LMS-Durham Symposium on the Mathematics of Data Assimilation

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Parameter Estimation and New Application Areas

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Joint work (recently) with
M. D. Chekroun, D. Kondrashov & Y. Shprits, UCLA; A. Carrassi, IRM, Brussels; L. Roques and S. Soubeyrand, INRA, Avignon; C.-J. Sun, CSIRO, Perth; A. Trevisan, ISAC-CNR, Bologna; and many others: please see
http://www.atmos.ucla.edu/tcd/ and http://www.environnement.ens.fr/

Outline

- Data in meteorology, oceanography and space physics
  - *in situ* & remotely sensed
- Basic ideas, data types, & issues
  - how to combine data with models
  - transfer of information
    – between variables & regions
  - filters & smoothers
  - stability of the forecast-assimilation cycle
- Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
  - paleoclimate
- Concluding remarks and bibliography

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Parameter Estimation

a) Dynamical model

$dx/dt = M(x, \mu) + \eta(t)$

$y^o = H(x) + \varepsilon(t)$

Simple (EKF) idea – augmented state vector

$d\mu/dt = 0$, $X = (x^T, \mu^T)^T$

b) Statistical model

$L(\rho)\eta = w(t)$, L – AR(MA) model, $\rho = (\rho_1, \rho_2, \ldots. \rho_M)$

Examples: 1) Dee *et al.* (*IEEE*, 1985) – estimate a few parameters in the covariance matrix $Q = E(\eta, \eta^T)$; also the bias $<\eta> = E\eta$;

2) POPs - Hasselmann (1982, Tellus); Penland (1989, *MWR*; 1996, *Physica D*); Penland & Ghil (1993, *MWR*)

3) $dx/dt = M(x, \mu) + \eta$: Estimate both M & Q from data (Dee, 1995, *QJ*), Nonlinear approach: Empirical mode reduction (EMR: Kravtsov *et al.*, *J. Clim.*, 2005; Kondrashov *et al.*, *J. Clim.*, 2005; Strounine *et al.*, *Physica D*, 2009)

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Estimating noise – I

$Q_1 = Q_{slow}\,,\quad Q_2 = Q_{fast}\,,\quad Q_3 = 0;$

$R_1 = 0,\quad R_2 = 0,\quad R_3 = R;$

$Q = \sum \alpha_i Q_i;\; R = \sum \alpha_i R_i\,;$

$\alpha(0) = (6.0, 4.0, 4.5)^T;$

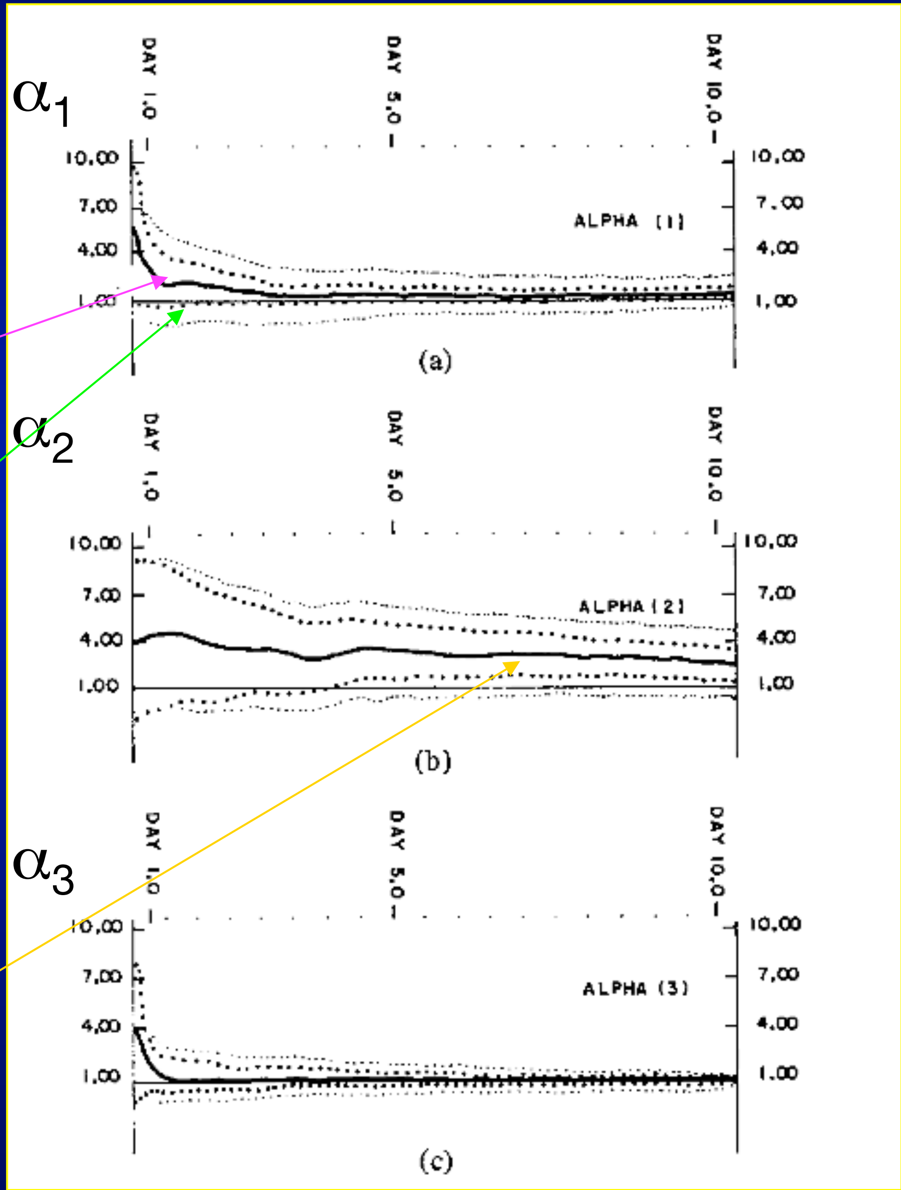
$Q(0) = 25^* I.$

Dee *et al*. (1985, *IEEE Trans. Autom. Control*, **AC-30**)

estimated

true (α =1)

Poor convergence for Q_{fast}?



