A particle physicist's perspective on topological insulators.

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Durham, July 22, 2010

based on: "Fractional topological insulators in three dimensions", with J.Maciejko, X.-L. Qi, S. Zhang

also: "A holographic fractional topological insulator" with C. Hoyos and K. Jensen as well as work in progress with T. Takayanagi and J. Maciejko

Outline.

- Review of topological insulators
  (focus on effective field theory)

- Fractional topological insulators
  (work with Maciejko, Qi and Zhang)

- Holographic realization
  (work with Hoyos and Jensen)

- Quantum Spin Hall Effect
  (work with Maciejko and Takayanagi)

Review of topological insulators

Effective theory on insulators.

What is the low energy description of a generic, time reversal invariant insulator?

Insulator = gapped spectrum

Low energy DOFs: only Maxwell field.

Task: Write down the most general action for E and B, with up to two derivatives, consistent with symmetries.

Low energy effective action.

Low energy DOFs: only Maxwell field.

$$S_0 = \int d^3x dt L_0 = \frac{1}{8\pi} \int d^3x dt \left(\epsilon \vec{E}^2 - \frac{1}{\mu} \vec{B}^2 \right) .$$

Permittivity and Permeability.

Rotations allow one extra term.

$$S_\theta = \frac{\theta}{2\pi}\frac{\alpha}{16\pi}\int d^3x dt \epsilon_{\mu\nu\rho\tau}F^{\mu\nu}F^{\rho\tau} = \frac{\theta}{2\pi}\frac{\alpha}{4\pi}\int d^3x dt \partial^\mu(\epsilon_{\mu\nu\rho\sigma}A^\nu\partial^\rho A^\tau)$$

$$= \frac{\theta\alpha}{4\pi}\int d^3x dt (E\cdot B)$$

But: Under time reversal E is even, B odd

So naively the most general description of a time reversal invariant insulator does not allow for a theta term.

Flux Quantization.

Dirac Quantization
of magnetic charge:

$$g = n \frac{e}{2\alpha}$$

Implies quantization of magnetic flux!

$$\int_S F = g$$

On any Euclidean closed 4-manifold M:

$$\frac{\alpha}{32\pi^2} \int_M d^4x F_{\mu\nu} F_{\sigma\tau} \epsilon^{\mu\nu\sigma\tau} = N \in \mathbf{Z}$$

Flux Quantization.

- Partition function

$$Z(\theta) = \exp\left\{i\frac{\alpha\theta}{32\pi^2}\int_M d^4x F_{\mu\nu}F_{\sigma\tau}\epsilon^{\mu\nu\sigma\tau}\right\} = e^{iN\theta}$$

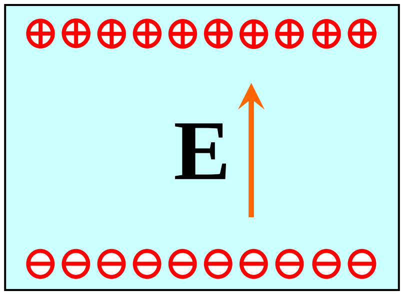
- is periodic in $\theta \rightarrow \theta+2\pi$ (Abelian version of the "Θ vacuum" (Callan, Dashen, Gross 1976, Jackiw&Rebbi, 1976))
- θ is time-reversal odd
- $\rightarrow$time-reversal invariant insulator can have $\theta=0$ or π
- Z_2 classification

Topological Insulators

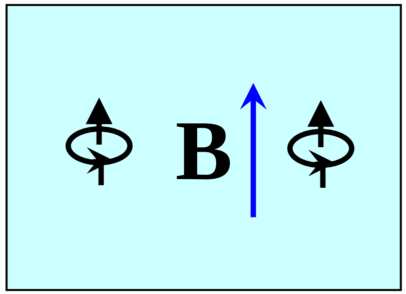
Low energy description of a T-invariant insulator described by 3 parameters: ε, μ, and:

$\theta = 0$	Topologically trivial insulators
$\theta = \pi$	Topologically non-trivial insulators

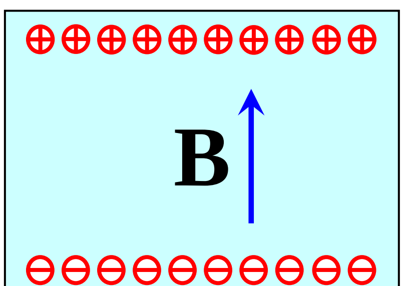
Physical Consequences.



