Use of Non-Maxwellian Distributions in Atomic and Nuclear Processes of Astrophysical Interest

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5 Interesting Processes

- Non-resonant nuclear reactions in stars
- Radiative Recombination (RR) in stars
- Resonant reactions in stars
- Dielectronic recombination (DR)
- Deuteron-deuteron fusion reactions in deuterated metals with deuteron beams

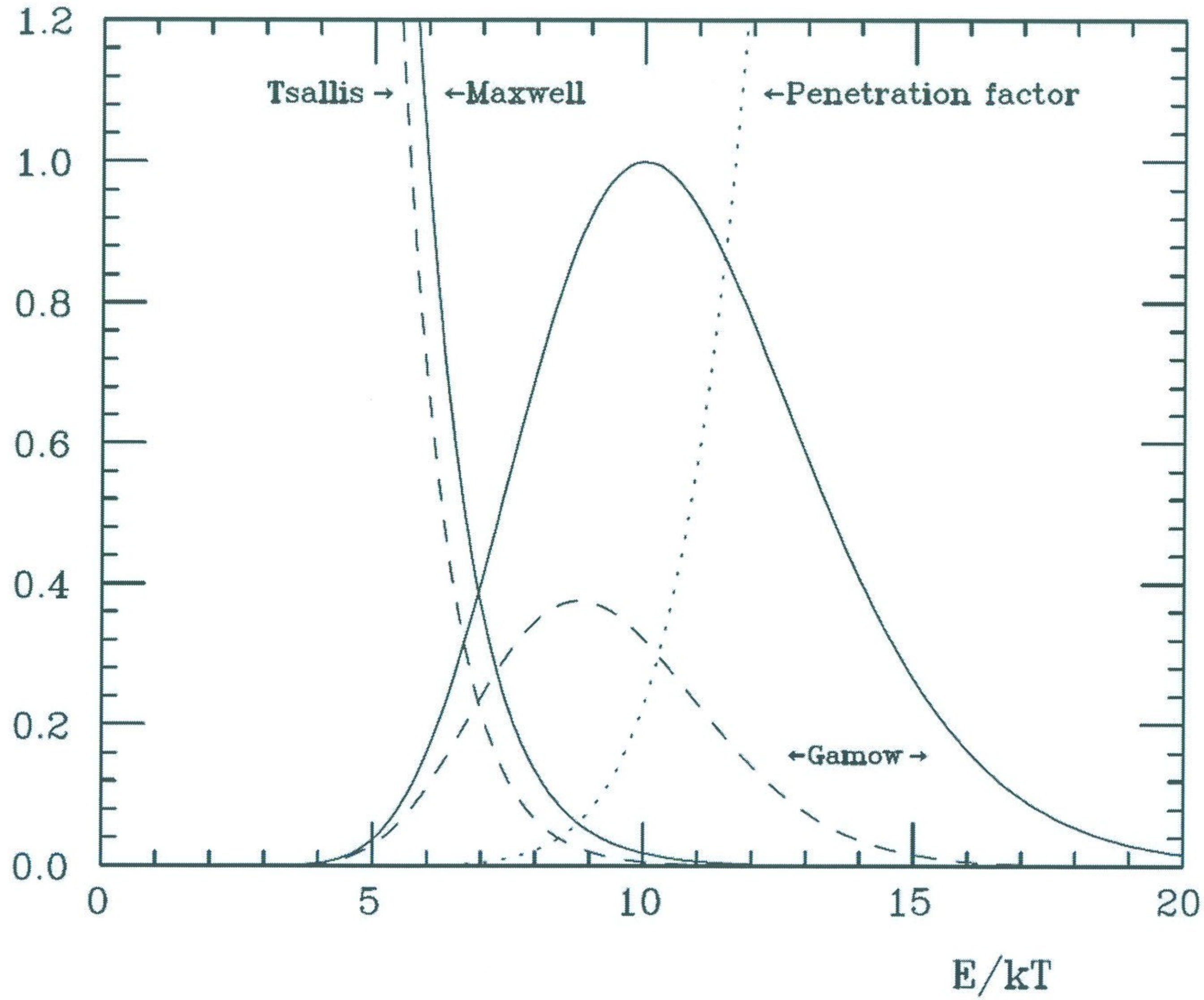
Non-Resonant Nuclear Reactions in Stars

$p + p$ in the Sun

Only very few particles responsible of energy production

- Coulomb repulsion makes fusion difficult
- Tunnel effect (Gamow peak)
- Rate very sensitive to the tail of the distribution
- Small enhancement or depletion leads to major consequences on rates
- Experiments on light ion fusion at Gran Sasso Lab