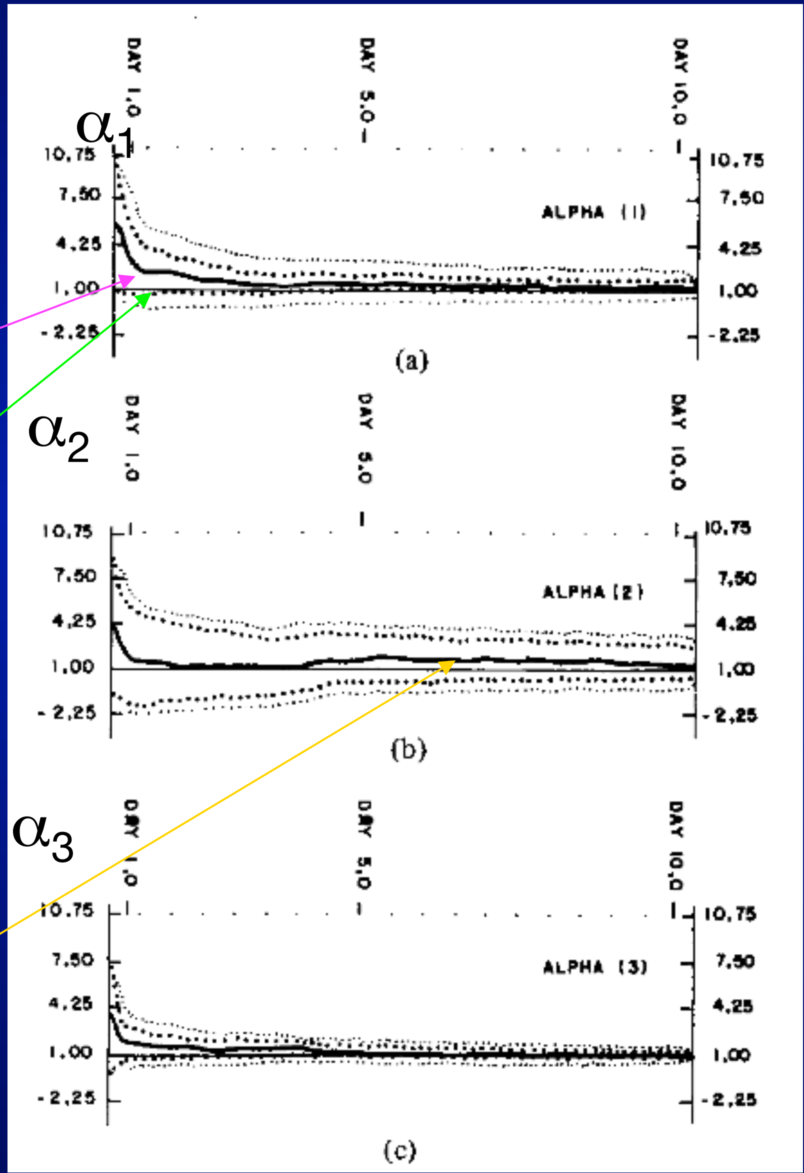
Estimating noise – II

Same choice of $\alpha(0)$, Q_i, and R_i but

$$\Theta(0) = 25 * \begin{bmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

estimated

true ($\alpha = 1$)



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Good convergence for Q_{fast}!