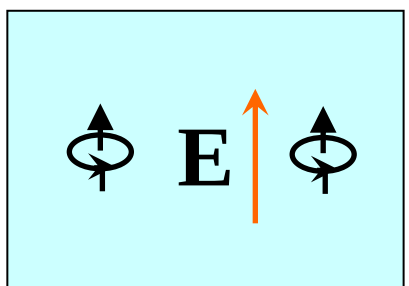
$4\pi\mathbf{P}=(\varepsilon-1)\mathbf{E}$



$4\pi\mathbf{M}=(1-1/\mu)\mathbf{B}$



$4\pi\mathbf{P}=(\alpha\theta/2\pi)\mathbf{B}$

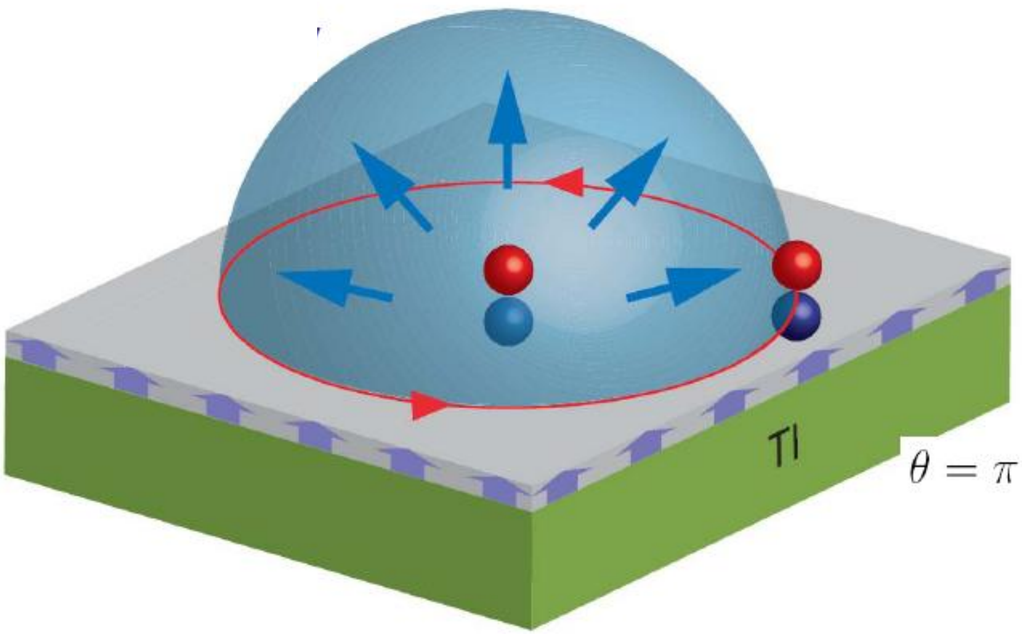


$4\pi\mathbf{M}=(\alpha\theta/2\pi)\mathbf{E}$

A topological term in the action

$$
\begin{aligned}
\nabla\cdot\mathbf{D} &= 4\pi\rho \\
\nabla\cdot\mathbf{B} &= 0 \\
\nabla\times\mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \\
\nabla\times\mathbf{H} &= \frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} \\
\mathbf{D} &= \mathbf{E}+4\pi\mathbf{P}-\frac{\theta}{\pi}\alpha\mathbf{B} \\
\mathbf{H} &= \mathbf{B}-4\pi\mathbf{M}+\frac{\theta}{\pi}\alpha\mathbf{E}
\end{aligned}
$$

