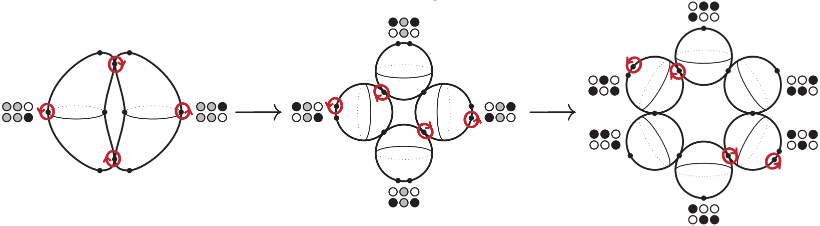
Maximal Unitarity at Two Loops

Kasper J. Larsen
Nikhef



Durham, LMS Symposium

Polylogarithms as a Bridge between Number Theory and Particle Physics

July 12, 2013

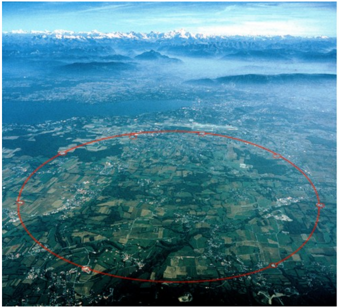
Based on 1108.1180, 1205.0801, 1208.1754
(with S. Caron-Huot, H. Johansson and D. Kosower)

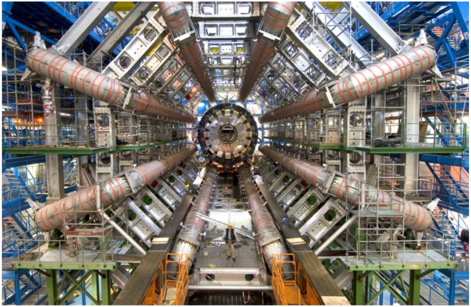
Part 1: Introduction

- motivations for studying amplitudes
- modern methods for computation at one loop

Obvious motivation: Large Hadron Collider

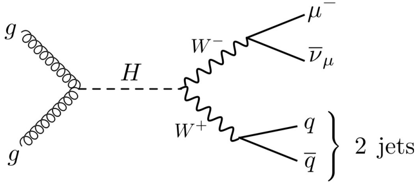
The searches at LHC for physics beyond the Standard Model require a detailed understanding of background, especially QCD, processes.



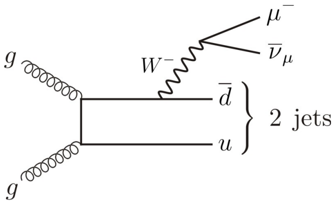


Examples of signals and QCD backgrounds

Signal: An example of a Higgs boson process:



Background: An example of a QCD background process:



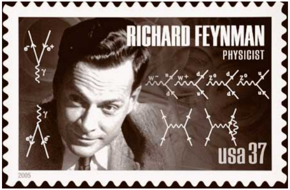
Two motivations, actually

In fact, there are two important motivations:

- **LHC phenomenology**
  Quantitative estimates of QCD background: needed for precision measurements, uncertainty estimates of NLO calculations, and reducing renormalization scale dependence.

- **Reveal fascinating structure in QFT**
  For $\mathcal{N} = 4$ SYM: hidden symmetries (integrability $\longrightarrow$ non-perturbative solution) and new dualities (to Wilson loops and correlators).

  For $\mathcal{N} \leq 4$: connection to multivariate complex analysis and algebraic geometry.



In practice, the Feynman diagram prescription produces a very large number of terms: e.g. for the five-gluon tree-level amplitude

$k_1 \cdot k_4 \, \varepsilon_2 \cdot k_1 \, \varepsilon_1 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5$

Feynman diagrams hide simplicity

Yet, the final result for five-gluon tree-level amplitude is simple,

$$A_5^{\text{tree}}(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$
$$A_5^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+) = \frac{i\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}\,.$$

This strongly suggests there should exist better methods for computing amplitudes.

At one-loop level, unitarity has proven very successful, allowing e.g. the calculation of $qg \to W +$ multi-jets.

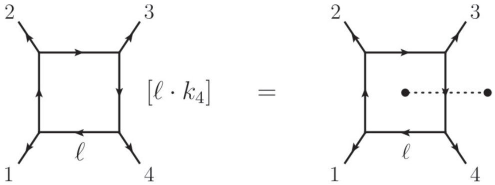
This talk is about extending generalized unitarity (systematically) to two loops.