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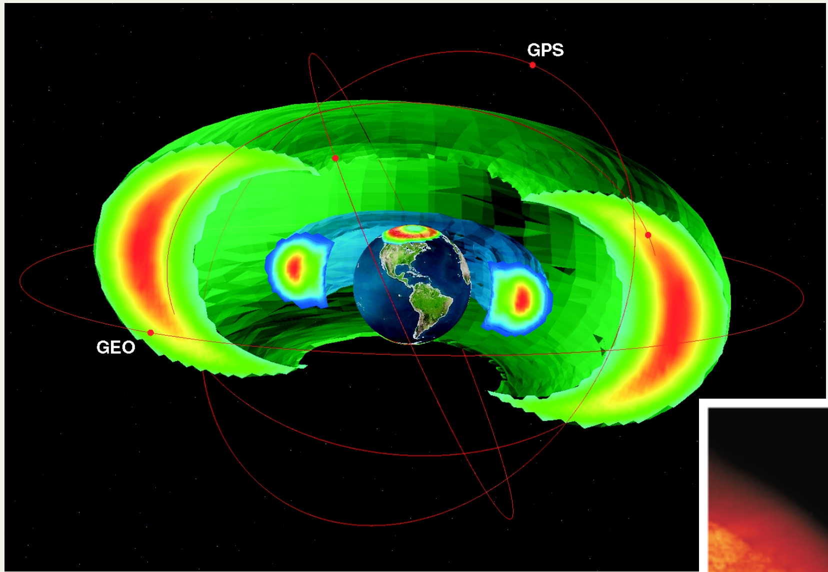
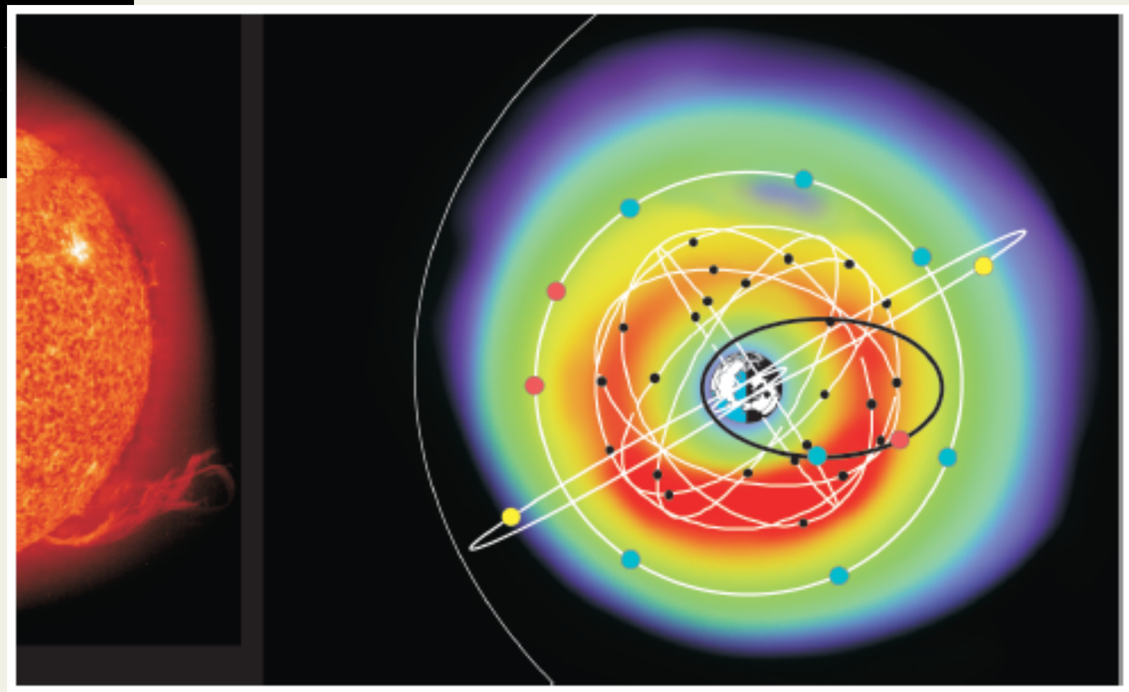
Space physics data