Magnetic Monopoles in TI



$\theta = \pi$

prediction: mirror charge of an electron is a **magnetic monopole**

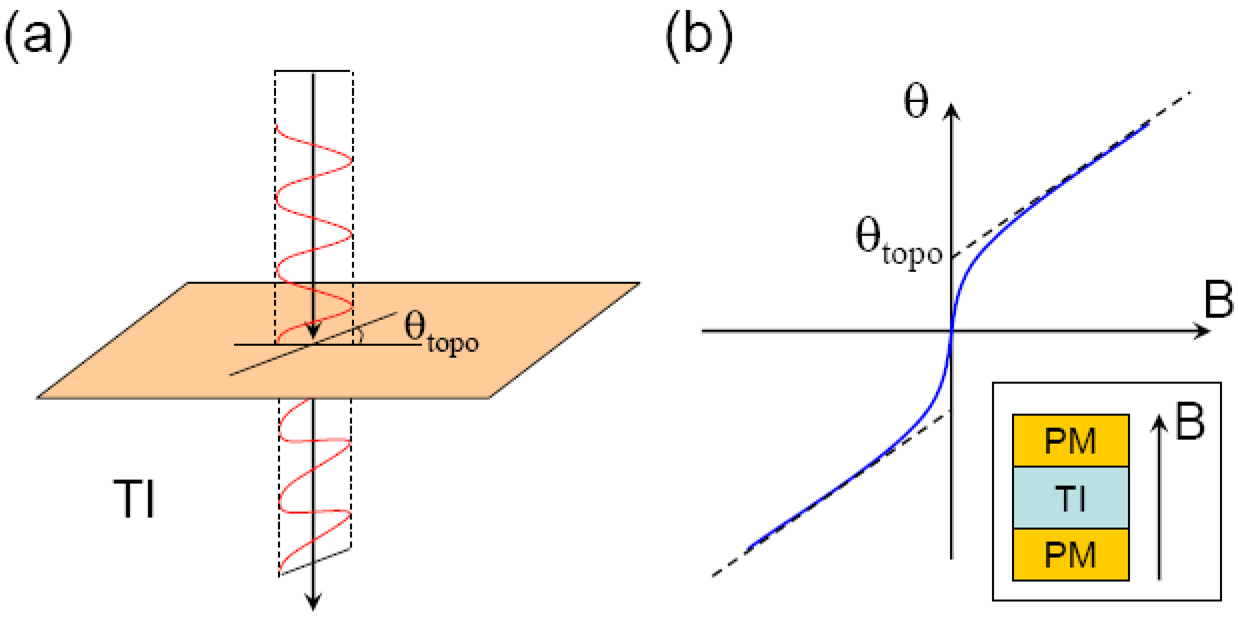
first pointed out by Lee and Sikivie, re-obtained in the TI context by Qi et. al.

Compact expression from requiring **E&M duality covariance** **(AK).**

$$g = \frac{\alpha\theta/2\pi}{1+\alpha^2\theta^2/4\pi^2} q$$

(for $\mu=\mu'$, $\varepsilon=\varepsilon'$)

Faraday and Kerr Effect



change in θ

B independent contribution to Kerr/Faraday **(Qi et al)**

(Polarization of transmitted and reflected wave rotated by angle θ)

$$\theta_{\rm topo} = \arctan \frac{2\alpha\Delta}{\sqrt{\epsilon/\mu} + \sqrt{\epsilon'/\mu'}}$$

A Microscopic Model

A microscopic model: Massive Dirac Fermion.

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu - M)\psi$$

Time Reversal: $M \longrightarrow M^*$

Time reversal system has real mass.
Two options: positive or negative.

Chiral rotation and ABJ anomaly.

Massless theory invariant under chiral rotations:

$$\psi \to e^{-i\phi\gamma_5/2}\psi$$

Symmetry of massive theory if mass transforms:

$$M \to e^{i\phi}M$$

Phase can be rotated away! Chose M positive.

Chiral rotation and ABJ anomaly.

But in the quantum theory chiral rotation is anomalous. Measure transforms.

$$\Delta\mathcal{L} = C\alpha\frac{\phi}{32\pi^2}\,\mathrm{tr}\;\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$C = \sum\nolimits_{fields} q^2 = 1\cdot 1^2 = 1$$

Single field with unit charge.

$$\theta \to \theta - C\phi$$

Chiral rotation and ABJ anomaly.

$$\theta \rightarrow \theta - C\phi$$

Axial rotation with $\Phi=\pi$:

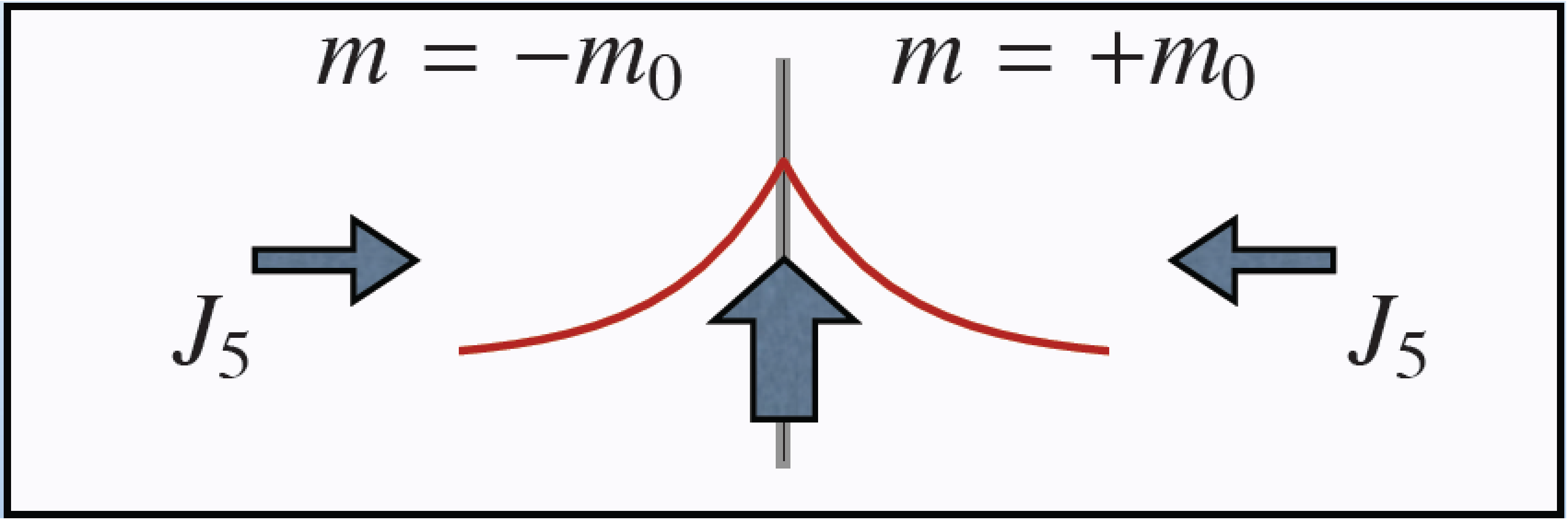
- Rotates real negative mass into positive mass.
- Generates $\theta=\pi$!