Integral reductions and integral basis

Feynman rules $\longrightarrow$ *numerator powers in integrals*

At one loop, all such integrals can be expanded in a *basis.*

For example, consider the box insertion

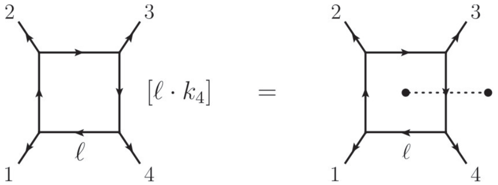


Integral reductions and integral basis

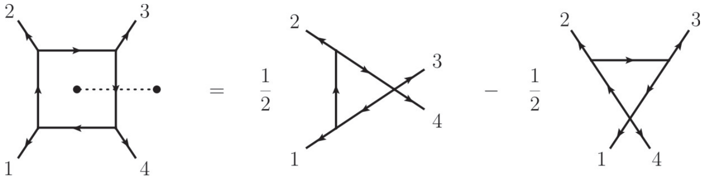
Feynman rules $\longrightarrow$ *numerator powers in integrals*

At one loop, all such integrals can be expanded in a *basis.*

For example, consider the box insertion

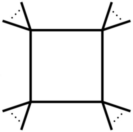
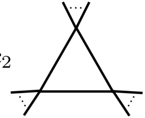
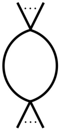
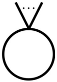


By using the identity $\ell \cdot k_4 = \frac{1}{2}\left((\ell + k_4)^2 - \ell^2\right)$, this can be reduced to



The modern unitarity approach (1/2)

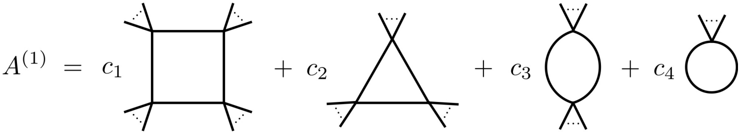
Use integral reductions to write the one-loop amplitude as a linear combination of *basis integrals*

$A^{(1)} = c_1$  $+ \; c_2$  $+ \; c_3$  $+ \; c_4$ 

$+ \;$ rational terms

The modern unitarity approach (1/2)

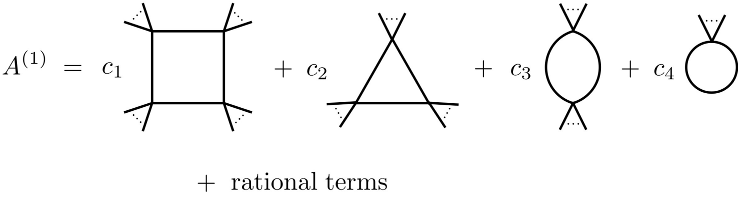
Use integral reductions to write the one-loop amplitude as a linear combination of *basis integrals*



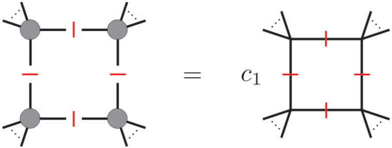
$$+ \text{ rational terms}$$

To determine c_i, apply cuts $\frac{1}{(\ell-K)^2} \longrightarrow \delta\big((\ell-K)^2\big)$ to both sides.

Use integral reductions to write the one-loop amplitude as a linear combination of *basis integrals*

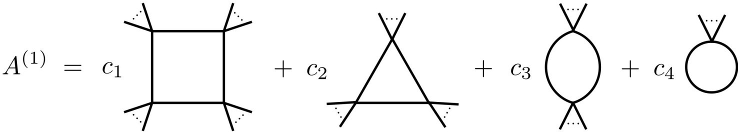


To determine c_i, apply cuts $\frac{1}{(\ell-K)^2} \longrightarrow \delta\big((\ell-K)^2\big)$ to both sides.
Applying a quadruple cut [Britto, Cachazo, Feng] isolates a single box integral:



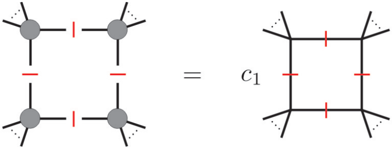
The modern unitarity approach (1/2)

Use integral reductions to write the one-loop amplitude as a linear combination of *basis integrals*



$$+ \text{ rational terms}$$

To determine c_i, apply cuts $\frac{1}{(\ell-K)^2} \longrightarrow \delta\big((\ell-K)^2\big)$ to both sides.
Applying a quadruple cut [Britto, Cachazo, Feng] isolates a single box integral:



$$\Longrightarrow \quad c_1 = \frac{1}{2} \sum_{\text{kin sols}} \prod_{j=1}^{4} A_j^{\text{tree}}$$

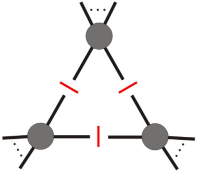
The modern unitarity approach (2/2)

A triple cut will leave $4 - 3 = 1$ free *complex* parameter z.
Parametrizing the loop momentum,

$$\ell^\mu = \alpha_1 K_1^{\flat\mu} + \alpha_2 K_2^{\flat\mu} + \frac{z}{2}\langle K_1^{\flat-}|\gamma^\mu|K_2^{\flat-}\rangle + \frac{\alpha_4(z)}{2}\langle K_2^{\flat-}|\gamma^\mu|K_1^{\flat-}\rangle$$

one obtains an explicit formula for the triangle coefficient [Forde]

$$c_\triangle = \oint_{C(\infty)} \frac{dz}{z}$$



$$(z)$$

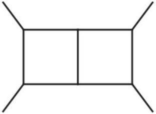
Part 2: From trees to two loops

- maximal cuts at two loops
- constructing two-loop amplitudes out of tree-level data
- elliptic integrals in $\mathcal{N} = 4$ SYM amplitudes

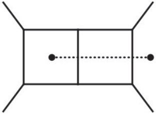
From trees to two loops

Expand the massless 4-point two-loop amplitude in a basis, e.g.