Two decades ago …

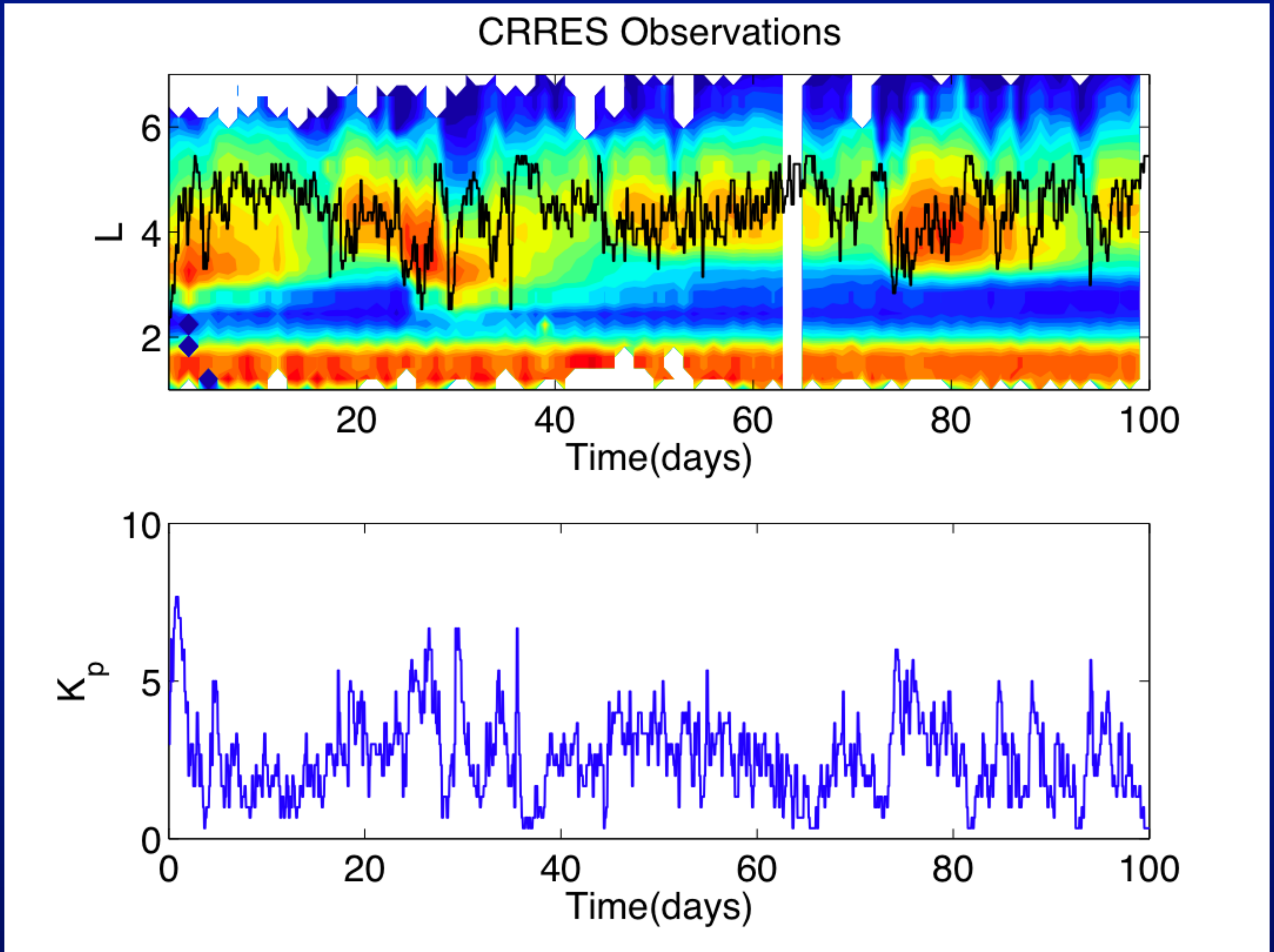
… and now



Space platforms in Earth’s magnetosphere

Parameter Estimation for Space Physics – I

Daily fluxes of 1 MeV relativistic electrons in Earth's outer radiation belt
(CRRES observations from 28 August 1990)
***K_p* - index of solar activity (external forcing) –** used to determine the position of the plasmapause L_{pp} (black) in the observations



Kondrashov, Shprits, Ghil & Thorne (*J. Geophys. Res.*, 2007)

Parameter estimation for space physics – II

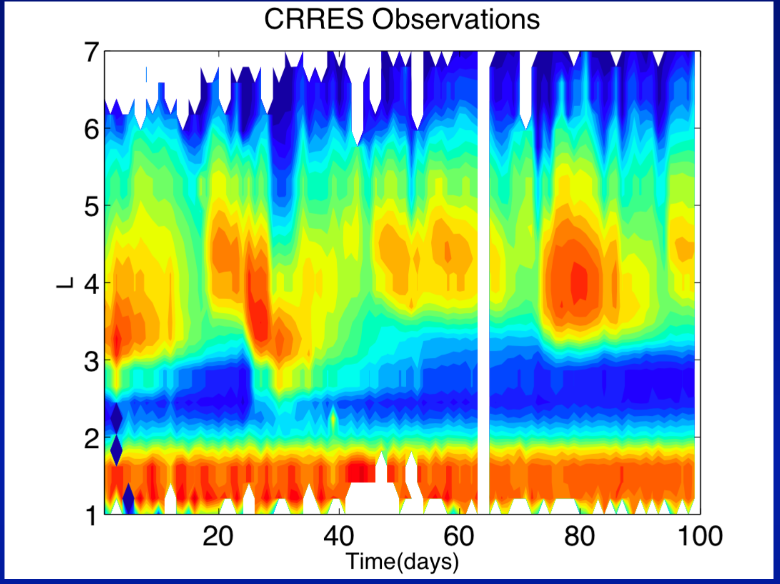
HERRB-1D code (Y. Shprits) – estimating phase-space density *f* and electron lifetime τ_L:

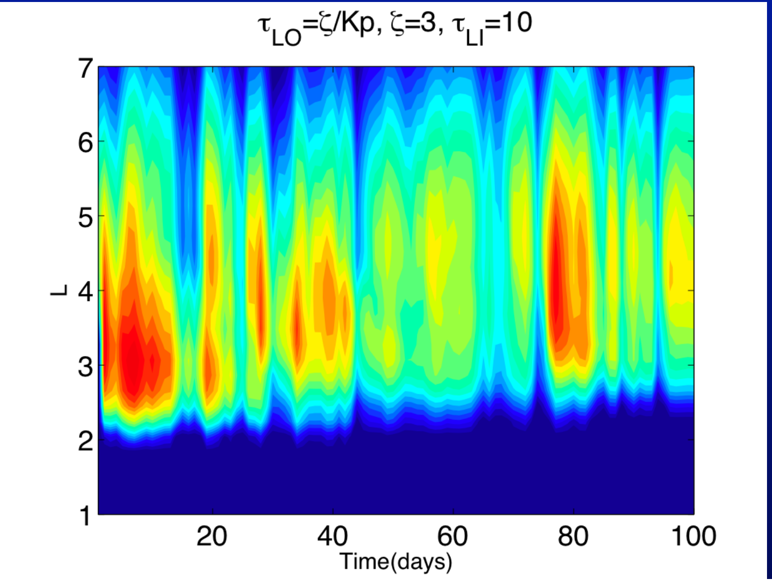
$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(L^{-2} D_{LL} \frac{\partial f}{\partial L}\right) - \frac{f}{\tau_L}$$

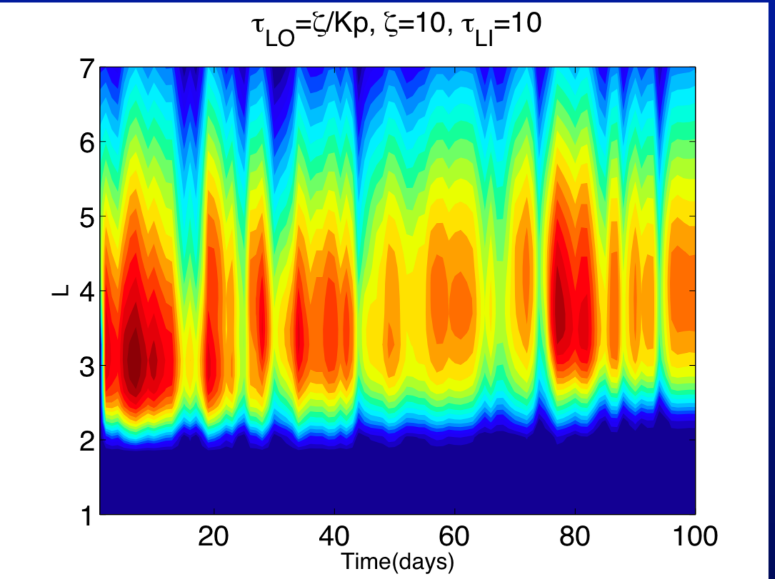
Different lifetime parameterizations for plasmasphere – out/in:

$\tau_{Lo} = \zeta / K_p(t)$; τ_{Li} = const.