Positive mass = Trivial Insulator.

Negative mass = Topological Insulator.

Localized Zero Mode on Interface.

Domain Wall has localized zero mode!



Domain Wall = TI/non-TI Interface

A Lattice Realization.

How to get $\theta = \pi$ from non-interacting electrons in periodic potential (Band-Insulator)?

Topology of Band Structure!

Define Z_2 valued topological invariant of bandstructure to distinguish trivial ("positive mass") from topologically non-trivial ("negative mass").

Band Structure Topology.

Multi-Band-Berry-Connection.

(Qi, Hughes, Zhang)

$$\theta \equiv 2\pi P_3(\theta) = \frac{1}{16\pi^2}\int d^3\mathbf{k}\epsilon^{ijk}\mathrm{Tr}\{[f_{ij}(\mathbf{k}) - \frac{2}{3}ia_i(\mathbf{k})\cdot a_j(\mathbf{k})]\cdot a_k(\mathbf{k})\}$$

$$f_{ij}^{\alpha\beta} = \partial_i a_j^{\alpha\beta} - \partial_j a_i^{\alpha\beta} + i\left[a_i, a_j\right]^{\alpha\beta},$$

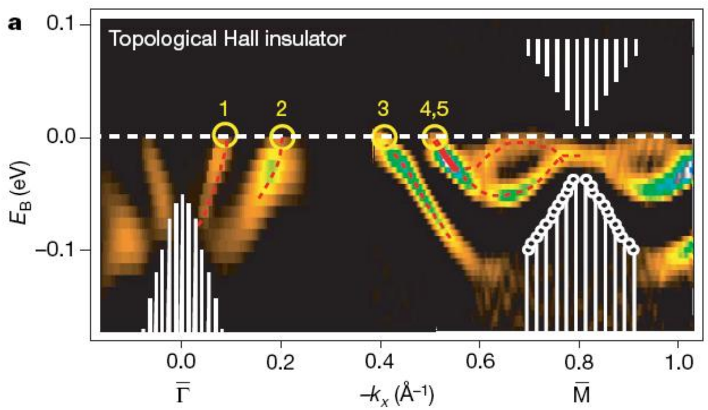
$$a_i^{\alpha\beta}(\mathbf{k}) = -i\left\langle \alpha, \mathbf{k}\right| \frac{\partial}{\partial k_i}\left|\beta, \mathbf{k}\right\rangle$$

$\theta=0$	Vacuum, …
$\theta=\pi$	$Bi_{1-x}Sb_x$, Bi2Se3, Bi2Te3, Sb2Te3

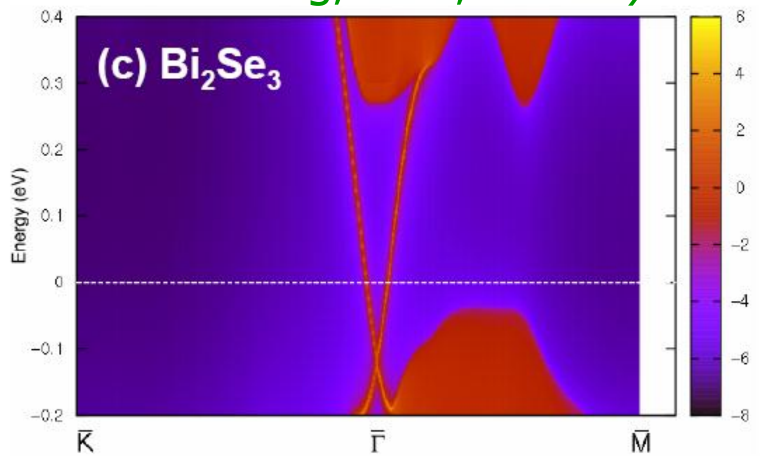
predicted: $TlBi(Sb)Te(Se, S)_2$, LaPtBi etc (Heusler compounds)