$A_4^{2-\text{loop}} = c_1(\epsilon)$



$+ \; c_2(\epsilon)$

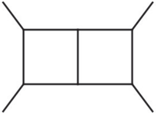
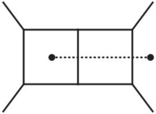


$+$ ints with fewer props

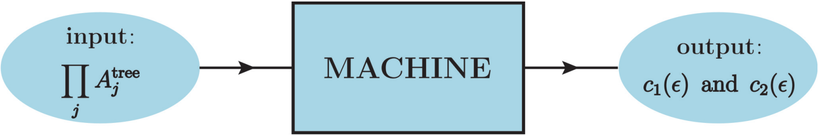
$+$ rational terms

From trees to two loops

Expand the massless 4-point two-loop amplitude in a basis, e.g.

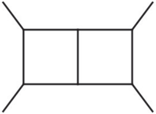
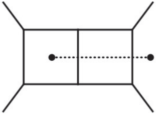
$A_4^{2-\text{loop}} = c_1(\epsilon)$  $+ \; c_2(\epsilon)$  $+$ ints with fewer props $+$ rational terms

Compute $c_1(\epsilon)$ and $c_2(\epsilon)$ according to

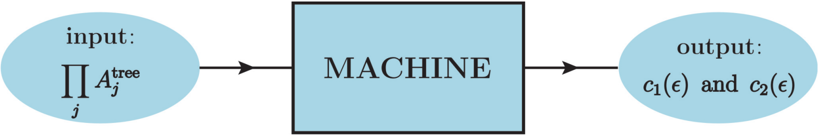


From trees to two loops

Expand the massless 4-point two-loop amplitude in a basis, e.g.

$A_4^{2-\text{loop}} = c_1(\epsilon)$  $+ \, c_2(\epsilon)$  + ints with fewer props + rational terms

Compute $c_1(\epsilon)$ and $c_2(\epsilon)$ according to

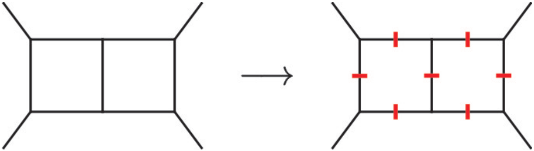


The machinery: *contour integrals* $\oint_{\Gamma_j}(\cdots)$

The philosophy: basis integral $I_j \longleftrightarrow$ unique Γ_j producing c_j

The anatomy of two-loop maximal cuts

Cutting all seven visible propagators in the double-box integral,



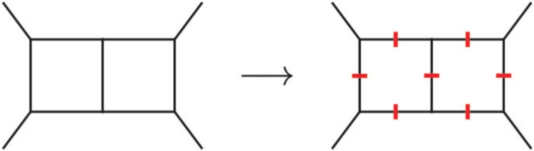
produces (cf. [Buchbinder, Cachazo]), setting $\chi \equiv \frac{t}{s}$,

$$\int d^4p\, d^4q \prod_{i=1}^{7} \frac{1}{\ell_i^2} \quad \longrightarrow \quad \int d^4p\, d^4q \prod_{i=1}^{7} \delta^{\mathbb{C}}(\ell_i^2) \;=\; \oint_{\Gamma} \frac{dz}{z(z+\chi)}\,,$$

a contour integral in the complex plane.

The anatomy of two-loop maximal cuts

Cutting all seven visible propagators in the double-box integral,



produces (cf. [Buchbinder, Cachazo]), setting $\chi \equiv \frac{t}{s}$,

$$\int d^4p\, d^4q \prod_{i=1}^{7} \frac{1}{\ell_i^2} \longrightarrow \int d^4p\, d^4q \prod_{i=1}^{7} \delta^{\mathbb{C}}(\ell_i^2) = \oint_{\Gamma} \frac{dz}{z(z+\chi)},$$

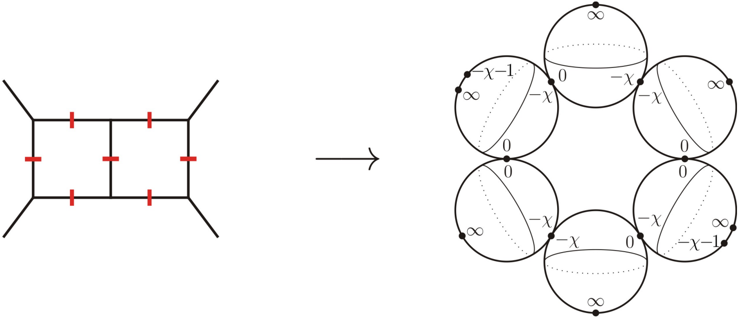
a contour integral in the complex plane.