The Gamow-Peak

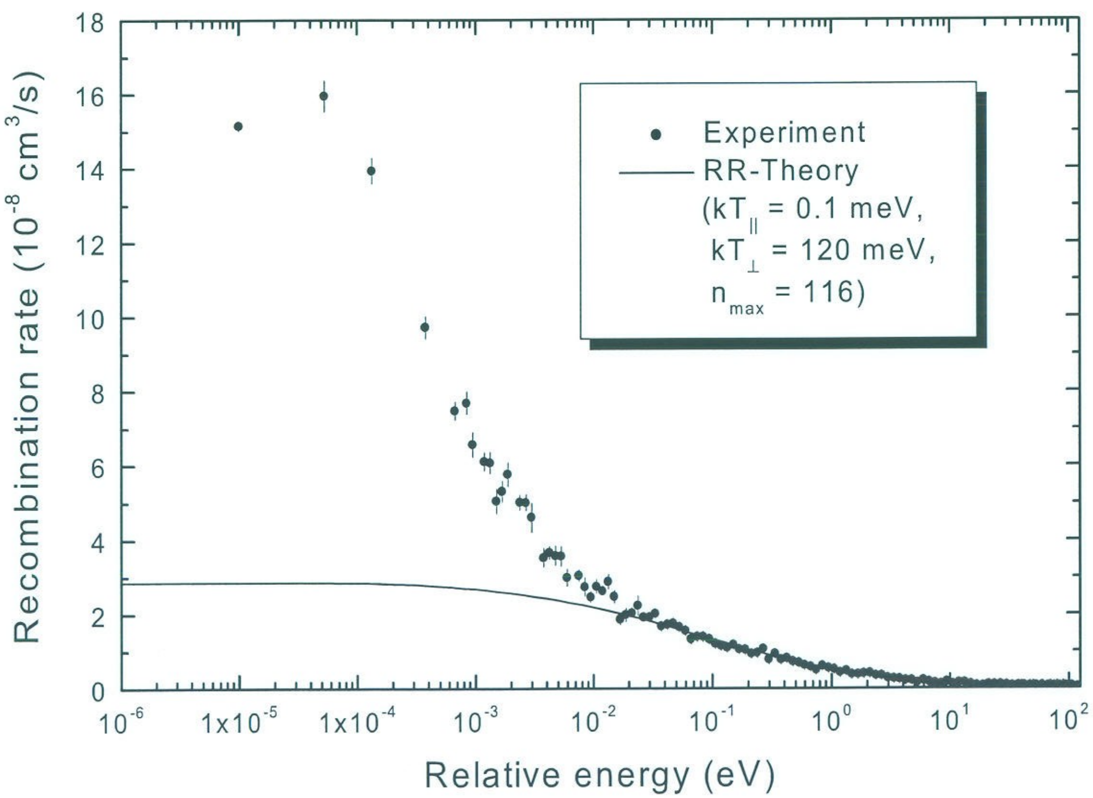


Radiative Recombination (RR) in Stars (Atomic)

Acts among electrons and ions in stellar systems

$$A^{Z+} + e^{-} \rightarrow A^{(Z-1)+} + h\nu$$

- Can be studied in Lab or cooling devices in storage rings
- Non-resonant process: high at $E_R \rightarrow 0$
- Very sensitive to the head of distribution



Strong increase of experimental rate respect to theory!

Resonant Fusion Reactions in Astrophysical Plasmas

CNO-cycle

Described by a Lorentzian cross section

- Rate sensitive to position and width of the resonance with respect to the Gamow peak
- Careful evaluation of $f(E_R)$ close to resonance required
- Small changes from MB lead to great discrepancies (*much more than for non-resonant reactions*)
- Studied in Lab, although measures are prohibitively difficult at stellar temperatures ($< 1\,\mathrm{MeV}$)

Dielectronic Recombination (DR) (Atomic)

Resonant process in star atmospheres and storage rings devices (e.g. GSI)

$$A^{Z+} + e^- \rightarrow (A^{(Z-1)+})^* \rightarrow A^{(Z-1)+} + h\nu$$

The rate depends on:

- DR peaks position respect to temperature
- Head and cutoff of the distribution (for $T < 100\,\mathrm{eV}$)
- Complicated atomic structure
  - 21 resonances for C^{3+} between $0.2 \div 0.6\,\mathrm{eV}$

Transfer of information from Lab experiment to astrophysical plasmas complicated because:

- In Lab, plasmas **are not** in a global equilibrium state
- Stellar plasmas are in other stationary states
- Hypothesis that everything is Maxwellian **hardly verified**

Deuteron-Deuteron Fusion Reactions in Metals

- Accelerated deuteron beams against deuterated metal matrix
- Very important for applications
- Related to problem of hydrogen stocking in cells