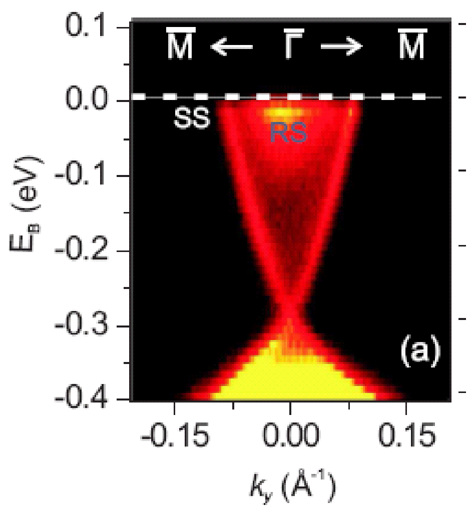
Experimentally found Zero Modes.

Hasan group, *Nature* 2008



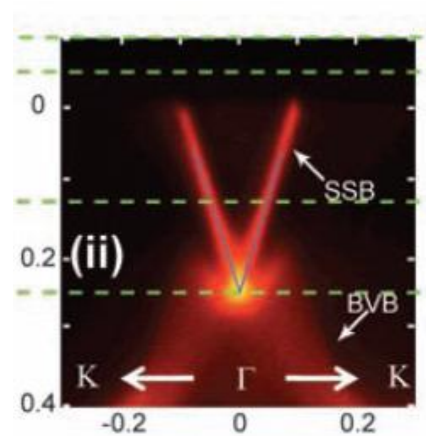
H. J. Zhang, et al, *Nat Phys* 2009





Hasan group,
Bi_2Se_3
Nat Phys 2009

Bi_2Te_3
Chen et al
Science 2009



Summary of Strategy:

Low Energy Effective Theory:

Dirac Quantization $\longrightarrow$ $\theta = \text{Integer} \cdot \pi$

Microscopic Model:

ABJ anomaly $\longrightarrow$ $\theta/\pi = \sum (\text{charge})^2$

Connection to Experiment:

Band Topology $\longrightarrow$ $\theta = \text{QHZ-invariant}$

IR

UV

Fractional topological insulators

(work with Maciejko, Qi and Zhang)

Fractional Topological Insulators?

Recall from Quantum Hall physics:

electron

$$\sigma_{xy} = n\frac{e^2}{h}$$

Quantum Hall

e$^-$ interactions →

(m odd for fermions)

fractionalizes into m partons

$$\sigma_{xy} = \frac{n}{m}\frac{e^2}{h}$$

Fractional Quantum Hall

Fractional Topological Insulators?

TI = half of an integer quantum hall state on the surface

expect: fractional TI = half a fractional QHS
Hall quantum = half of 1/odd integer.

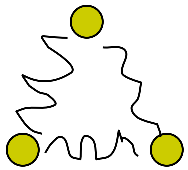
Can we get this from charge fractionalization?

Partons.

Microscopic Model:

ABJ anomaly $\longrightarrow$ $\theta/\pi = \sum (\text{charge})^2$

e^- =



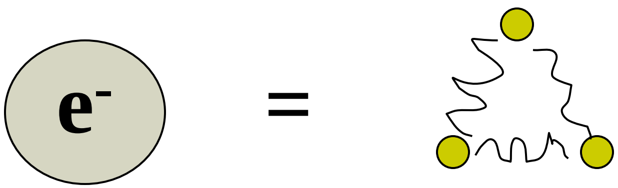
electron breaks up into m partons.

$$\theta / \pi = \sum (\text{charge})^2 = m \cdot \left(\frac{1}{m}\right)^2 = \frac{1}{m}$$

(m odd so e^- is fermion)

(if partons form a TI = have negative mass)

Partons.



To ensure that physical (= gauge invariant) states carry integer electron charge add "statistical" gauge field.

Simplest models: $U(1)^3/U(1)$ quiver gauge theory

$SU(3)$ with $N_f=1$

Important dynamical question.

Is the gauge theory in a confining or deconfining phase?

We need: deconfined! Favors abelian models.
(or N=4 SYM with N=2 massive hyper)

Gapless modes present; charged fields all gapped.

