MAXIMAL SUPERGRAVITIES AND M-THEORY DEGREES OF FREEDOM

Twistors, Strings and Scattering Amplitudes

London Mathematical Society Durham Symposium 2007

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Maximal supergravities:

- are very nontrivial classical field theories
  - 32 supersymmetries
  - combine gravity, gauge fields with non-abelian extensions, scalar fields and a variety of antisymmetric tensor fields
  - non-trivial duality symmetries
  - improved short distance behaviour

- they could be well defined and consistent

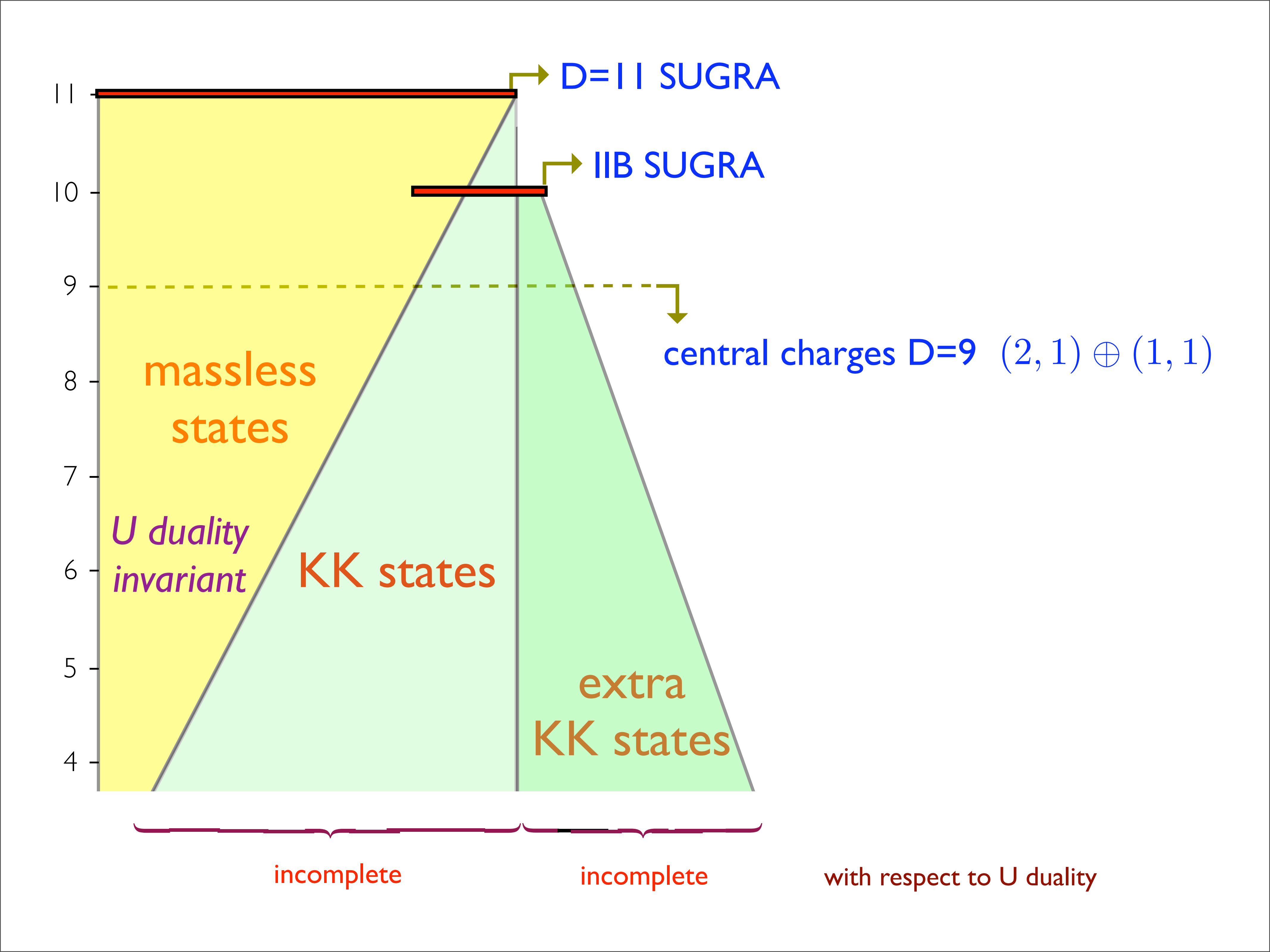
- or are they (necessarily) part of a bigger theory ?