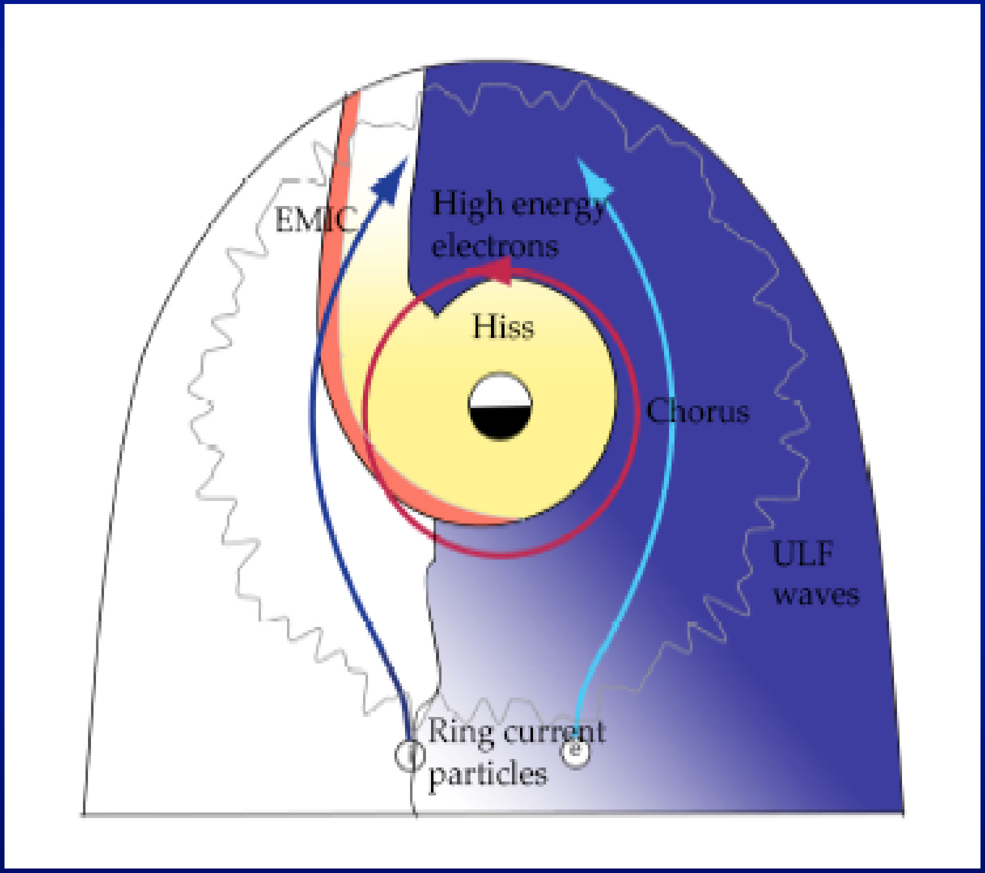
What are the **optimal** lifetimes to match the observations best?







Dominant loss mechanisms



Pitch angle scattering due to resonance interactions with :

1) Plasmaspheric hiss (whistler mode waves) loss time on the scale of 5-10 days (Lyons & Thorne, 1973; Abel & Thorne, 1998; Meredith et al., 2006)
2) Chorus waves outside plasmapause provide fast losses on the scale of a day (Horne et al., 2005; Albert et al., 2005; O'Brien, 2004; Thorne et al., 2005)
3) EMIC waves mostly in plumes on the dusk side – very fast localized losses (Millan et al., 2002; Summers & Thorne, 2003; Albert, 2003, Bortnik et al., 2006; Shprits et al., 2006a)
4) Combined effect of losses to magnetopause and outward radial diffusion (Shprits et al., 2006b).