Jacobian poles $z = 0$ and $z = -\chi$: composite leading singularities

encircle $z = 0$ and $z = -\chi$ with $\Gamma = \omega_1 C_\epsilon(0) + \omega_2 C_\epsilon(-\chi)$
$\longrightarrow$ freeze z ("$8^{\rm th}$ cut")

Choosing contours: *die Qual der Wahl*

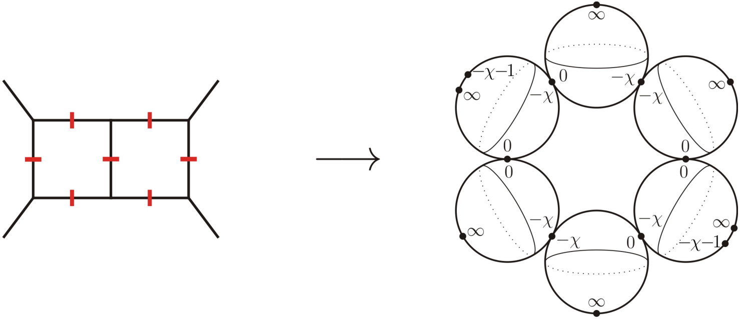
Six inequivalent classes of solutions to on-shell constraints



4 massless external states $\longrightarrow$ 8 independent leading singularities

Choosing contours: *die Qual der Wahl*

Six inequivalent classes of solutions to on-shell constraints



4 massless external states $\longrightarrow$ 8 independent leading singularities

How do we select contours within this variety of possibilities?

Principle for selecting contours

To fix the contours, insist that

vanishing Feynman integrals must have vanishing heptacuts.

This ensures that

$$\mathrm{I}_1 = \mathrm{I}_2 \quad \Longrightarrow \quad \text{cut}\,(\mathrm{I}_1) = \text{cut}\,(\mathrm{I}_2)\,.$$

Principle for selecting contours

To fix the contours, insist that

vanishing Feynman integrals must have vanishing heptacuts.

This ensures that

$$\mathrm{I}_1 = \mathrm{I}_2 \quad \Longrightarrow \quad \mathrm{cut}\,(\mathrm{I}_1) = \mathrm{cut}\,(\mathrm{I}_2)\,.$$

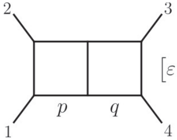
Origin of terms with vanishing $\mathbb{R}^D \times \mathbb{R}^D$ integration:
reduction of Feynman diagram expansion to a *basis of integrals*
(including use of integration-by-parts identities).

Remarkable simplification:

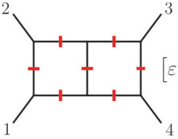
- 4 massless external states: 22 $\longrightarrow$ 2 double-box integrals
- 5 massless external states: 160 $\longrightarrow$ 2 "turtle-box" integrals
- 5 massless external states: 76 $\longrightarrow$ 1 pentagon-box integral

There are two classes of constraints on Γ's:

1) Levi-Civita integrals. For example,



$$[\varepsilon(p,1,2,4)] \;=\; 0 \quad \Longrightarrow$$

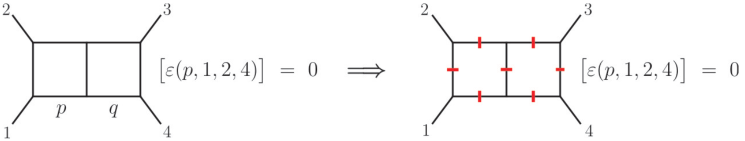


$$[\varepsilon(p,1,2,4)] \;=\; 0$$

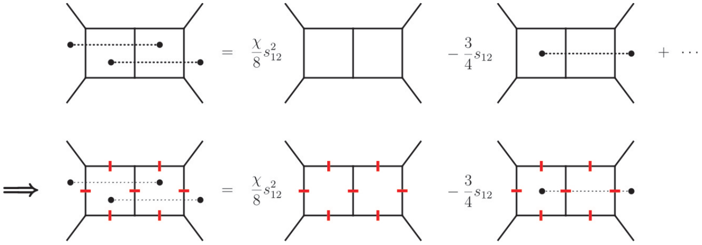
Contour constraints, part 1/2

There are two classes of constraints on Γ's:

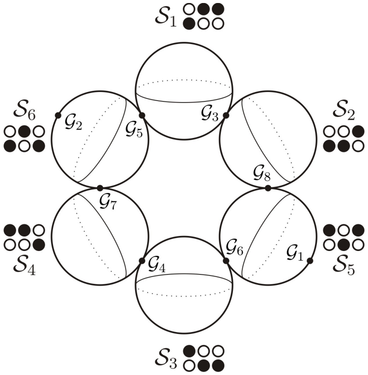
1) Levi-Civita integrals. For example,



2) integration by parts (IBP) identities must be preserved. For example,



The constraints in the case of four massless external momenta:

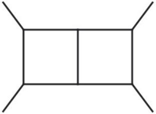
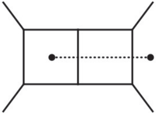


$$\begin{aligned}
\omega_1 - \omega_2 &= 0 \\
\omega_3 - \omega_4 &= 0 \\
\omega_5 - \omega_6 &= 0 \\
\omega_7 - \omega_8 &= 0 \\
\omega_3 + \omega_4 - \omega_5 - \omega_6 &= 0 \\
\omega_1 + \omega_2 - \omega_5 - \omega_6 + \omega_7 + \omega_8 &= 0
\end{aligned}$$

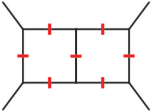
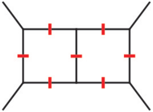
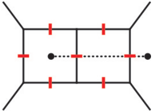
leaving $8 - 4 - 2 = 2$ free winding numbers.