M **?**

Definition of M-Theory

- ? 11-dimensional supergravity
- ? toroidal compactifications thereof with $E_{n(n)}(\mathbb{Z})$
- ? + Kaluza-Klein states (1/2-BPS)
- ? + branes + etcetera
- ? what about IIB theory
- ? Matrix theory
- ? Membrane theory

We start from the (effective) field theory perspective with **32** supersymmetries



D=11 SUGRA
IIB SUGRA
central charges D=9 $(2,1) \oplus (1,1)$
11
10
9
8
7
6
5
4
massless states
U duality invariant
KK states
extra KK states
incomplete
incomplete
with respect to U duality

11D - IIA - IIB PERSPECTIVE

(*9* space-time dimensions)

KKA
- *IIA* momentum — KK states + *D0* branes
- *IIB* winding — strings + *D1* branes

$(2,1)$

implied by U duality

KKB
- *IIB* momentum — KK states
- *IIA* winding — strings

$(1,1)$

9D SUGRA contains 2+1 gauge fields ⟵ central charges

Supergravity / supermembrane perspective ?

Schwarz, 1996

Aspinwall, 1996

Abou-Zeid, dW, Lüst, Nicolai, 1999-2001

11D - SUPERMEMBRANE PERSPECTIVE

KKA *11D* momentum KK states $(2, 1)$

KKB Supermembrane wrapped on T^2 $(1, 1)$

DICHOTOMIC FIELD THEORY

9D SUGRA coupled to KK states of both
11D SUGRA and *IIB* SUGRA

indication of higher-dimensional origin (without full decompactification)

D=11	IIA	D=9	IIB	SO(1,1)
$\hat{G}_{\mu\nu}$	$G_{\mu\nu}$	$g_{\mu\nu}$	$G_{\mu\nu}$	0
$\hat{A}_{\mu\,9\,10}$	$C_{\mu\,9}$	B_μ	$G_{\mu\,9}$	-4
$\hat{G}_{\mu\,9}$, $\hat{G}_{\mu\,10}$	$G_{\mu\,9}$, C_μ	A^α_μ	$A^\alpha_{\mu\,9}$	3
$\hat{A}_{\mu\nu\,9}$, $\hat{A}_{\mu\nu\,10}$	$C_{\mu\nu\,9}, C_{\mu\nu}$	$A^\alpha_{\mu\nu}$	$A^\alpha_{\mu\nu}$	-1
$\hat{A}_{\mu\nu\rho}$	$C_{\mu\nu\rho}$	$A_{\mu\nu\rho}$	$A_{\mu\nu\rho\sigma}$	2
$\hat{G}_{9\,10}$, $\hat{G}_{9\,9}$, $\hat{G}_{10\,10}$	ϕ, $G_{9\,9}$, C_9	$\left\{\begin{array}{l}\phi^\alpha \\ \exp(\sigma)\end{array}\right.$	ϕ^α $G_{9\,9}$	0 7

$$M_{\mathrm{BPS}}(q_1, q_2, p) = m_{\mathrm{KKA}}\, \mathrm{e}^{3\sigma/7}\, |q_\alpha \phi^\alpha| + m_{\mathrm{KKB}}\, \mathrm{e}^{-4\sigma/7}\, |p| \qquad m^2_{\mathrm{KKA}}\, m_{\mathrm{KKB}} \propto T_{\mathrm{m}}$$

more generally:

SUPERSYMMETRY ANTI-COMMUTATOR

$$\{Q_\alpha, \bar{Q}_\beta\} = \Gamma^M_{\alpha\beta} P_M + \frac{1}{2}\Gamma^{MN}_{\alpha\beta} Z_{MN} + \frac{1}{5!}\Gamma^{MNPQR}_{\alpha\beta} Z_{MNPQR}$$

CENTRAL CHARGES (pointlike)

9	$\mathrm{SL}(2) \times \mathrm{SO}(1,1)$	$\mathrm{SO}(2)$	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1})$
8	$\mathrm{SL}(3) \times \mathrm{SL}(2)$	$\mathrm{U}(2)$	$(\mathbf{3},\mathbf{2})$
7	$\mathrm{E}_{4(4)} \equiv \mathrm{SL}(5)$	$\mathrm{USp}(4)$	$\mathbf{10}$
6	$\mathrm{E}_{5(5)} \equiv \mathrm{SO}(5,5)$	$\mathrm{USp}(4) \times \mathrm{USp}(4)$	$\mathbf{16} \rightarrow (\mathbf{4},\mathbf{4})$
5	$\mathrm{E}_{6(6)}$	$\mathrm{USp}(8)$	$\mathbf{27} \oplus \mathbf{1}$
4	$\mathrm{E}_{7(7)}$	$\mathrm{SU}(8)$	$\mathbf{56} \rightarrow \mathbf{28} \oplus \overline{\mathbf{28}}$
3	$\mathrm{E}_{8(8)}$	$\mathrm{SO}(16)$	$\mathbf{120}$
2	$\mathrm{E}_{9(9)}$	$\mathrm{SO}(16)$	$\mathbf{1} \oplus \mathbf{120} \oplus \mathbf{135}$

compare to vector fields!

CENTRAL CHARGES (stringlike)

9	$\mathrm{SL}(2) \times \mathrm{SO}(1,1)$	$\mathbf{2}$
8	$\mathrm{SL}(3) \times \mathrm{SL}(2)$	$(\mathbf{3},\mathbf{1})$
7	$\mathrm{SL}(5)$	$\mathbf{5}$
6	$\mathrm{SO}(5,5)$	$\mathbf{10} \oplus \mathbf{1} \rightarrow (\mathbf{5},\mathbf{1}) \oplus (\mathbf{1},\mathbf{5}) \oplus (\mathbf{1},\mathbf{1})$
5	$\mathrm{E}_{6(6)}$	$\overline{\mathbf{27}}$
4	$\mathrm{E}_{7(7)}$	$\mathbf{63}$
3	$\mathrm{E}_{8(8)}$	$\mathbf{135}$
2	$\mathrm{E}_{9(9)}$	$\mathbf{135}$

compare to tensor fields!

another perspective ....

GAUGINGS

class of deformations of maximal supergravities

gauging versus scalar-vector-tensor duality

first: 3 space-time dimensions

128 scalars and 128 spinors, but no vectors !

obtained by dualizing vectors in order to realize the symmetry $E_{8(8)}(\mathbb{R})$

solution:

introduce 248 vector gauge fields with Chern-Simons terms

$$\mathcal{L}_{\text{CS}} \propto g\,\varepsilon^{\mu\nu\rho}\,A_\mu{}^M\,\Theta_{MN}\left[\partial_\nu A_\rho{}^N - \frac{1}{3}g\,f_{PQ}{}^N A_\nu{}^P A_\rho{}^Q\right]$$

EMBEDDING TENSOR

'invisible' at the level of the toroidal truncation

Nicolai, Samtleben, 2000

another example: 5 space-time dimensions

42 scalars and 27 vectors, and no tensors !

in order to realize the symmetry $E_{6(6)}^{\rm rigid} \times \mathrm{USp}(8)^{\rm local}$.

introduce a local subgroup such as $E_{6(6)} \to \mathrm{SO}(6)^{\rm local} \times \mathrm{SL}(2)$

inconsistent!