Experimental increase (Bochum) **not** explained by atomic screening or other classical effects

- Energy-momentum uncertainty increases the $f(E_R)$ tail
- Looks like a non-extensive deformation, although it is a quantum effect
- The energy-momentum dispersion is Lorentzian
- Enhancement if deformation close to Lorentzian peak

Approach for Treating the Previous 5 Problems

Usually it is based on assumptions of:

- **Global or local thermodynamical equilibrium** (TE, LTE)
- **Maxwellian** distribution of particles (*also when radiation is not described by a Planck distribution*)

MB is a good approximation for describing the 5 problems

Let us consider the Sun. After solving the neutrino oscillation puzzle:

- We forgot about **structural problems**, still to be solved
- We forgot the problem of **choosing a suitable distribution function** $f(v)$

The Standard Solar Model resurrected

The Choice of the Distribution Function

For passing from MB to another distribution with more particle either in the head or in the tail we must:

- either receive energy from the system
- or give energy

If the system is isolated this energy is *exchanged with the system itself* owing to

- **correlations** among particles (non-linearizable effects)
- presence of an internal **finite energy bath** (related to a cutoff in the distribution)

Reasons for Choosing a Slightly Deformed Maxwellian

In addition to experimental facts, other few reasons call for a deformed Maxwellian distribution:

- Electric *microfields*
- Correlations
- Fluctuations
- Random forces

All previous effects are non-linearizable!

A departure from TE and LTE can go towards **Non-Extensive (NE) generalized statistics**

Stationary Distribution Function Under Random Forces

We start from a kinetic equation in presence of an external/internal random force $\mathcal{F}$, with collision frequency ν:

$$\pm\frac{2}{3}\frac{\mathcal{F}^2}{\mu^2\nu^2}\frac{\mathrm{d}f}{\mathrm{d}v}+\kappa\left(vf+\frac{k_\mathrm{B}T}{\mu}\frac{\mathrm{d}f}{\mathrm{d}v}\right)=0$$

$$f(v)\propto\exp\left[-\int_0^v \mathrm{d}v'\frac{\mu v'}{k_\mathrm{B}T\pm\frac{2}{3}\frac{\mathcal{F}^2}{\mu\kappa\nu^2}}\right]$$

- $\mathcal{F}$ due to electric *microfields* or random forces
- $\nu^2\equiv\nu_0^2+\nu_1^2+\nu_2^2+\ldots$ if many competing interactions are present
- $\pm$ due to sub-/super-diffusivity

Interaction Cross Sections

$$\nu(v) \equiv nv\sigma(v)$$

- $\sigma_0(v) = \alpha_0 v^{-1}$: interaction between and ion and an induced-dipole
  - gives MB even in the presence of the external fi eld $\mathcal{F}$
- $\sigma_1(v) = \alpha_1$: reinforced Coulomb interaction
- $\sigma_2(v) = \alpha_2 v$: we shall show it is related to quantum effect

The Long-Life Stationary State ($\frac{T_{\mathrm{eff}}}{T} \leq \frac{\alpha_1^4}{4\alpha_0^2\alpha_2^2}$)

$$f(v) \propto \exp\left(-\frac{\mu v^2}{2k_{\mathrm{B}}T}\right) \times \left(\frac{2c_2v^2 + c_1 - 2\sqrt{|K|c_2}}{2c_2v^2 + c_1 + 2\sqrt{|K|c_2}}\right)^{\frac{\mu\tau}{4k_{\mathrm{B}}T\sqrt{|K|c_2}}}$$

- $c_1 \equiv \left(\frac{\alpha_1}{\alpha_0}\right)^2$ and $c_2 \equiv \left(\frac{\alpha_2}{\alpha_0}\right)^2$
- $\tau \equiv T_{\mathrm{eff}}/T - 1$ and $K \equiv -\frac{c_1^2}{4c_2} + \tau + 1$
- $k_{\mathrm{B}}T_{\mathrm{eff}} = k_{\mathrm{B}}T \pm \frac{2}{3}\frac{\mathcal{F}^2}{\kappa\mu n^2\alpha_0^2}$