Master contours: the concept

Going back to the two-loop basis expansion

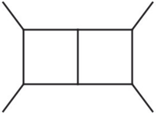
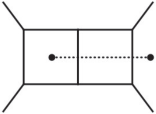
$A_4^{2-\text{loop}} = c_1(\epsilon)$  $+\ c_2(\epsilon)$  + ints with fewer props + rational terms

and applying a heptacut one finds

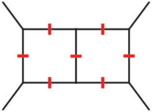
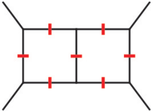
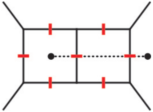
 $\left[\prod_{j=1}^{6} A_j^{\text{tree}}\right] = c_1(\epsilon)$  $+\ c_2(\epsilon)$ 

Master contours: the concept

Going back to the two-loop basis expansion

$A_4^{2-\text{loop}} \;=\; c_1(\epsilon)$  $+\; c_2(\epsilon)$  $+$ ints with fewer props $+$ rational terms

and applying a heptacut one finds

 $\left[\prod_{j=1}^{6} A_j^{\text{tree}}\right] \;=\; c_1(\epsilon)$  $+\; c_2(\epsilon)$ 

Exploit free parameters $\longrightarrow$ $\exists$ contours with

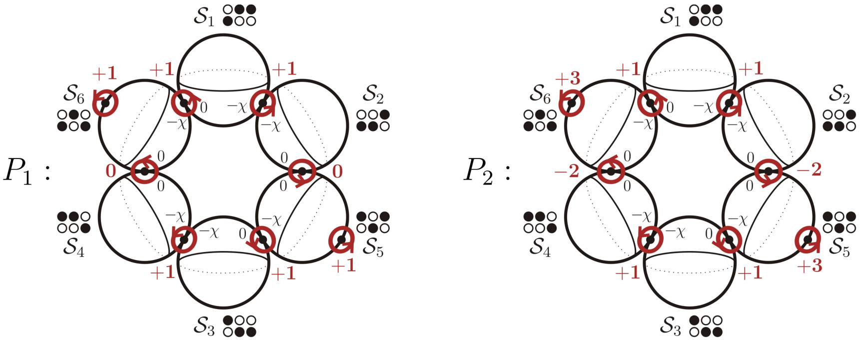
$$P_1 : \;\big(\text{cut}\,(\mathrm{I}_1),\, \text{cut}\,(\mathrm{I}_2)\big) \;=\; (1,0)$$
$$P_2 : \;\big(\text{cut}\,(\mathrm{I}_1),\, \text{cut}\,(\mathrm{I}_2)\big) \;=\; (0,1)\,.$$

We call such P_i *master contours*.

With four massless external states,

$$c_1 = \frac{i\chi}{8} \oint_{P_1} \frac{dz}{z(z+\chi)} \prod_{j=1}^{6} A_j^{\text{tree}}(z) \qquad c_2 = -\frac{i}{4s_{12}} \oint_{P_2} \frac{dz}{z(z+\chi)} \prod_{j=1}^{6} A_j^{\text{tree}}(z)$$

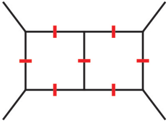
With our choice of basis integrals, the P_i are



n = winding number

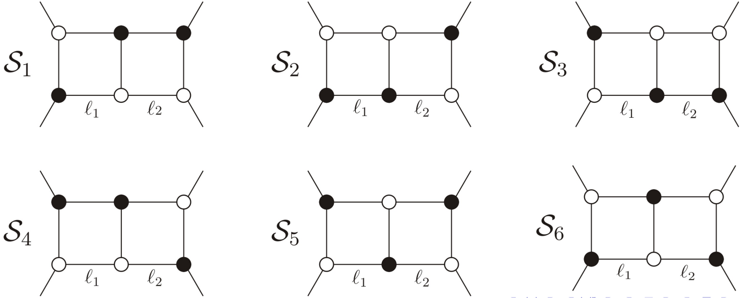
Characterizing the on-shell solutions

There are six solutions for the heptacut loop momenta



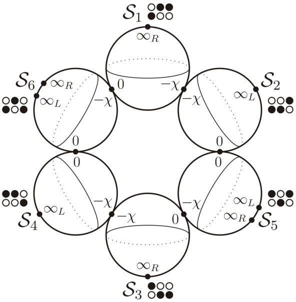
Set $k_i^\mu = \lambda_i \sigma^\mu \widetilde{\lambda}_i$ and classify each vertex according to

$$\lambda_a \propto \lambda_b \propto \lambda_c \quad (\overline{\text{MHV}}) \quad \longrightarrow \quad \bullet$$
$$\widetilde{\lambda}_a \propto \widetilde{\lambda}_b \propto \widetilde{\lambda}_c \quad (\text{MHV}) \quad \longrightarrow \quad \circ$$