Günaydin, Romans, Warner, 1986

vectors decompose according to: $\overline{\mathbf{27}} \to (\mathbf{15},\mathbf{1}) + (\overline{\mathbf{6}},\mathbf{2})$

charged vector fields
must be (re)converted to tensor fields !

gauge group encoded into the **EMBEDDING TENSOR** $\Theta_M{}^\alpha$

- ☞ treated as spurionic order parameter $\in E_{6(6)}$
- ☞ probes new M-theory degrees of freedom

$$X_M = \Theta_M{}^\alpha\, t_\alpha$$

gauge group generators

$E_{6(6)}$ generators

The embedding tensor is subject to constraints !

- closure: $[X_M, X_N] = f_{MN}{}^P\, X_P$

$$\Theta_M{}^\beta\, \Theta_N{}^\gamma\, f_{\beta\gamma}{}^\alpha = f_{MN}{}^P\, \Theta_P{}^\alpha = -\,\boxed{\Theta_M{}^\beta\, t_{\beta N}{}^P}\, \Theta_P{}^\alpha$$

$$\hookrightarrow \quad X_{MN}{}^P \ \in E_{6(6)}$$

$$[X_M, X_N] = -X_{MN}{}^P\, X_P$$

$X_{MN}{}^P$ contains the gauge group structure constants, but is not symmetric in lower indices, unless contracted with the embedding tensor !!!!

- supersymmetry: $\Theta_M{}^\alpha \in \mathbf{351}$

$$\longrightarrow \quad \mathbf{27} \times \mathbf{78} = \cancel{\mathbf{27}} + \mathbf{351} + \cancel{\overline{\mathbf{1728}}}$$

$$(\mathbf{351} \times \mathbf{351})_{\rm s} = \cancel{\overline{\mathbf{27}}} + \cancel{\mathbf{1728}} + \mathbf{351}' + \mathbf{7722} + \mathbf{17550} + \mathbf{34398}$$

(closure)

dW, Samtleben, Trigiante, 2005

EMBEDDING TENSORS FOR *D = 3,4,5,6,7*

7	$\mathrm{SL}(5)$	$\mathbf{10}\times\mathbf{24}=\mathbf{10}+\boxed{\mathbf{15}}+\boxed{\mathbf{40}}+\mathbf{175}$
6	$\mathrm{SO}(5,5)$	$\mathbf{16}\times\mathbf{45}=\mathbf{16}+\boxed{\mathbf{144}}+\mathbf{560}$
5	$\mathrm{E}_{6(6)}$	$\mathbf{27}\times\mathbf{78}=\mathbf{27}+\boxed{\mathbf{351}}+\mathbf{1728}$
4	$\mathrm{E}_{7(7)}$	$\mathbf{56}\times\mathbf{133}=\mathbf{56}+\boxed{\mathbf{912}}+\mathbf{6480}$
3	$\mathrm{E}_{8(8)}$	$\mathbf{248}\times\mathbf{248}=\boxed{\mathbf{1}}+\mathbf{248}+\boxed{\mathbf{3875}}+\mathbf{27000}+\mathbf{30380}$

dW, Samtleben, Trigiante, 2002

- characterize all possible gaugings
- group-theoretical classification
- universal Lagrangians

applications in *D = 2,3,4,5,7* space-time dimensions
in *D=4*, for *N = 2,4,8* supergravities
in *D=3*, for *N = 1,...,6,8,9,10,12,16* supergravities

de Vroome, dW, Herger, Nicolai, Samtleben, Schön, Trigiante, Weidner

digression:

consider the representations appearing in $(\mathbf{27}\times\mathbf{27})_{\rm s} = (\overline{\mathbf{27}} + \mathbf{351}')$

$$X_{(MN)}{}^{P} = d_{I,MN}\, Z^{P,I} \qquad\qquad d_{MNI} \;:\; E_{6(6)}$$ invariant tensor(s)

two possible representations can be associated with the new index $\begin{cases} \overline{\mathbf{27}} \\ \cancel{\mathbf{351}'} \end{cases}$