Useful, for instance, for low-energy atomic physics

The Long-Life Stationary State ($\frac{T_{\rm eff}}{T} > \frac{\alpha_1^4}{4\alpha_0^2\alpha_2^2}$)

$$f(\varepsilon_p) \propto \exp\left[-\frac{\varepsilon_p}{k_{\rm B} T_{\rm eff}}\right] \exp\left[-\delta\left(\frac{\varepsilon_p}{k_{\rm B} T_{\rm eff}}\right)^2\right] \exp\left[-\gamma\left(\frac{\varepsilon_p}{k_{\rm B} T_{\rm eff}}\right)^3\right]$$

$$\delta = \pm\frac{2}{3}\frac{\mathcal{F}^2}{\kappa\mu^2 n^2}\frac{\alpha_1^2}{\alpha_0^4}$$

$$\gamma = \pm\frac{8}{9}\frac{\mathcal{F}^2 k_{\rm B} T}{\kappa\mu^3 n^2}\frac{\alpha_2^2}{\alpha_0^4}\left(1 - \frac{\alpha_1^4}{\alpha_0^2\alpha_2^2}\right) + \frac{16}{27}\frac{\mathcal{F}^4}{\kappa^2\mu^4 n^4}\frac{\alpha_2^2}{\alpha_0^6}$$

- δ-exp (Druyvenstein): if $\varepsilon_p \sim k_{\rm B} T_{\rm eff}/|\delta|$
- γ-exp: if $\varepsilon_p \sim |\delta/\gamma|\, k_{\rm B} T_{\rm eff}$

Connection with Non-Extensive Statistics

Our $f(\varepsilon_p)$ reduces to the NE distribution:

- In the limit $(q-1)\frac{\varepsilon_p}{k_B T_{\mathrm{eff}}} \to 0$
- With the position $\delta = (1-q)/2$
  - $q = 1 \mp \frac{4}{3}\frac{\mathcal{F}^2}{\kappa\mu^2 n^2}\frac{\alpha_1^2}{\alpha_0^4}$

In the case of the electric microfields we get $\delta \simeq 12\Gamma^2\alpha^4$:

- Γ, plasma parameter (generally $\Gamma \lesssim 1$ or $\Gamma \geq 1$)
- $0.4 < \alpha < 1$ (for dense stellar plasmas)
  - α parameter related to ion-ion correlation function

Quantum Effects in Stellar Plasmas

The quantum energy-momentum uncertainty with a Lorentz dispersion $\mathcal{D}(E, \varepsilon_p)$ gives a power-like tail on the $f(\varepsilon_p)$ distribution

$$f(\varepsilon_p) \equiv \int \mathrm{d}\varepsilon \mathcal{D}(\varepsilon, \varepsilon_p) \propto \frac{\sqrt{\varepsilon_p}}{(k_\mathrm{B} T)^{3/2}} \left[\exp\left(-\frac{\varepsilon_p}{k_\mathrm{B} T} \right) + \mathrm{const} \cdot \frac{(k_\mathrm{B} T)^{3/2}}{\varepsilon_p^4} \right]$$

$$f(\varepsilon_p) \sim f_{MB}(\varepsilon_p) + \mathrm{const} \cdot \frac{\sqrt{\varepsilon_p}}{(k_\mathrm{B} T)^{3/2}} \frac{(k_\mathrm{B} T)^{3/2}}{\varepsilon_p^4}$$

This behaviour is obtained from the kinetic equation if $\sigma_2 \propto \sqrt{\varepsilon_p}$ is assumed

From Which Interaction Does $\sigma_2 \propto \sqrt{\varepsilon_p}$ Come?

From a dimensional analysis, the interaction that hypothetically links quantum and non-extensive effects is tidal-like

$$F_Q(r) = f_{Q_0} \left(\frac{r}{R_0}\right)^3 \quad r \leq R_0$$

Assuming:

- An entropic parameter $q \sim 0.1$
- A proton plasma
- Density $n \approx 10^{-14}\ \mathrm{fm}^{-3}$
- $R_0 \approx 10^5\ \mathrm{fm}$

We obtain $f_{Q_0} \approx 10^{-12}\ \mathrm{MeV/fm}$

Non-Extensive Approach to Non-Resonant Fusion Reactions in the Sun

Schism between helioseismology and models with revised composition arisen because:

- Abundant elements (C, N, O, Ne) provide major contributions to the opacity of the solar interior
- This in turn influences internal the structure and the depth at which the interior becomes convective

Problems with SSM:

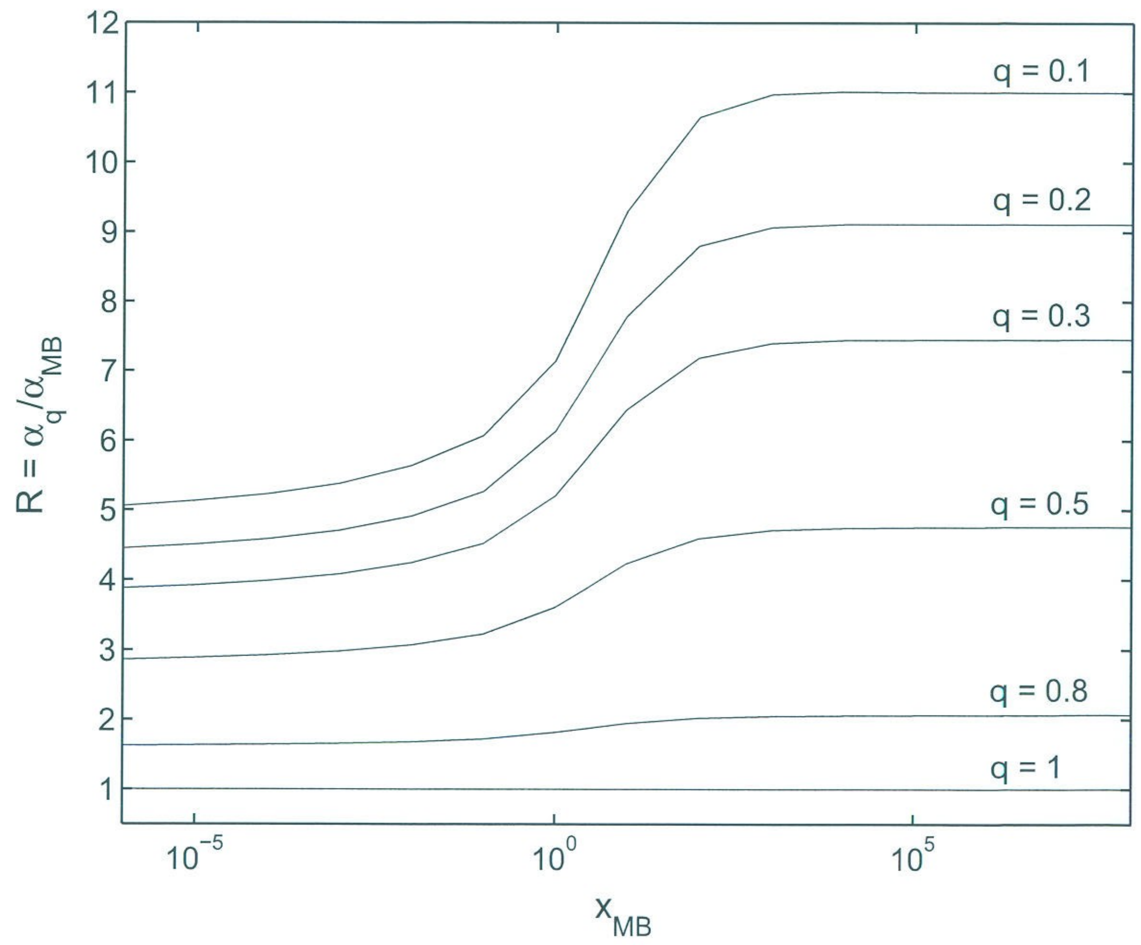
- Revision of abundances
- Neutrino fluxes (B^8, Be, hep)
- CNO flux

Little cracks in the solar neutrino physics?

Non-Extensive Approach to Radiative Recombination

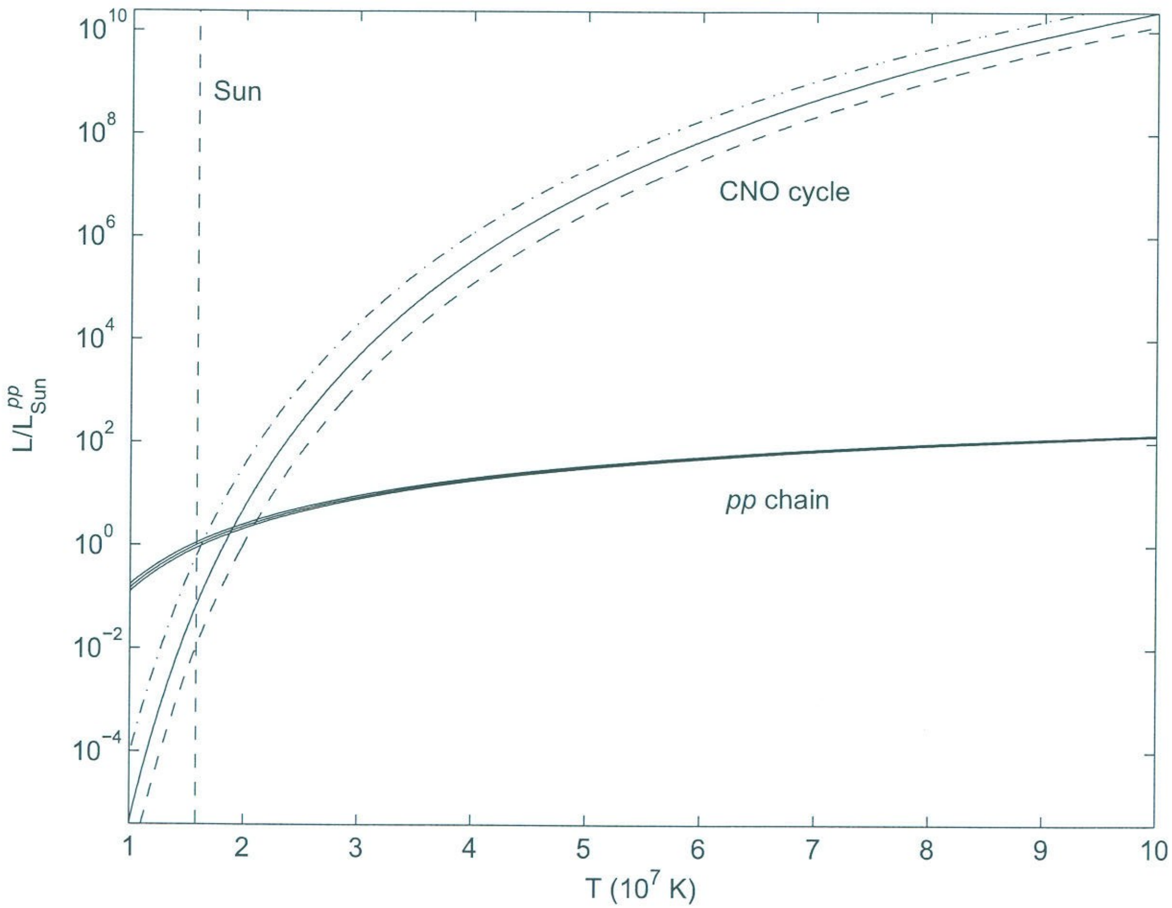
Experimental results require many particles in the head of energy distribution $f(E_R)$