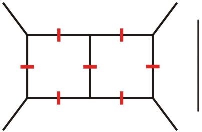


Two-loop leading singularities

heptacut solutions $\longrightarrow$ Riemann spheres
(e.g., $c_\triangle = \oint_{C_\epsilon(\infty)} \frac{dz}{z} \prod_{j=1}^{3} A_j^{\text{tree}}(z)$)



$\longleftarrow$



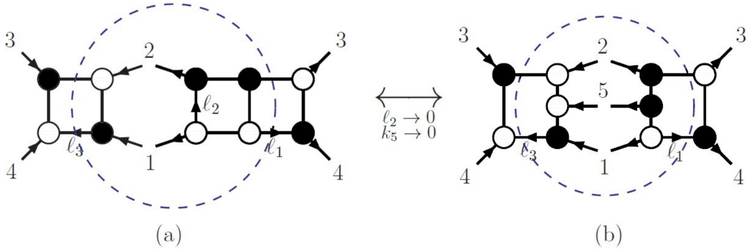
$\Big|_{\mathcal{S}_i} = \oint_{\Gamma_i} \frac{dz}{z(z+\chi)}$

points $\in \mathcal{S}_i \cap \mathcal{S}_j \longrightarrow$ no notion of ● or ○ $\longrightarrow$ resp. prop. is soft
also: $\mathcal{S}_i \cap \mathcal{S}_j \subset \{$leading singularities$\}$!

two-loop leading singularities $\longrightarrow$ IR singularities of integral

Two-loop leading vs. IR singularities

Observation: leading-singularity residues cancel between virtual (a) and real (b) contributions to cross section



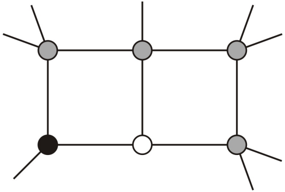
in complete analogy with the KLN theorem on IR cancelations.

Classification of heptacut solutions

Arbitrary # of external states. Define

$$\mu_i \equiv \begin{cases} \text{m} & \text{if } i^{\text{th}} \text{ vertical prop. } \in \text{3-pt. vertex} \\ \text{M} & \text{if } i^{\text{th}} \text{ vertical prop. } \notin \text{3-pt. vertex} \end{cases}$$

e.g.,



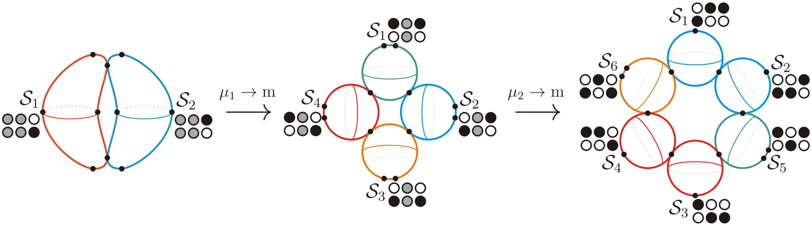
is (m,m,M)

The solution to $\ell_i^2 = 0$, $i = 1, \ldots, 7$ is

- case 1 (M,M,M): 1 torus
- case 2 (M,M,m) etc.: 2 $\mathbb{CP}^1$ with $\mathcal{S}_i \longleftrightarrow$ distrib. of ●, ○
- case 3 (M,m,m) etc.: 4 $\mathbb{CP}^1$ with $\mathcal{S}_i \longleftrightarrow$ distrib. of ●, ○
- case 4 (m,m,m): 6 $\mathbb{CP}^1$ with $\mathcal{S}_i \longleftrightarrow$ distrib. of ●, ○

Uniqueness of master contours

Limits $\mu_i \to \mathrm{m} \implies$ chiral branchings: torus $\xrightarrow{\mu_3 \to \mathrm{m}}$



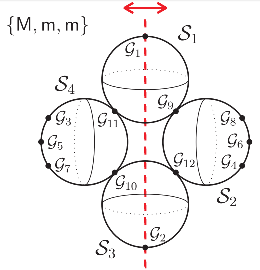
Each torus-pinching: new IR-pole + new residue thm

$\implies$ # of lead. sing. same in all cases

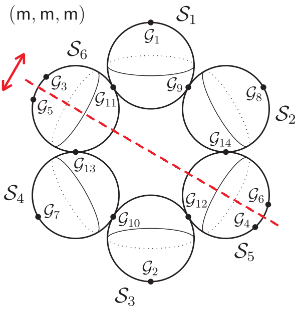
In all cases: **# of master Γ's = # of basis integrals**

$\implies$ all linear relations are preserved

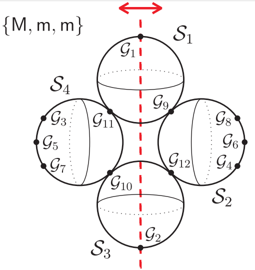
$\implies$ perfect analogy with one-loop generalized unitarity



The IBP constraints are invariant under flips.



