrices modulo scalars (rather than groups in division algebras). I will discuss how we showed there were no groups Π in these cases, confirming a conjecture of Prasad and Yeung.

M. Deraux (Université de Grenoble I)

Non-arithmetic lattices in complex hyperbolic geometry, part 1

I will give a quick overview of the geometry of complex hyperbolic space, with emphasis on the construction of (possibly non-arithmetic) lattices. We will discuss the recent discovery of a new family of non-arithmetic lattices in the isometry group of the complex hyperbolic plane (joint work with J. Parker and J. Paupert).

V. Emery (Université de Genève)

Covolumes of non-uniform arithmetic lattices in $\mathrm{PU}(n,1)$

Using Prasad's volume formula and techniques developed by Borel-Prasad, we can determine the smallest covolume among non-uniform arithmetic lattices in $\mathrm{PU}(n,1)$. For even n this gives the smallest covolume in each commensurability class of such lattices. Moreover, using local-to-global methods, we determine the number of lattices (up to conjugacy in $\mathrm{PU}(n,1)$) realizing these minimal covolumes. This is a joint work with Matthew Stover.

T. Gelander (Hebrew University)

On the growth of Betti numbers of arithmetic groups

We study the asymptotic behavior of the Betti numbers of higher rank locally symmetric manifolds as their volumes tend to infinity. We prove a uniform version of the Lück Approximation Theorem, which is much stronger than the linear upper bounds on Betti numbers proved by Gromov. The basic idea is to adapt the theory of local convergence, originally introduced for sequences of graphs of bounded degree by Benjamini and Schramm, to sequences of Riemannian manifolds. Using rigidity theory we are able to show that when the volume tends to infinity, the manifolds locally converge to the universal cover in a sufficiently strong manner that allows us to derive the convergence of the normalized Betti numbers.

Joint work with M. Abert, N. Bergeron, I. Biringer, N. Nikolov, J. Raimbault and I. Samet.

W.M. Goldman (University of Maryland)

Geometric structures on manifolds

A. Gorodnik (University of Bristol)

Large sieve for arithmetic groups

We develop a version of the large sieve for sets in arithmetic groups indexed by height functions, and discuss several applications. In particular, we explain how to compute the asymptotics of the number of elements with generic Galois groups. This is a joint work with Amos Nevo.

C.H. Grossi (Universidade de São Paulo)

Constructing (complex) hyperbolic manifolds

In the talk, we present new ideas concerning the construction of hyperbolic manifolds, mainly of real or complex hyperbolic ones. Such construction is especially important in the complex hyperbolic case, where there is a sensible lack of explicit examples. The most interesting types of hyperbolic manifolds are of course the compact ones and the disc bundles over closed surfaces.

Many new examples of complex hyperbolic disc bundles are constructed in a joint work with S. Anan′in and N. Gusevskii. Developing these methods provides a series of complex hyperbolic disc bundles which is much wider with respect to possible values of discrete invariants. This includes a new construction of a complex hyperbolic trivial bundle. A trivial bundle has been found earlier by S. Anan′in in the context of certain complex hyperbolic representations called “pentagons” (“pentagons” enjoy a sort of rigidity à la Toledo) and their Teichmüller space. (The existence of a complex hyperbolic trivial bundle was a long-standing problem raised by W. M. Goldman.) All the above constructions use a local version of Poincaré’s Polyhedron Theorem.

We will also discuss a project concerning compact complex hyperbolic manifolds (that is, compact $\mathbb{C}$-surfaces of general type with $c_1^2 = 3c_2$). Such manifolds are known to be rigid and our approach to studying the Dirichlet polyhedra explores a rigidity stronger than that observed by G. Giraud. As a first step in the project, a local version of Poincaré’s Polyhedron Theorem for compact manifolds is already developed.

The above intensively uses the approach and jargon of the so called “classic geometries”.