Parameter estimation for space physics – III

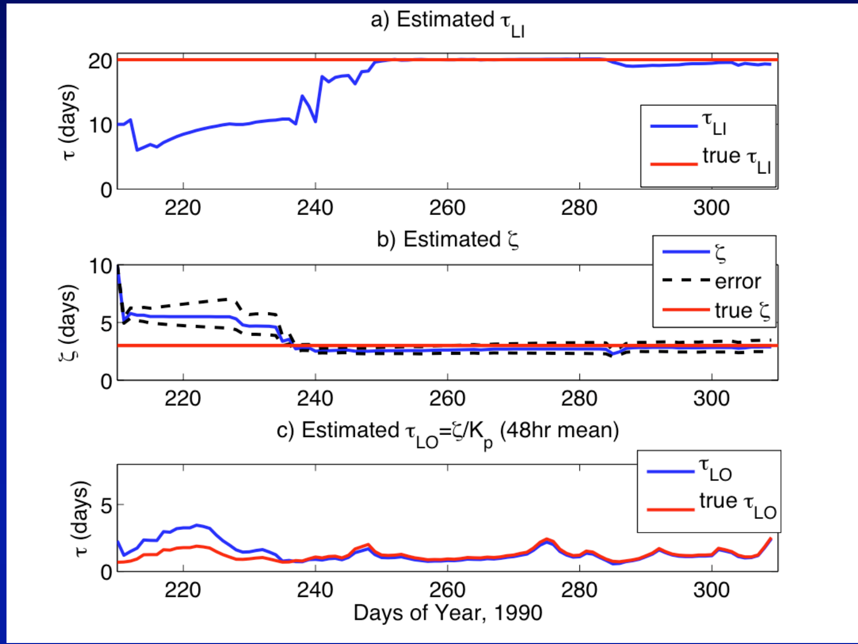
Daily observations from the "truth" —

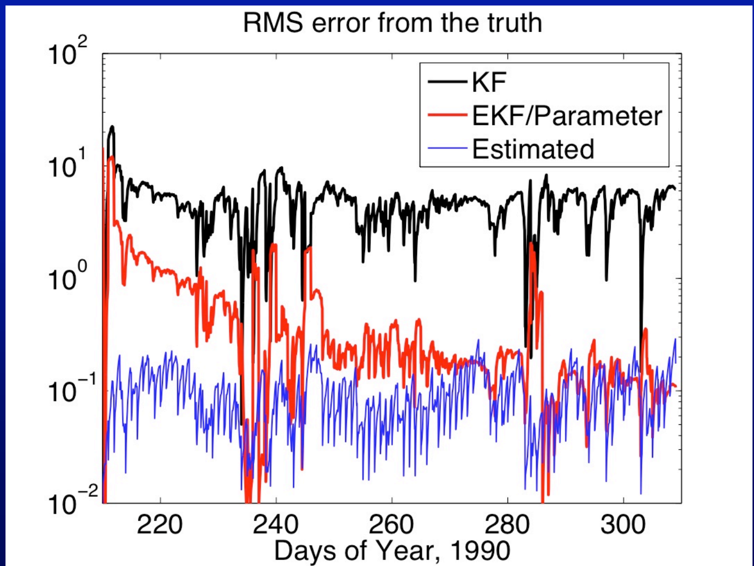
$\tau_{Lo} = \zeta/K_p$, $\zeta = 3$, and $\tau_{LI} = 20$ —

are used to correct the model's "wrong" parameters, $\zeta = 10$ and $\tau_{LI} = 10$.

The estimated error $tr(P^f) \approx$ actual.

When the parameters' assumed uncertainty is large enough, their EKF estimates converge rapidly to the "truth".





Black – actual errors for state estimation only

Red – actual errors for state and parameter estimation

Blue – EKF-estimated error (tr P_k^f)

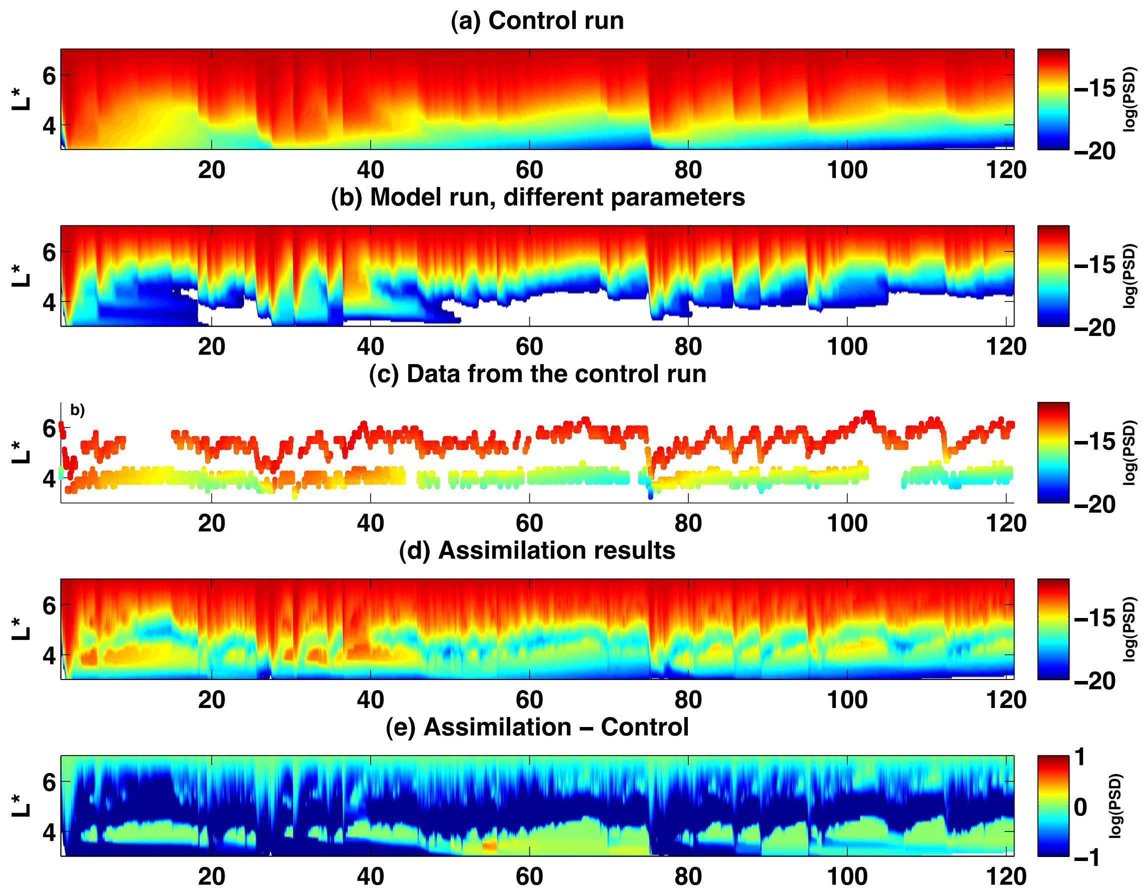
Log-normal EKF for Order-of-Magnitude Changes in Dependent Variables: Space Plasmas – I

Phase-space densities (PSDs) in the Van Allen radiation belts vary by several orders of magnitude over the interval $1 \leq L \leq 6R_{\mathrm{E}}$, where $R_{\mathrm{E}} =$ Earth's radius. This interval includes sharp gradients at the time-varying plasmapause:

$$L_{\mathrm{PP}} = 2R_{\mathrm{E}} - 6R_{\mathrm{E}}.$$

Not good for standard sequential (or control) methods that assume normally distributed errors ➔ Change of variables!