Alternative: Higgsed Phase with unbroken discrete gauge invariance.

Why not confined phase?

Need chiral symmetry to be unbroken.
("Confinement w/o chiral symmetry" breaking ok
--- but also has extra light, neutral states.)

MIT theorem (basically Dirac quantization):
Need either extra massless degrees of freedom
(e.g deconfined phase or SUSY QCD with $N_f = N_c + 1$)
or degenerate groundstate (e.g. Higgs phase)

How to make a fractional TI?

Need: Strong electron/electron interactions

(so electrons can potentially fractionalize)

Strong spin/orbit coupling

(so partons can form topological insulator)

How can one tell if a given material is a fractional TI (in theory/in practice)?

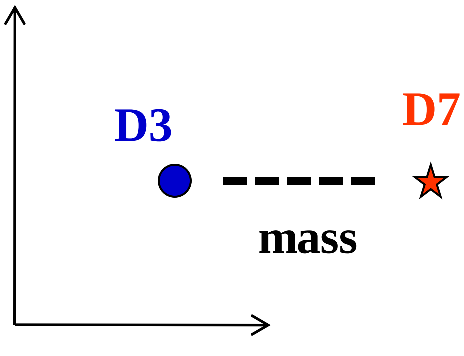
Transport! Fractional Hall + Kerr.

Holographic Realization

(work with Hoyos and Jensen)

fTI in N=4 SYM and AdS/CFT

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x	-	-	-	-	-	-
D7	x	x	x	x	x	x	x	x		

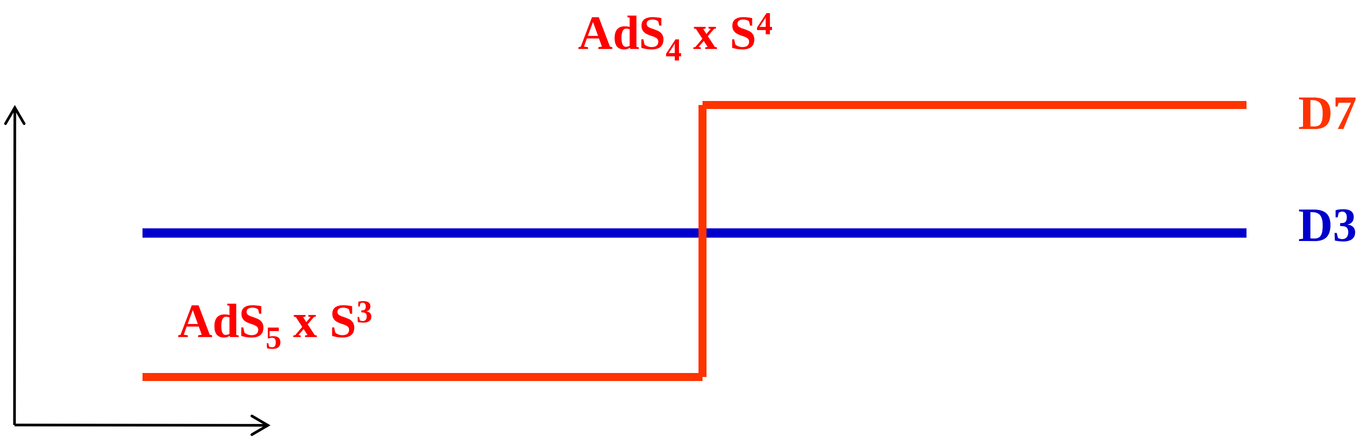


(8,9) space

T-invariant =real mass
= D7 at x_9=0, any x_8

(here this is a choice to preserve T,
Takayanagi and Ryu impose orientifold that
makes T-violating mass inconsistent)