N. Gusevskii (Universidade Federal de Minas Gerais)

A characterization of R-Fuchsian groups acting on the complex hyperbolic plane

We prove that a complex hyperbolic non-elementary Kleinian group G acting on two-dimensional complex hyperbolic space $\mathbf{H}^2_{\mathbb{C}}$ is $\mathbb{R}$-Fuchsian, that is, G leaves invariant a totally real plane in $\mathbf{H}^2_{\mathbb{C}}$, if and only if every loxodromic element of G is either hyperbolic or loxodromic whose elliptic part is of order 2. Also, we study the question of which complex hyperbolic surfaces admit simple closed geodesics. This is a joint work with Heleno Cunha.

R. Kellerhals (Université de Fribourg)

Hyperbolic orbifolds of small volume

We provide a survey on small volume (real) hyperbolic orbifolds and discuss several related aspects of combinatorial, algebraic and analytical nature.

B. Klingler (Université Paris 7)

Rigidity for complex hyperbolic lattices

I will explain how Hodge theory (Abelian and non-Abelian) forces many rigidity properties for finite dimensional representations of complex hyperbolic lattices.

A. Lubotzky (Hebrew University)

Counting arithmetic subgroups, surfaces and manifolds

We will present some new and old results concerning the question of estimating the number of (arithmetic) lattices of covolume at most v in a simple Lie group G (or equivalently, the number of manifolds of volume at most v covered by the symmetric space associated with G). We will try also to map what are the problems which are still open in this area.

E. Leuzinger (Karlsruhe Institute of Technology)

A geometric characterization of arithmetic Fuchsian groups

We discuss a conjecture of Sarnak asserting that arithmetic Fuchsian groups are characterized by the so called "bounded clustering" property of their trace set.

V. Markovic (University of Warwick)

Immersing nearly geodesic surfaces in hyperbolic manifolds and applications

I will discuss my work with Jeremy Kahn on the proof of Surface Subgroup Theorem and our most recent work on the proof of the Ehrenpreis conjecture. Several related results will be stated and some new directions outlined.

J. Millson (University of Maryland)

The geometric theta correspondence and totally geodesic cycles

This talk will be a continuation of that given by Nicolas Bergeron at this conference. The point of the two talks is to present an outline of our joint work with Colette Moeglin entitled "Hodge type theorems for arithmetic manifolds associated to orthogonal groups". The main point is to prove that *for n small* the homology of codimension nq of the standard compact arithmetic quotients of $\mathrm{SO}(p,q)/\mathrm{SO}(p) \times \mathrm{SO}(q)$, with $p \geq q$, is spanned by certain totally geodesic cycles. In particular for $p+1>5$ the next-to-top homology group of a compact hyperbolic p-manifold is spanned by totally geodesic hypersurfaces. Similar results hold for homology with local coefficients – so for $p+1>5$ the space of infinitesimal conformal and projective deformations on the above hyperbolic manifolds is spanned by infinitesimal bendings.

N. Nikolov (Imperial College London)

Local convergence of congruence subgroups in an arithmetic lattice

Motivated by the Lück's approximation theorem (on the homology growth of subgroups in finitely presented groups) we study homology growth of lattices in a simple Lie group G. One is lead to the notion of local convergence of lattices or more generally of invariant random subgroups of G. The situation is very nice when G has $\mathbb{R}$-rank at least 2: any infinite sequence of lattices converges locally to the identity subgroup! When G has rank one this is no longer true but at least one has local convergence of congruence subgroups inside a given cocompact arithmetic subgroup of G.

Joint work with Abert, Bergeron, Biringer, Gelander, Raimbault and Samet.

D. Panov (King's College London)

Building complex manifolds using hyperbolic geometry

A question of Gromov asks: Is it true that every compact manifold is a quotient of a hyperbolic space by a discrete group of isometries? I will explain how one can use this question to give an elementary proof a the following theorem of Taubes: Compact complex 3-folds can have arbitrary finitely presented fundamental group. This is a joint work with Anton Petrunin.

F. Paulin (Université Paris-Sud 11)

Representation of integers by product of linear forms and diagonal flows

We prove an asymptotic formula as $n \to +\infty$ for the number of equivalence classes of representations of integers with absolute value at most n by a $\mathbb{Q}$-irreducible integral polynomial which is the product of d real linear forms in d variables. Our arguments use the mixing property with exponential decay of correlations of diagonal flows acting on the space of lattices in $\mathbb{R}^d$. An application is given to the asymptotic counting of elements (modulo units) with bounded norm in totally real number fields. This is a joint work with A. Gorodnick.