- Use of the non-extensive $q < 1$ distribution
- Need of cutoff (high E_R)
- Major effects at high temperature ($x_{\mathrm{MB}} = k_B T / E_{\mathrm{Ryd}}$) on deformed-to-MB rate ratio ($R = \alpha_q / \alpha_{MB}$)



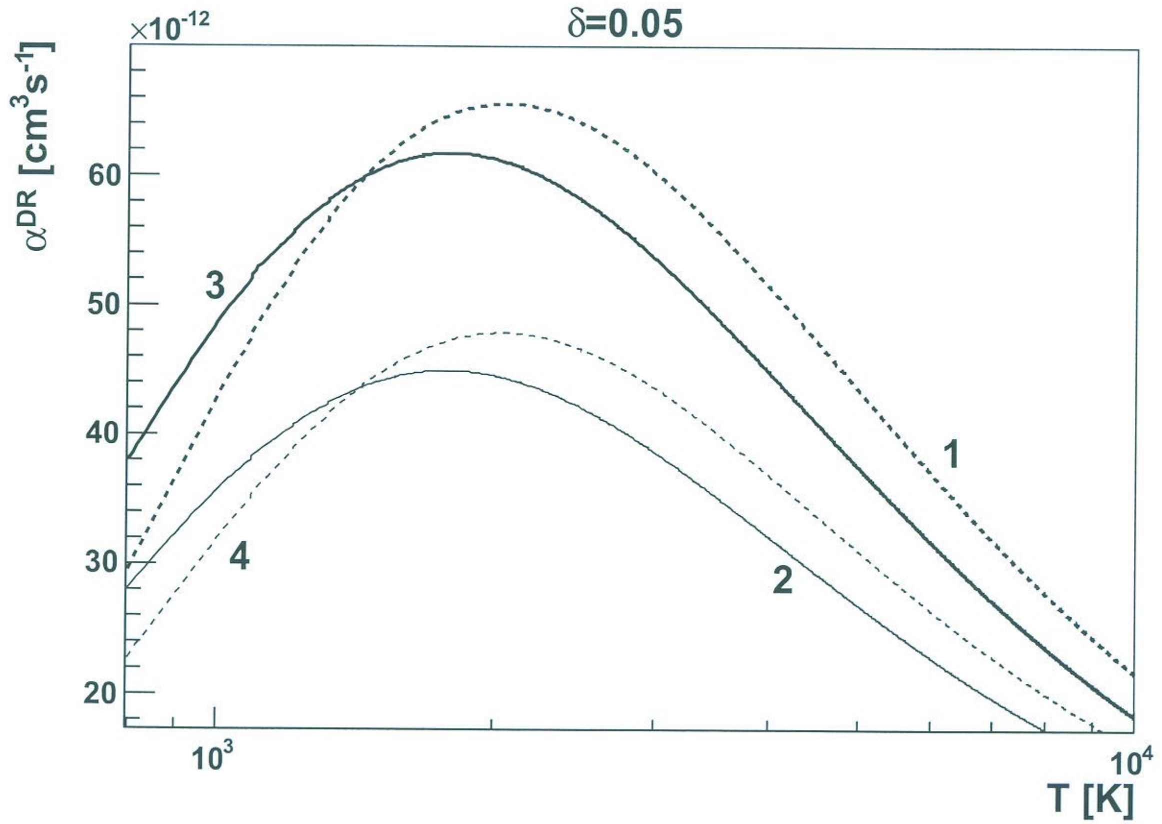
Non-Extensive Approach to CNO-Cycle Reactions