$$\overline{\mathbf{27}}\times(\mathbf{27}\times\mathbf{27})_{\rm s} = \boxed{\mathbf{351}} + \mathbf{27} + \mathbf{27} + \overline{\mathbf{351}}' + \overline{\mathbf{1728}} + \overline{\mathbf{7722}}$$

indeed: $(\overline{\mathbf{27}}\times\overline{\mathbf{27}})_{\rm a} = \mathbf{351} \longrightarrow X_{(MN)}{}^{P} = d_{MNQ}\, Z^{PQ}$

from the closure constraint:

$$Z^{MN}\,\Theta_N{}^{\alpha} = 0 \quad \to \quad Z^{MN}\,X_N = 0$$ orthogonality

$$X_{MN}{}^{[P}\,Z^{Q]N} = 0$$ gauge invariant tensor

this structure is generic (at least, for the groups of interest) and we will exploit it later !

rather than converting and tensors into vectors and reconverting some of them them when a gauging is switched on, we introduce both vectors and tensors from the start, transforming into the representations $\overline{\mathbf{27}}$ and $\mathbf{27}$, respectively

extra gauge invariance

$$\delta A_\mu^M = \partial_\mu \Lambda^M - g\, X_{[PQ]}{}^M\, \Lambda^P\, A_\mu^Q - g\, Z^{MN}\, \Xi_{\mu\, N}$$

$$\mathcal{F}_{\mu\nu}{}^M = \partial_\mu A_\nu{}^M - \partial_\nu A_\mu{}^M + g\, X_{[NP]}{}^M\, A_\mu{}^N A_\nu{}^P$$

not fully covariant

introduce fully covariant field strength $\boxed{\mathcal{H}_{\mu\nu}{}^M = \mathcal{F}_{\mu\nu}{}^M + g\, Z^{MN}\, B_{\mu\nu\, N}}$

to compensate for lack of closure:

$$\begin{aligned}\delta B_{\mu\nu\, M} \;=\;& 2\,\partial_{[\mu}\Xi_{\nu]N} - g\, X_{PN}{}^Q\, A_{[\mu}{}^P\, \Xi_{\nu]Q} + g\, Z^{MN}\, \Lambda^P\, X_{PN}{}^Q\, B_{\mu\nu\, Q}\\ & - g\left(2\, d_{MPQ}\, \partial_{[\mu} A_{\nu]}{}^P - g\, X_{RM}{}^P\, d_{PQS} A_{[\mu}{}^R\, A_{\nu]}{}^S\right)\Lambda^Q\end{aligned}$$

because of the extra gauge invariance, the degrees of freedom remain unchanged

upon switching on the gauging there will be a balanced decomposition of vector and tensor fields

Universal invariant Lagrangian containing
kinetic terms for the tensor fields combined with a
Chern-Simons term for the vector fields

$$
\begin{aligned}
\mathcal{L}_{\rm VT} \;=\;& \frac{1}{2} i\varepsilon^{\mu\nu\rho\sigma\tau}\Big\{ g Z^{MN} B_{\mu\nu\,M}\Big[D_\rho B_{\sigma\tau\,N} + 4\, d_{NPQ}\, A_\rho{}^P \Big(\partial_\sigma A_\tau{}^Q + \frac{1}{3} g\, X_{[RS]}{}^Q\, A_\sigma{}^R\, A_\tau{}^S\Big)\Big] \\
& - \frac{8}{3} d_{MNP}\Big[A_\mu{}^M\, \partial_\nu A_\rho{}^N\, \partial_\sigma A_\tau{}^P \\
& + \frac{3}{4} g\, X_{[QR]}{}^M\, A_\mu{}^N A_\nu{}^Q A_\rho{}^R \Big(\partial_\sigma A_\tau{}^P + \frac{1}{5} g\, X_{[ST]}{}^P A_\sigma{}^S A_\tau{}^T\Big)\Big]\Big\}
\end{aligned}
$$

this term is present for ALL gaugings
there is no other restriction than the constraints on
the embedding tensor

dW, Samtleben, Trigiante, 2005

Can this be generalized?