J. Paupert (Université de Fribourg)

Non-arithmetic lattices in complex hyperbolic geometry, part 2

G. Prasad (University of Michigan)

Volumes of arithmetic quotients of semi-simple groups, finiteness theorems, and fake projective spaces

I will give an exposition of the formula for the volume of quotient of a semi-simple simply connected group by a principal S-arithmetic subgroup, the finiteness results obtained jointly with Armand Borel using this formula, and the joint work with Sai-Kee Yeung on classification of fake projective planes and the existence and nonexistence of their higher dimensional arithmetic analogues.

M.S. Raghunathan (Tata Institute of Fundamental Research)

The first Betti number of compact manifolds of constant curvature

I will outline in these lectures some known results on the first Betti number of locally symmetric compact spaces. I will begin with a sketch proof of Kazhdan's theorem on the vanishing of the first Betti number of compact locally symmetric spaces of higher rank. After this I will describe the results of Matsushima relating Betti numbers with representation theory and the use of this and the Weil representation by Kazhdan to obtain non-vanishing of the first Betti number for certain arithmetic subgroups in $\mathrm{SU}(n,1)$. Finally I will relate the non-vanishing of the Betti number for arithmetic subgroups in $\mathrm{SO}(n,1)$ to the congruence subgroup problem and combining this with Kazhdan's result for $\mathrm{SU}(n,1)$, deduce the non-vanishing of the first Betti number for practically all arithmetic subgroups in $\mathrm{SO}(n,1)$, $n > 3$. If time permits I will describe an approach (as yet unsuccessful) for dealing with the case $n = 3$ (the Thurston conjecture).

A. Rahm (Weizmann Institute)

Implications of hyperbolic geometry to operator K-theory of arithmetic groups

As a class of examples of arithmetic groups acting as orientation preserving isometries on hyperbolic 3-space, we consider the Bianchi groups: The $(\mathrm{P})\mathrm{SL}_2$ matrix groups over rings of integers in imaginary quadratic number fields.

This is a representative class of such arithmetic groups because every noncocompact arithmetic Kleinian group is commensurable with some Bianchi group.

With a recently released open access computer program, we can explicitly compute the quotient space and stabiliser subgroups for the action of any of the Bianchi groups.

Feeding the complex representation rings of these stabiliser subgroups along the quotient space into a spectral sequence converging to the Bredon homology of the Bianchi groups, we obtain the equivariant K-homology of the latter. And by the Baum/Connes assembly map, which is an isomorphism for the Bianchi groups (and conjectured to be so for all finitely presented groups), we obtain operator K-theory groups for the Bianchi groups, namely the K-theory of their reduced C^*-algebras, which is very hard to compute directly.

We give an explanation for the occurring results in terms of torsion subcomplex reduction, a novel technique that has been very recently submitted for publication.

Another application of the information about the action of the Bianchi groups is the determination of the cusp form spaces which contain Bianchi modular forms that are not lifted by the Langlands base change procedure, as investigated in a joint project with Mehmet Haluk Sengun.

A. Rapinchuk (University of Virginia)

Weakly commensurable arithmetic groups and length-commensurable locally symmetric spaces

I will report on a series of joint papers with Gopal Prasad in which we introduced the notion of weak commensurability for Zariski-dense subgroups of semi-simple algebraic groups and applied our results on the classification of weakly commensurable arithmetic subgroups to understanding isospectral and length-commensurable locally symmetric spaces. Our analysis of weak commensurability hinges on the existence of special elements, called generic, in any finitely generated Zariski-dense subgroup of the group of rational points of a semi-simple algebraic group over a field of characteristic zero. So, in the beginning of the talk I will outline the techniques that yield the existence of generic elements and also demonstrate their applications to other problems. I will then define what it means for two Zariski-dense subgroups to be weakly commensurable, and discuss some basic consequences of weak commensurability. For S-arithmetic subgroups of absolutely almost simple algebraic groups, we in fact have a complete understanding of when weak commensurability implies commensurability; we have also proven that in all situations the S-arithmetic subgroups that are weakly commensurable to a given S-arithmetic subgroup split into finitely many commensurability classes. Finally, we will discuss applications of weak commensurability to the analysis of isospectral and length-commensurable locally symmetric spaces. Time permitting, I will also talk about connections with the classical question as to what extent a simple algebraic group is determined by the set of isomorphism classes of its maximal tori (a very interesting particular case of this general question is whether two quaternion algebras having the same maximal subfields are necessarily isomorphic).