The star luminosity ratio $L/L_{\odot}$ versus plasma temperature
($q = 0.991 \div 1.009$)



- Larger NE effects on CNO than on *pp*
- Only slight deformations allowed in the Sun
- CNO provides nearly all luminosity at higher T

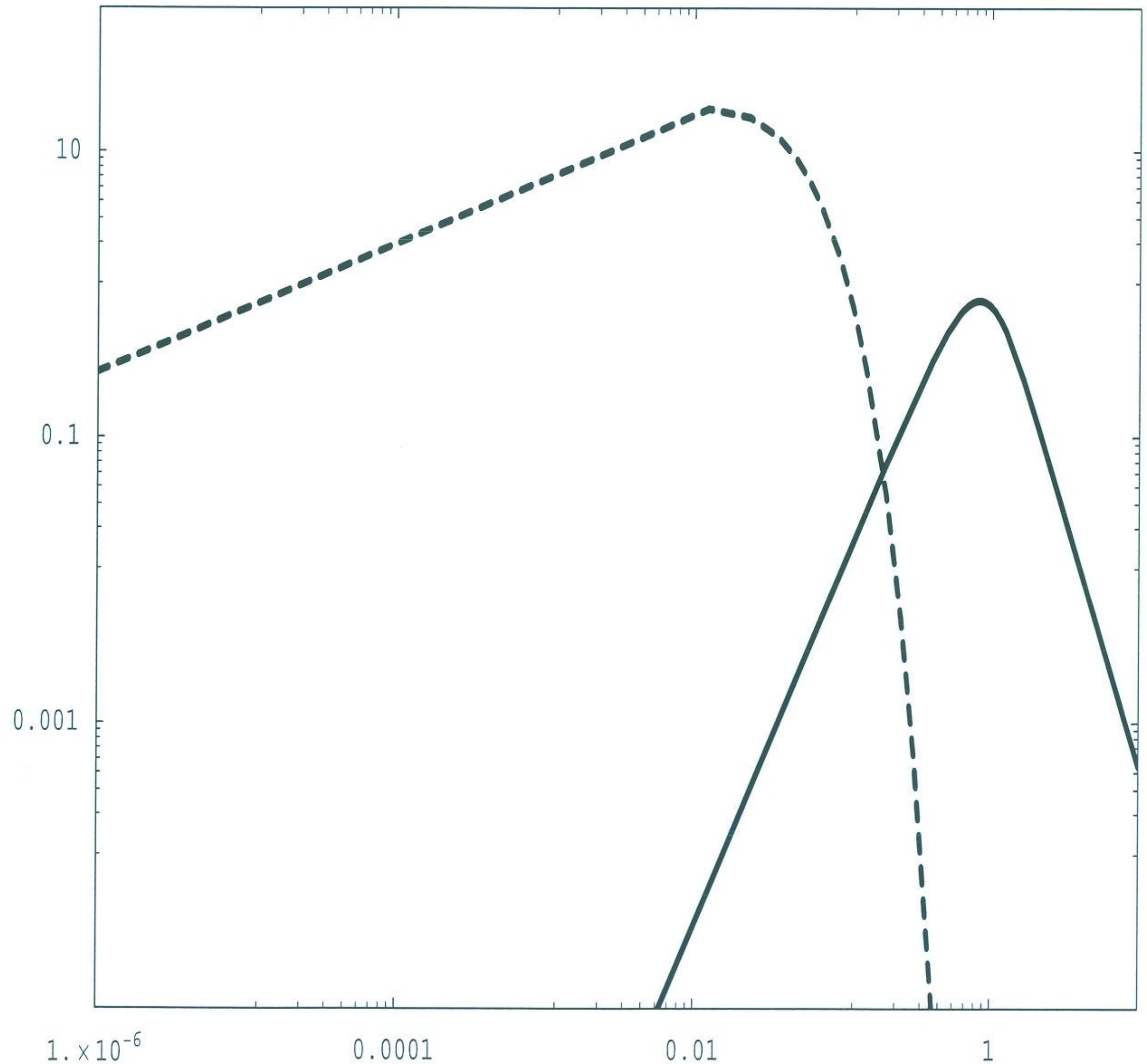
Non-Extensive Approach to DR



1. α_{NE}
2. Experimental fit
3. α_{MB}
4. Non-extensive fit

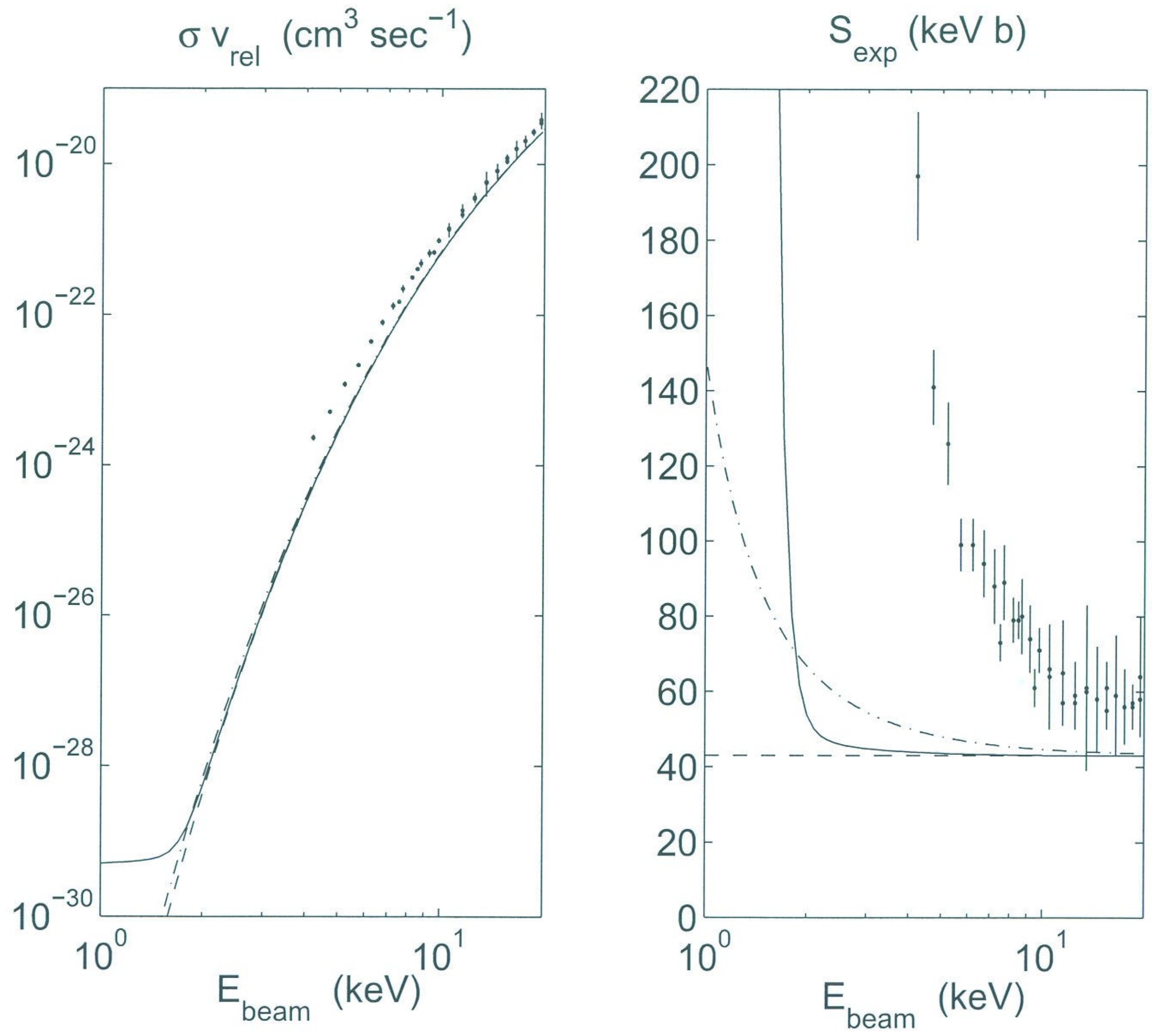
MB and NE Momentum Distributions Vs. Energy

$T = 0.0244\,\mathrm{eV}$



1. MB: dashed
2. NE: solid

Rate and S-Factor for $d(d,p)t$



Conclusions

- In stellar plasmas, MB is only a first-order approximation and corrections originate from the microscopical dynamics
- Quantum corrections may be related to a $\sigma(\varepsilon_p) \propto \sqrt{\varepsilon_p}$ cross section
- All deformations may be understood within NE statistical mechanics
- Momentum distributions other than MB may be interpreted as long-life stationary states
- If both Lab and stellar plasmas are not in a Maxwellian state, one must be very careful in transferring info obtained in Lab to interpret astrophysical observations