Non-abelian vector-tensor hierarchies

Generalize the combined gauge algebra

☛ algebra closes on $\Theta_M{}^\alpha A_\mu{}^M$

$$\delta A_\mu{}^M = \partial_\mu \Lambda^M - g\, X_{[PQ]}{}^M\, \Lambda^P\, A_\mu{}^Q - g\, \overbrace{Z^{M,I}\,\Xi_{\mu\, I}}^{\text{non-closure}}$$

$$\delta B_{\mu\nu\, I} = 2\, D_{[\mu}\Xi_{\nu]I} + \cdots$$

☛ algebra closes on $Z^{M,I} B_{\mu\nu\, I}$

$$\delta B_{\mu\nu\, I} = 2\, D_{[\mu}\Xi_{\nu]I} + \cdots - g\, \overbrace{Y_{IM}{}^J\, \Phi_{\mu\nu\, J}{}^M}^{\text{non-closure}}$$

with $\quad Z^{M,I}\, Y_{IN}{}^J = 0 \quad \longrightarrow \quad Y_{IM}{}^J \equiv X_{MI}{}^J + 2\, d_{I,MN}\, Z^{N,J}$

$$\delta S_{\mu\nu\rho\, I}{}^M = 3\, D_{[\mu}\Phi_{\nu\rho]I}{}^M + \cdots$$

☛ algebra closes on $Y_{IM}{}^J S_{\mu\nu\rho\, J}$

etcetera

dW, Samtleben, 2005

explicit results are complicated:

$$\mathcal{H}_{\mu\nu\rho\,I} \;\equiv\; 3\Big[D_{[\mu}B_{\nu\rho]\,I} + 2\,d_{I,MN}\,A_{[\mu}{}^{M}(\partial_\nu A_{\rho]}{}^{N} + \frac{1}{3}gX_{[PQ]}{}^{N}A_\nu{}^{P}A_{\rho]}{}^{Q})\Big]$$
$$+\,g\,Y_{IM}{}^{J}\,S_{\mu\nu\rho\,I}{}^{M}$$

$$\begin{aligned}\delta S_{\mu\nu\rho\,I}{}^{M} \;=\;& g\,\Lambda^{N}X_{NI}{}^{J}\,S_{\mu\nu\rho J}{}^{M} - g\,\Lambda^{N}X_{NP}{}^{M}\,S_{\mu\nu\rho I}{}^{P}\\ &+3\,D_{[\mu}\Phi_{\nu\rho]I}{}^{M} + 3\,A_{[\mu}{}^{M}\,D_\nu\Xi_{\rho]I} + 3\,\partial_{[\mu}A_\nu{}^{M}\,\Xi_{\rho]\,I}\\ &-2g\,d_{I,NP}\,Z^{P,J}A_{[\mu}{}^{M}A_\nu{}^{N}\Xi_{\rho]J}\\ &+4\,d_{I,NP}\,\Lambda^{[M}\,A_{[\mu}{}^{N]}\,\partial_\nu A_{\rho]}{}^{P} + 2g\,X_{NI}{}^{J}\,d_{J,PQ}\,\Lambda^{Q}\,A_{[\mu}{}^{M}A_\nu{}^{N}A_{\rho]}{}^{P}\end{aligned}$$

Plumbing strategy: repair the lack of closure iteratively by introducing tensor gauge fields of increasing rank

$$\begin{array}{ccccccc} A_\mu{}^{M} & \longrightarrow & B_{\mu\nu}{}^{I} & \longrightarrow & S_{\mu\nu\rho I}{}^{M} & \longrightarrow & \text{etc}\\ \Lambda^{M} & & \Xi_{\mu I} & & \Phi_{\mu\nu I}{}^{M} & & \end{array}$$

encoded by the embedding tensor !