holographic fTI



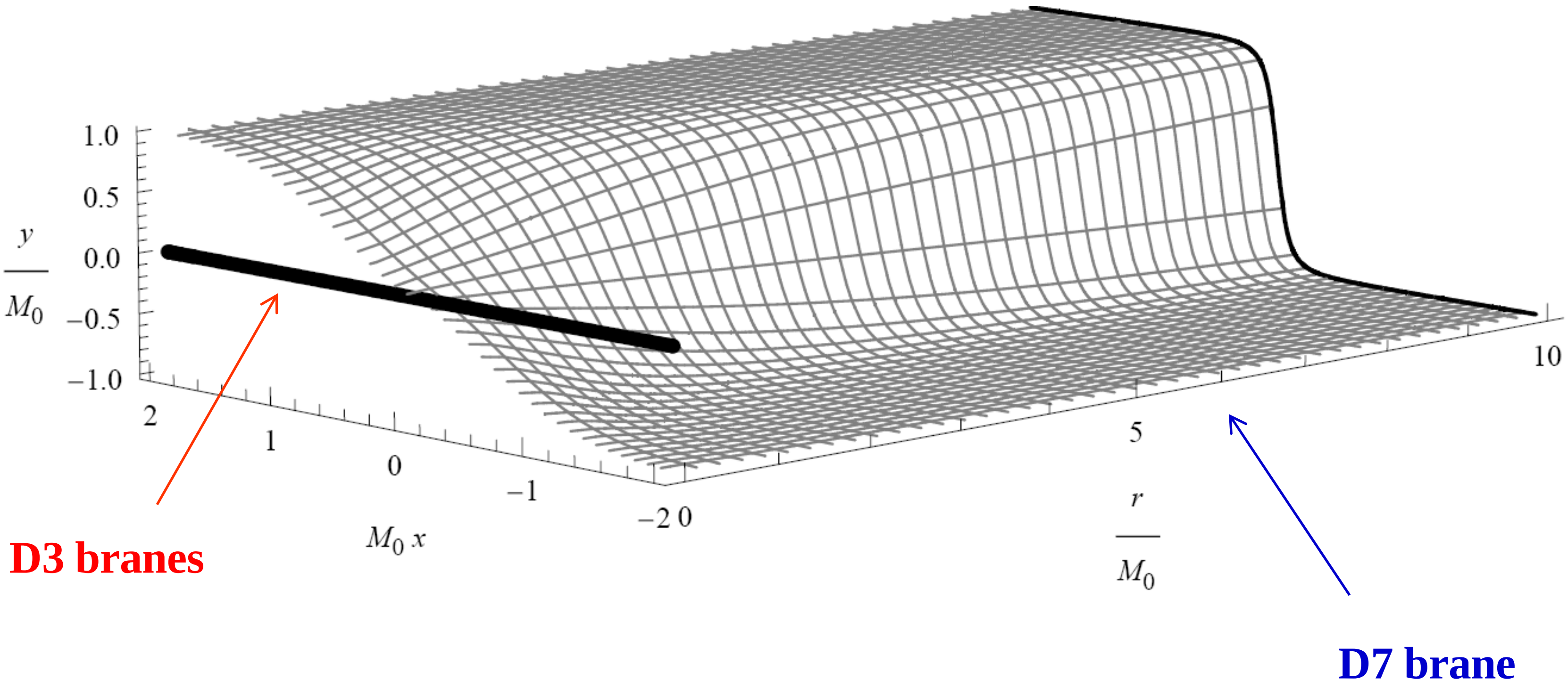
(3,8) space

$$ds^2_{S^5} = \cos^2\theta d\Omega_3^2 + d\theta^2 + \sin^2\theta d\phi^2$$

Find: $\theta(x,r)$

Smooth embedding. Approaches step at r=∞ (boundary of AdS)

holographic fTI



holographic fTI

- Other mass profiles possible; expect Hall current with filling fraction 1/(2m) for any zero crossing profile.
- This can easily be verified from AdS. Independent of details of embedding, the Hall current is uniquely determined by WZ term.