Symmetries and systematics of IBP constraints



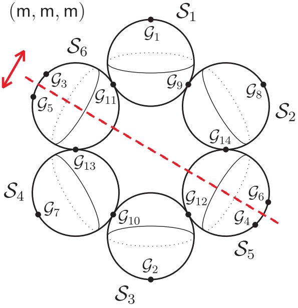
The IBP constraints are invariant under flips.

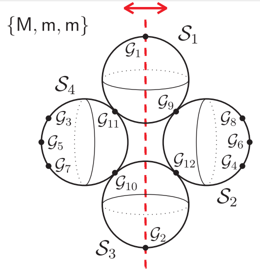
Reverse logic $\longrightarrow$ demand constraints to be invariant under flips and π-rotations.

$\{$M,m,m$\}$ case: choose basis, e.g. $\omega_{1,2,5,6} = 0$

$$r_1^{(b)}(\omega_3 + \omega_4 + \omega_7 + \omega_8) + r_2^{(b)}(\omega_9 + \omega_{10} - \omega_{11} - \omega_{12}) = 0$$

where, in fact, $r_1^{(b)} = r_2^{(b)} \neq 0$.





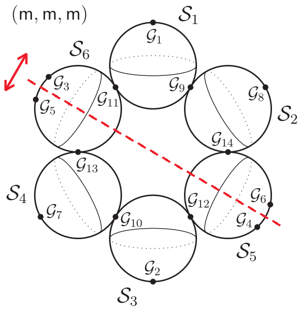
The IBP constraints are invariant under flips.

Reverse logic $\longrightarrow$ demand constraints to be invariant under flips and π-rotations.

{M,m,m} case: choose basis, e.g. $\omega_{1,2,5,6} = 0$

$$r_1^{(b)}(\omega_3 + \omega_4 + \omega_7 + \omega_8) + r_2^{(b)}(\omega_9 + \omega_{10} - \omega_{11} - \omega_{12}) = 0$$

where, in fact, $r_1^{(b)} = r_2^{(b)} \neq 0$.



(m,m,m) case:

1) constraint from {M,m,m} case inherited.
2) new flip symmetry $\longrightarrow$ new constraint:

$$r_1^{(c)}(\omega_3 + \omega_4) + r_2^{(c)}(\omega_{11} + \omega_{12} - \omega_{13} - \omega_{14}) = 0$$

as expressed in the basis $\omega_{1,2,5,6,7,8} = 0$.

In fact, $r_1^{(c)} = -r_2^{(c)} \neq 0$.

Integrals with fewer propagators

Solution to slashed-box on-shell constraints:

On-shell constraints leave $8 - 5 = 3$ free complex parameters.

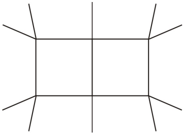
Multivariate residues depend on the order of integration.

Example: $f(z_i) = \frac{z_1}{z_2(a_1 z_1 + a_2 z_2)(b_1 z_1 + b_2 z_2)}$. Residues at $(z_1, z_2) = (0, 0)$:

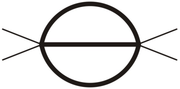
$$\frac{1}{(2\pi i)^2} \int_{C_\epsilon(0) \times C_{\epsilon^2}(0)} dz_1\, dz_2\; f(z_i) \;=\; \frac{1}{a_1 b_1}$$

$$\frac{1}{(2\pi i)^2} \int_{C_\epsilon(0) \times C_{\epsilon^2}\left(-\frac{a_1}{a_2} z_1\right)} dz_1\, dz_2\; f(z_i) \;=\; \frac{a_2}{a_1(a_1 b_2 - a_2 b_1)}$$

Elliptic curves vs. polylogs



$$= \int_u^\infty \frac{du'}{\sqrt{\tilde{Q}(u')}} \times (\mathrm{Li}_3(\cdots) + \cdots)$$



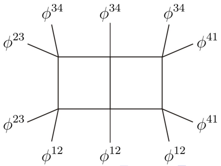
$$= \int_u^\infty \frac{du'}{\sqrt{\tilde{Q}(u')}} \times (\log(\cdots) + \cdots)$$

sunrise integral not expressible through polylogs
$\longrightarrow$ neither should 10-point integral be

Analytic expression $\longleftrightarrow$ maximal cut?

Wilson-loop amplitude correspondence $\Longrightarrow$

$$\mathcal{N} = 4 \text{ SYM:} \quad A^{(2)}(10\text{–scalar N}^3\text{MHV}) \quad \propto$$



Conclusions and outlook

- First steps towards fully automatized two-loop amplitudes
- Integration-by-parts identities $\longrightarrow$ reduce # of Feynman integrals by factor of 10-100

- Two-loop master contours are unique
  $\longrightarrow$ perfect analogy with one-loop unitarity
- Classification of maximal-cut solutions

- Maximal cuts contain vital information:
  pinches/punctures $\longrightarrow$ IR/UV divergences
  branch cuts $\longrightarrow$ non-polylogs in uncut integral

- Underlying algebraic geometry $\longrightarrow$ deeper understanding of maximal cuts (i.e., contour constraints)

Backup slides

Integrals and integral bases

- ideal two-loop basis: chiral integrals
- evaluate 4-point chiral integrals analytically

Maximally IR-finite basis

The two-loop integral coefficients c_i have $\mathcal{O}(\epsilon)$ corrections.