Leads to :

rank ⇨		1	2	3	4	5	6
7	$SL(5)$	$\mathbf{10}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\boxed{\mathbf{24}}$	$\mathbf{15}+\mathbf{40}$
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\boxed{\mathbf{45}}$	$\mathbf{144}$	
5	$E_{6(+6)}$	$\overline{\mathbf{27}}$	$\mathbf{27}$	$\boxed{\mathbf{78}}$	$\mathbf{351}$	$\mathbf{27}+\mathbf{1728}$	
4	$E_{7(+7)}$	$\mathbf{56}$	$\boxed{\mathbf{133}}$	$\mathbf{912}$	$\mathbf{133}+\mathbf{8165}$		
3	$E_{8(+8)}$	$\boxed{\mathbf{248}}$	$\mathbf{3875}$	$\mathbf{3875}+\mathbf{147250}$			

Striking feature:

rank *D-2* : adjoint representation of the duality group

dW, Samtleben, Nicolai, work in progress

rank ⇨		1	2	3	4	5	6
7	$SL(5)$	$\mathbf{10}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\boxed{\mathbf{15}+\mathbf{40}}$
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\boxed{\mathbf{144}}$	
5	$E_{6(+6)}$	$\overline{\mathbf{27}}$	$\mathbf{27}$	$\mathbf{78}$	$\boxed{\mathbf{351}}$	$\mathbf{27}+\mathbf{1728}$	
4	$E_{7(+7)}$	$\mathbf{56}$	$\mathbf{133}$	$\boxed{\mathbf{912}}$	$\mathbf{133}+\mathbf{8165}$		
3	$E_{8(+8)}$	$\mathbf{248}$	$\boxed{\mathbf{3875}}$	$\mathbf{3875}+\mathbf{147250}$			

Striking feature:

rank *D-1* : embedding tensor

rank ⇨		1	2	3	4	5	6
7	$\mathrm{SL}(5)$	$\mathbf{10}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\mathbf{15}+\mathbf{40}$
6	$\mathrm{SO}(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\boxed{}$
5	$\mathrm{E}_{6(+6)}$	$\overline{\mathbf{27}}$	$\mathbf{27}$	$\mathbf{78}$	$\mathbf{351}$	$\boxed{\mathbf{27}+\mathbf{1728}}$	
4	$\mathrm{E}_{7(+7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\boxed{\mathbf{133}+\mathbf{8165}}$		
3	$\mathrm{E}_{8(+8)}$	$\mathbf{248}$	$\mathbf{3875}$	$\boxed{\mathbf{3875}+\mathbf{147250}}$			

Striking feature:

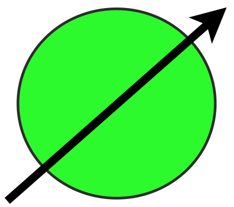
rank D : closure constraint on the embedding tensor

	rank ⇨	1	2	3	4	5	6
7	$\mathrm{SL}(5)$	$\boxed{\mathbf{10}}$	$\boxed{\mathbf{5}}$	$\boxed{\overline{\mathbf{5}}}$	$\boxed{\mathbf{10}}$	$\mathbf{24}$	$\mathbf{15}+\mathbf{40}$
6	$\mathrm{SO}(5,5)$	$\boxed{\mathbf{16}}$	$\mathbf{10}$	$\boxed{\overline{\mathbf{16}}}$	$\mathbf{45}$	$\mathbf{144}$	
5	$\mathrm{E}_{6(+6)}$	$\boxed{\overline{\mathbf{27}}}$	$\boxed{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\mathbf{27}+\mathbf{1728}$	
4	$\mathrm{E}_{7(+7)}$	$\boxed{\mathbf{56}}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{133}+\mathbf{8165}$		
3	$\mathrm{E}_{8(+8)}$	$\mathbf{248}$	$\mathbf{3875}$	$\mathbf{3875}+\mathbf{147250}$			

Perhaps most striking:

implicit connection between space-time Hodge duality and the U-duality group

Probes new states in M-Theory!