J. Ratcliffe (Vanderbilt University)

Right-angled Coxeter polytopes, hyperbolic 6-manifolds, and a problem of Siegel

By gluing together the sides of eight copies of an all-right angled hyperbolic 6-dimensional polytope, two orientable, arithmetic, hyperbolic 6-manifolds with Euler characteristic -1 are constructed. They are the first known examples of orientable hyperbolic 6-manifolds having the smallest possible volume. This is a joint work with B. Everitt, and S. Tschantz.

B. Remy (Université Claude Bernard Lyon 1)

Quasi-isometry classes of twin building lattices

We will explain that some well-chosen lattices in products of buildings provide infinitely many quasi-isometry classes of finitely presented simple groups. This is a consequence of a non-distortion result. This is joint work with Pierre-Emmanuel Caprace.

A. Salehi-Golsefidy (Princeton University)

Counting lattices in a simple Lie group

I will talk about a proof of Lubotzky's conjecture on the quantitative version of Wang's theorem. Roughly the conjecture says that the asymptotic growth of the number of lattices in G a simple Lie group with covolume at most x, up to an automorphism of G, is the same as the subgroup growth of any lattice in G.

M.H. Sengun (Universitat de Barcelona)

On the integral homology of Kleinian groups

I will report on my computations on the growth of torsion in the integral homology of (both arithmetic and non-arithmetic) Kleinian groups. The motivation behind is the recent work of Bergeron-Venkatesh and Marshall-Mueller.

M. Stover (University of Michigan)

Arithmeticity and complex reflection groups

n the past several years, there has been a wave of activity on finiteness theorems for arithmetic lattices in $\mathrm{SO}(n,1)$ generated by reflections. Complex reflection groups, first used by Mostow to build nonarithmetic lattices in $\mathrm{SU}(2,1)$, have also been of significant interest recently. I will discuss some finiteness theorems for arithmetic complex reflection groups, particularly so-called complex hyperbolic triangle groups, inspired by Takeuchi's finiteness theorem for arithmetic triangle groups.

J. Thompson (Durham University)

Deformed non-equilateral triangle groups

D. Toledo (University of Utah)

Periods of cubic surfaces

The moduli space of cubic surfaces is isomorphic to the a quotient of the unit ball in $\mathbb{C}^4$ by an arithmetic group. This was proved In previous work with Allcock and Carlson by constructing a map from the moduli space to the ball quotient, called the period map. We review this construction, then explain how to construct an explicit inverse to the period map by using suitable theta functions. This gives a new proof of the isomorphism between the two spaces. We explain what is known about explicit values of the period map and pose problems on special periods (CM points).

T.N. Venkataramana (Tata Institute of Fundamental Research)

On Zariski dense discrete subgroups of $\mathrm{SL}_n(\mathbb{R})$

We first show how a lattice in a real rank one simple group H may be extended to a Zariski dense (non-lattice) discrete subgroup of a larger real rank one simple Lie group. However, we have no examples of such a phenomenon when the real rank of the smaller group is strictly bigger than one; in fact, we show in a large number of cases that the larger Zariski dense discrete subgroup is a "super-rigid" subgroup.

P. Will (Université Grenoble 1)

Teichmüller slices in the $\mathrm{PU}(2,1)$ **representation variety of surface groups**

Organising Committee:

Misha Belolipetsky (Durham), Martin Bridson (Oxford), Marc Lackenby (Oxford), John Parker (Durham)

Scientific advisers:

Misha Kapovich (UC Davis), Alan Reid (UT Austin)