Important to know, as the integrals have poles in ϵ.

Maximally IR-finite basis

The two-loop integral coefficients c_i have $\mathcal{O}(\epsilon)$ corrections.

Important to know, as the integrals have poles in ϵ.

IR-finite integrals $\longrightarrow$ $\mathcal{O}(\epsilon)$ corrections not needed for amplitude

Candidates: num. insertions $\to 0$ in collinear int. regions, e.g.

$$\begin{aligned} I_{++} &\equiv I\big[[1|\ell_1|2\rangle\langle 3|\ell_2|4]\big] \times [2\,3]\langle 1\,4\rangle \\ I_{+-} &\equiv I\big[[1|\ell_1|2\rangle\langle 4|\ell_2|3]\big] \times [2\,4]\langle 1\,3\rangle \end{aligned}$$

Essentially the chiral integrals of [Arkani-Hamed et al.]

I_{++} and I_{+-} lin. independent $\longrightarrow$ use in *any gauge theory*

Philosophy: maximally IR-finite basis
$\longrightarrow$ minimize need for cuts in $D = 4 - 2\epsilon$

Evaluation of chiral integrals (1/3)

$I_{+\pm}$ are finite $\longrightarrow$ can be computed in $D = 4$

1) Feynman parametrize

$$I_{++} = -\chi^2 \left(1 + (1+\chi)\frac{\partial}{\partial \chi}\right) I_1(\chi) \quad \text{and} \quad I_{+-} = -(1+\chi)^2 \left(1 + \chi \frac{\partial}{\partial \chi}\right) I_1(\chi)$$

where

$$I_1(\chi) = \int \frac{d^3a \; d^3b \; dc \; c \; \delta(1 - c - \sum_i a_i - \sum_i b_i)\left(\sum_i a_i \sum_i b_i + c(\sum_i a_i + \sum_i b_i)\right)^{-1}}{\left(a_1 a_3 (c + \sum_i b_i) + (a_1 b_4 + a_3 b_6 + a_2 b_5 \chi)c + b_4 b_6 (c + \sum_i a_i)\right)^2}$$

2) "Projectivize"

$$I_1(\chi) = 6 \int_1^\infty dc \int_0^\infty \frac{d^7(a_1 a_2 a_3 a_{\mathcal{I}} b_1 b_2 b_3 b_{\mathcal{I}})}{\mathrm{vol}(\mathrm{GL}(1))} \frac{1}{(cA^2 + A.B + B^2)^4}$$

3) Obtain symbol

Integrate projective form one variable at the time, at the level of the symbol.

$$\mathcal{S}[I_1(\chi)] = \frac{2}{\chi}\,[\chi \otimes \chi \otimes (1+\chi) \otimes (1+\chi)] - \frac{2}{1+\chi}\,[\chi \otimes \chi \otimes (1+\chi) \otimes \chi]$$

4) "Integrate" symbol, using

a) I_1 has transcendentality 4 (fact, not a conjecture)

b) I_1 has no u-channel discontinuity

c) Regge limits:

$$I_1(\chi) \;\to\; \frac{\pi^2}{6}\log^2\chi + \left(4\zeta(3) - \frac{\pi^2}{3}\right)\log\chi + \mathcal{O}(1) \quad \text{as} \quad \chi \to 0$$

$$I_1(\chi) \;\to\; 6\zeta(3)\frac{\log\chi}{\chi} + \mathcal{O}(\chi^{-1}) \quad \text{as} \quad \chi \to \infty$$

In conclusion, for the "chiral" integrals

$$I_{++} \equiv I\big[[1|\ell_1|2\rangle\langle 3|\ell_2|4]\big] \times [2\,3]\langle 1\,4\rangle$$
$$I_{+-} \equiv I\big[[1|\ell_1|2\rangle\langle 4|\ell_2|3]\big] \times [2\,4]\langle 1\,3\rangle$$

we find the results

$$\begin{aligned} I_{++}(\chi) &= 2H_{-1,-1,0,0}(\chi) - \frac{\pi^2}{3}\mathrm{Li}_2(-\chi) \\ &\quad + \left(\frac{\pi^2}{2}\log(1+\chi) - \frac{\pi^2}{3}\log\chi + 2\zeta(3)\right)\log(1+\chi) - 6\chi\zeta(3) \\ I_{+-}(\chi) &= 2H_{0,-1,0,0}(\chi) - \pi^2\mathrm{Li}_2(-\chi) - \frac{\pi^2}{6}\log^2\chi - 4\zeta(3)\log\chi - \frac{\pi^4}{10} - 6(1+\chi)\zeta(3) \end{aligned}$$

Actual chiral integrals: transcendentality-breaking terms cancel.