Θ dial

Implications:

		1	2	3	4	5	6
7	$\mathrm{SL}(5)$	$\mathbf{10}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\mathbf{15}+\mathbf{40}$
6	$\mathrm{SO}(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	
5	$\mathrm{E}_{6(+6)}$	$\overline{\mathbf{27}}$	$\mathbf{27}$	$\mathbf{78}$	$\mathbf{351}$	$\mathbf{27}+\mathbf{1728}$	
4	$\mathrm{E}_{7(+7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{133}+\mathbf{8165}$		
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The table coincides substantially with results based on several rather different conceptual starting points:

- M(atrix)-Theory compactified on a torus: duality representations of states
- Correspondence between toroidal compactifications of M-Theory and del Pezzo surfaces
- ***E11*** decompositions

• Algebraic Aspects of Matrix Theory on T^d

Elitzur, Giveon, Kutasov, Rabinovici, 1997

Based on the correspondence between super-Yang-Mills on $\tilde{T}^d$ and M-Theory on T^d, a rectangular torus with radii $R_1, R_2, \ldots, R_d$ in the infinite-momentum frame.

Invariance group consist of permutations of the R_i combined with the T-duality relations ($i \neq j \neq k$) :

$$\boxed{R_i \to \frac{l_\mathrm{p}^3}{R_j R_k} \qquad R_j \to \frac{l_\mathrm{p}^3}{R_k R_i} \qquad R_k \to \frac{l_\mathrm{p}^3}{R_i R_j} \qquad l_\mathrm{p}^3 \to \frac{l_\mathrm{p}^6}{R_i R_j R_k}}$$

generate a group isomorphic with the Weyl group of $\mathrm{E}_{d(d)}$

The explicit duality multiplets arise as representations of this group.

Example d=3 :

3 KK states on T^d $\qquad M \sim \dfrac{1}{R_i}$

3 2-brane states wrapped on T^d $\qquad M \sim \dfrac{R_j R_k}{l_{\mathrm{p}}^3} \qquad j \neq k$

3 2-brane states wrapped on $T^d \times x^{11}$ $\qquad M \sim \dfrac{R_{11} R_i}{l_{\mathrm{p}}^3}$

the dimensions of these two multiplets coincide with the multiplets presented previously for the scalar and vector central charges.

for higher *d* the multiplets are sometimes incomplete, because they are not generated as a single orbit by the Weyl group.

		pointlike		stringlike	
		central charge	matrix theory	central charge	matrix theory
8	$SL(3) \times SL(2)$	6	6	3	3
7	$E_{4(4)} \equiv SL(5)$	10	10	5	5
6	$E_{5(5)} \equiv SO(5,5)$	16	16	$10 + 1$	10
5	$E_{6(6)}$	$27 + 1$	27	27	27
4	$E_{7(7)}$	56	56	63	$126 + 7$
3	$E_{8(8)}$	120	$240 + 8$	135	$2160 + 1715$
2	$E_{9(9)}$	256	∞	135	∞
			gauging		gauging

A Mysterious Duality

Iqbal, Neitzke, Vafa, 2001

This cannot be a coincidence!

It is important to uncover the physical interpretation of these duality relations. One possibility is that the del Pezzo surface is the moduli space of some probe in M-Theory. It must be a U-duality invariant probe ....

Such probe is the gauging encoded in the embedding tensor!

E11 decomposition

Based on the conjecture that *E11* is the underlying symmetry of M-Theory. Decomposing the relevant *E11* representation to dimensions $D<11$ yields representations that substantially overlap with those generated for the gaugings.

West et. al., 2001-2007
Bergshoeff et. al.,2005-2007

Conclusions

✦ Gaugings probe new degrees of freedom of M-Theory

✦ Maximal supergravity theories contain subtle information about M-Theory. This may be interpreted as an indication that supergravity needs to be extended towards string/M-theory. This is also indicated by comparing degrees of freedom originating from the maximal theories in various dimensions.

✦ There are unexpected connections with other results derived on the basis of rather different concepts

✦ More work needs to be done on clarifying these connections

✦ The group-theoretical properties of the tensor classification (in particular the global structure of the table) needs to be clarified