Structural Optimization via SAND and NAND

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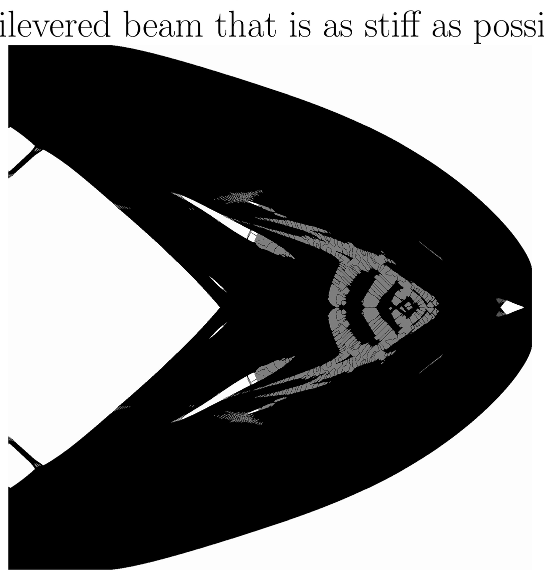
1 Introduction to stuctural optimization

A man walks into a bar, hands you some isotropic material and says design me a cantilevered beam that is as stiff as possible.



This is your reply as you just so happened to have glanced at this poster on structural optimization.

2 Simultaneous Analysis and Design (SAND)

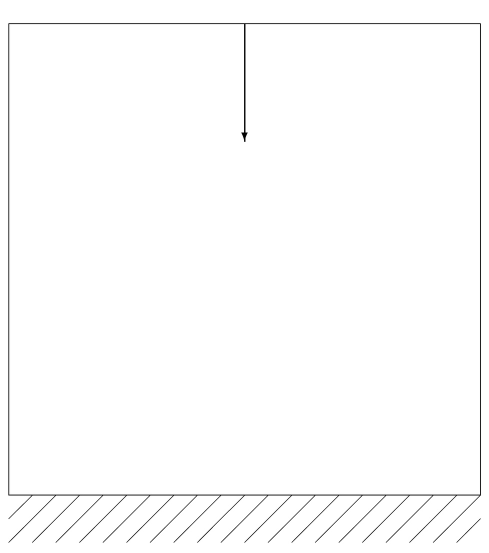
Write a large optimization problem as follows.

$\min_{\rho,u} f^T u$	minimising the compliance
subject to	
$K(\rho)u = f$	linear elasticity equations
$\sum \rho \le V_{\max}$	volume constraints on material
$\rho_{\min} \le \rho \le \rho_{\max}$	bounds on the density values

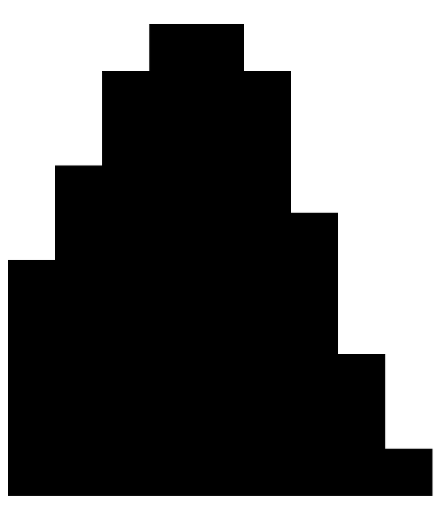
ρ are the densities, f is a given load, $V_{\max}$ is a given amount of material, and u are the displacements that need to satisfy the equilibrium equations.

Implemented using SQP algorithms.

3 SAND results



Considered problem



Resulting SAND solution

The SAND approach currently has to be used on a very coarse discretization due to the difficulty of satisfying the nonlinear constraints; i.e. the elasticity equations.
10×10 mesh of 10^2 elements.

4 Nested Analysis and Design (NAND)

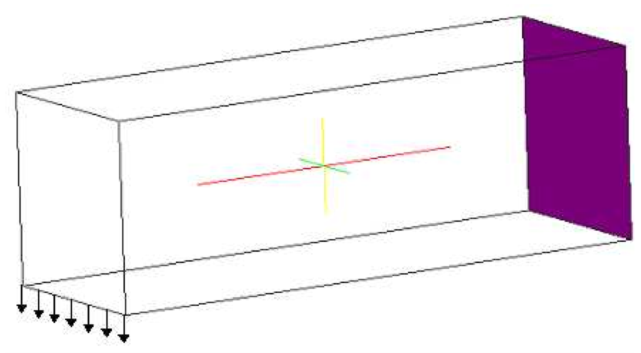
Do until convergence

- Solve the linear system
$$K(\rho)u = f$$
- Solve the local optimization problem

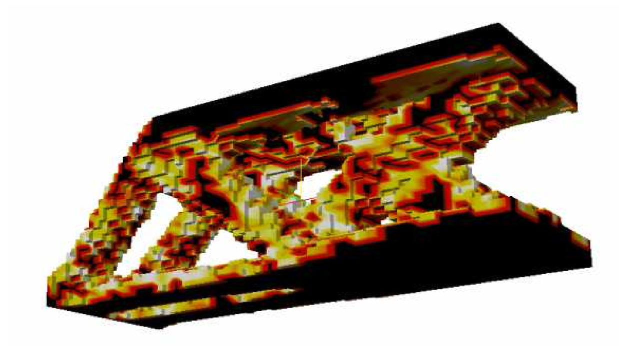
$\min_{\rho} f^T K(\rho)^{-1} f$	minimising the compliance
subject to	
$\sum \rho \le V_{\max}$	volume constraints on material
$\rho_{\min} \le \rho \le \rho_{\max}$	bounds on the density values

This formulation means we always have a physically meaningful structure.