D. Kondrashov, Y. Shprits & M. Ghil (*Space Weather*, 2011, submitted)



Log-normal EKF for Order-of-Magnitude Changes in Dependent Variables: Space Plasmas – II

Introduce the new variable $S = \log(f)$ to yield the nonlinear PDE in *S*:

$$\frac{\partial S}{\partial t} = L^2 \frac{\partial}{\partial L}\left(\frac{1}{L^2} D_{LL} \frac{\partial S}{\partial L}\right) - \frac{1}{\tau_L} + D_{LL}\left(\frac{\partial S}{\partial L}\right)^2 .$$

To deal with the nonlinearity and the sharp gradients, we use a total-variation diminishing, second-order scheme (A. Harten, *JCP*, 1983).

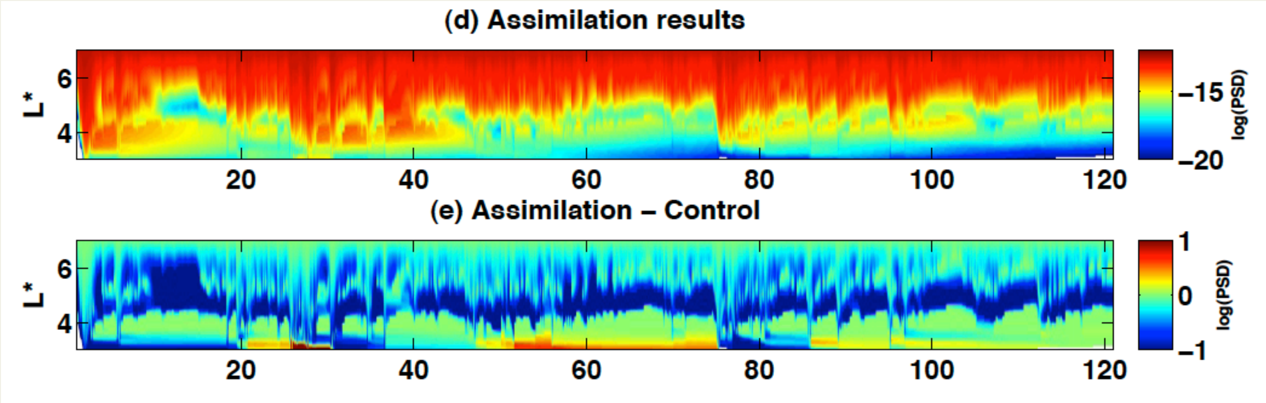
The linear Kalman filter for the original PDE in *f* has to be replaced by an EKF. Results are definitely better with the modified PDE & the log-EKF, as shown by the plot below for "fraternal (dizygotous)-twin" experiments. This is especially so when the observational error covariances ***R*** are much larger than the model errors ***Q***.

Another way of evaluating assimilation scheme performance is by considering the variance of the innovation sequence residuals:

$E\mathbf{z}_k^{\mathrm{T}}\mathbf{z}_k,$ where

$\mathbf{z}_k \equiv \mathbf{y}_k^{\mathrm{o}} - \mathbf{H}x_k^{\mathrm{f}}.$

D. Kondrashov, Y. Shprits & M. Ghil (*Space Weather*, 2011, submitted)



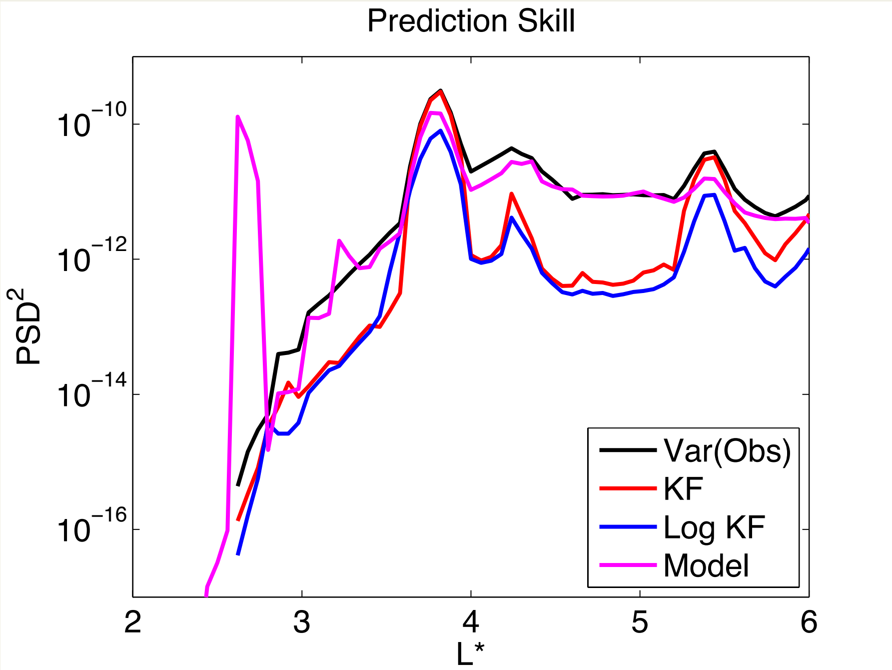
Log-normal EKF for Order-of-Magnitude Changes in Dependent Variables: Space Plasmas – III

We have used real observational data sets from 4 spacecraft missions: the Combined Release and Radiation Effects Satellite (CRRES), GEO-1989 (GEO), GPS NS18 (GPS), and Akebono.