Was expected: WZ term = anomalies

Explicit Realization of a non-abelian fTI

The Quantum Spin Hall Effect

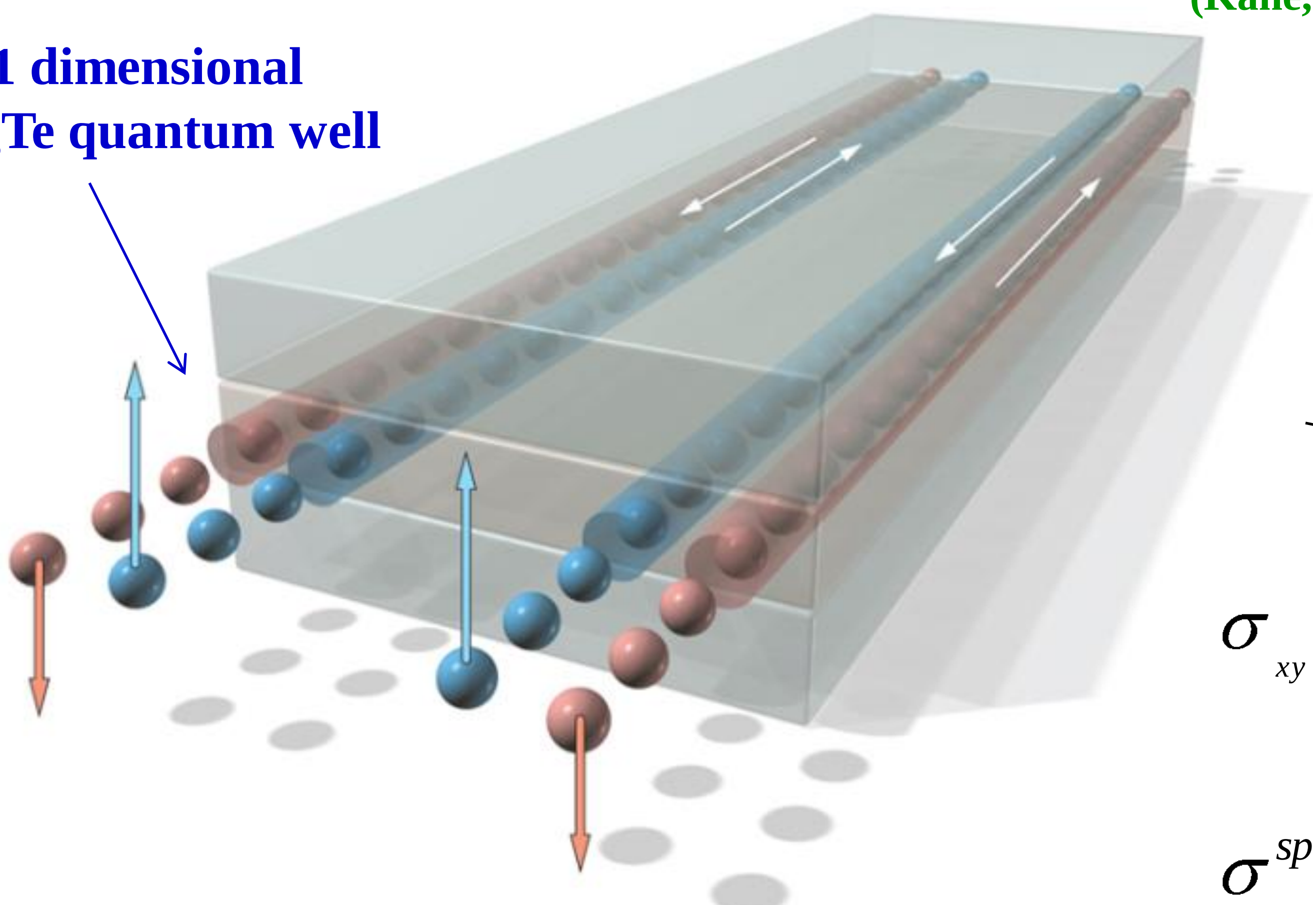
(or better: the quantum R-current-Hall effect)

(work with Maciejko and Takayanagi)

Quantum Spin Hall Effect in HgTe

(Kane, Mele; Bernevig Zhang)

2+1 dimensional HgTe quantum well



electric field

$$\sigma_{xy} = 0$$

$$\sigma_{xy}^{spin} = \frac{e}{2\pi}$$

Continuum Description:

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu - M)\psi$$

2+1: only one mass M
T odd. Single fermion massless!

But: **(Jackiw-Templeton)**

opposite sign!

$$\mathcal{L}_M = M\bar{\Psi}\Psi = M(\bar{\psi}_2\psi_2 - \bar{\psi}_1\psi_1)$$

T invariant mass possible for two fermions!

Coupling to Electromagnetism:

Symmetry: $U(2) \longrightarrow U(1)_{EM} \times U(1)_R$

symmetry of free fermions

gauged

remaining global symmetry

$\boxed{U(1)_R \text{ plays role of spin!}}$

	$U(1)_{EM}$	$U(1)_R$	Sign(mass)
ψ_1	+1	+1	+1
ψ_2	+1	-1	-1

Integrate out fermions:

Induced CS: $$k_{ab} = \frac{1}{2} \sum_i q_{i,a} q_{i,b} \ \mathrm{sign}(M)$$

Electromagnetic: Contributions from the two fermions cancel.
No CS term. No Hall current.

Mixed EM/R: Contributions from the two fermions add. k=1.

The A_R F_{EM} gives rise to "Quantum-R-Hall-Effect"

Applications.

- As in 3+1, given the continuum description it is trivial to construct low energy description of fractional quantum spin hall effect. e splits into m partons. CS term picks up factor of 1/m.

- Holographic realization in terms of probe brane system also straightforward. This time it's the D3/D5 system. Again, topological properties independent of embedding.

Summary.

- Effective field theory for fractional TI can be constructed
- Basic ingredient: fractionalization
- Effective θ follows from anomaly/Dirac quantization
- Requires strong LS coupling and strong interactions.
- Experimental signatures: transport + Kerr/Faraday
- Holographic realization straight forward