5 NAND 3D results

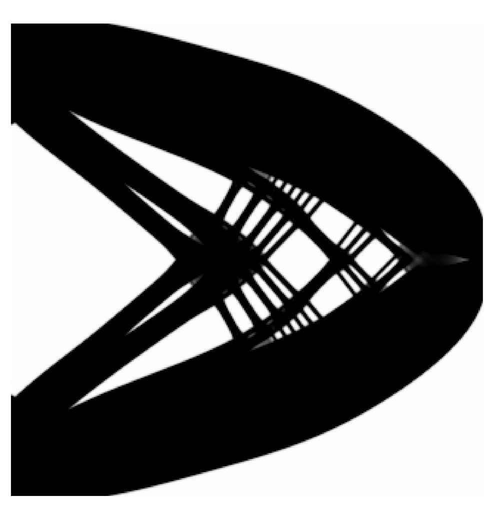


Cantilevered beam fixed on the right hand side (purple face) with a vertical load applied to the bottom left edge.

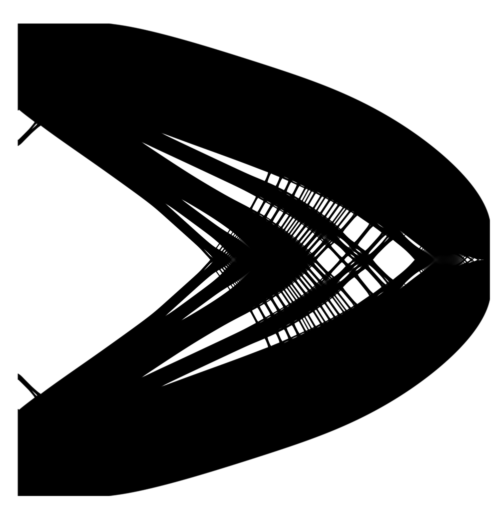


Optimized structure

6 NAND 2D results



320×320 mesh of 1.024×10^5 variables



1600×1600 mesh of 2.56×10^6 variables

A 2D cantilevered beam fixed on the left hand side with a vertical load in the in the middle on the right hand side.

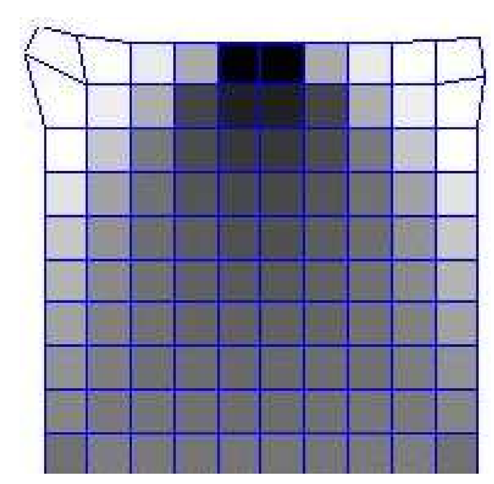
7 Stability problem

"A process of optimization leads almost inevitably to designs which exhibit the notorious failure characteristics often associated with the buckling of thin elastic shells", Hunt 1973.
To overcome this, we use linear buckling analysis and include this in the formulation of the optimization algorithm as a constraint.

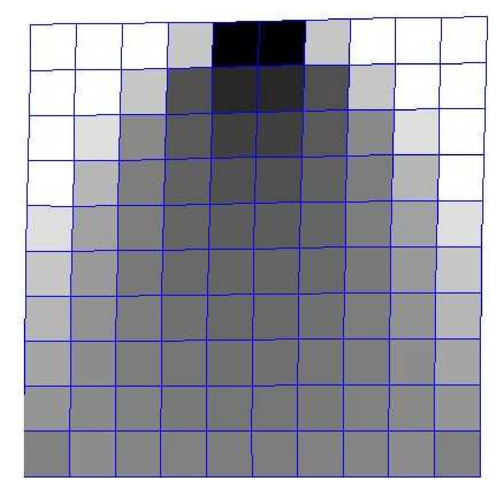
$\min \sum \rho$	minimising the weight
subject to	
$K(\rho)u = f; f^T u < c$	upper bound on compliance
$\lvert K(\rho) + \lambda K_\sigma(\rho)\rvert = 0; \lambda > s$	lower bound on the buckling load

λ is the smallest positive value that satisfies the equation on the left hand side. It is the multiplier of the applied loads that then gives the critical load. s is the safety factor which we use to ensure the structure is stable.

8 Buckling analysis results



Directly computed buckling mode



Adjusted eigenpair computation

Left shows spurious localised eigenvectors that occur when the density of elements drops. These regions represent voids so we must adjust our method to ensure we find the physical mode. Right shows the result of our adjusted method.

9 Future work

- Full convergence analysis comparing the two different SAND and NAND formulations.
- Determine why SQP solvers have problems in satisfying the equilibrium equations as nonlinear constraints.
- Solve the buckling problem efficiently using semi-definite programming methods, specifically designed to deal with coalescing eigenvalues.

CASE award, Rutherford Appleton Laboratory

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