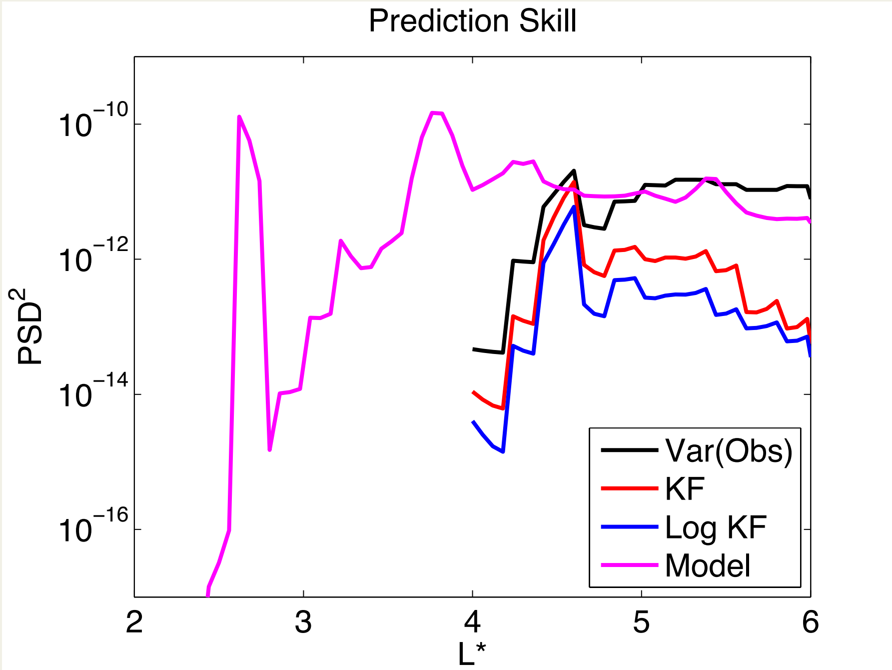
CRESS has the best coverage and accuracy, and was used as a benchmark.

Assimilation was performed with the Akebono and GEO observations, separately.

Plotted are the results for $E\mathbf{z}_k^{\mathrm{T}}\mathbf{z}_k,\ \mathbf{z}_k \equiv \mathbf{y}_k^{\mathrm{o}} - \mathbf{H}x_k^{\mathrm{f}}.$



Akebono



GEO

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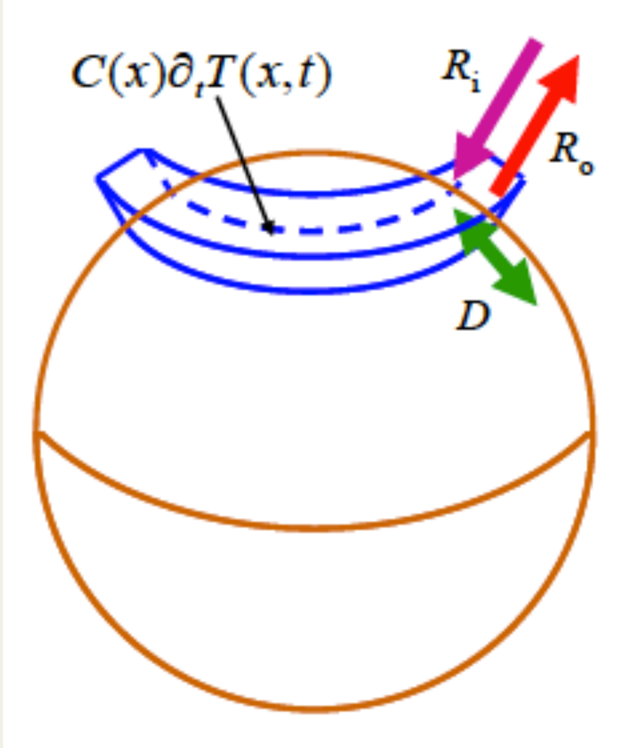
Parameter estimation for energy balance models with memory (EBMMs) – I

One considers a 1-D paleoclimate model governed by an EBM for zonally averaged surface air temperatures $T(t, x)$:

$$c(x)\,\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\,\frac{\partial T}{\partial x}\right) + \mu\,Q(x)[1 - a(x,T)] - g(x,T);$$

here $R_\mathrm{i} = \mu\,Q(x)[1 - a(x,T)]$ is the absorbed solar radiation, with $a = a(x, T)$ the planetary albedo, and $R_\mathrm{o} = g(x, T)$ is the terrestrial radiation, modified by the greenhouse effect, while $0 \leq x \leq 1$ is a meridional variable. The albedo depends on past temperatures, because of the long time needed to build up and melt ice sheets.

Ghil (*JAS*, 1976), Bhattacharya, Ghil & Vulis (*JAS*, 1983), Roques *et al.* (*Phil. Trans.*, 2011, submitted)



Zonal belt with heat capacity $C(x)$ and temperature $T(t, x)$, subject to incoming radiation R_i, outgoing radiation R_o, and meridional diffusion D.

Parameter estimation for energy balance models with memory (EBMMs) – II

The memory effects are represented by a history function $H = H(t, x, T)$,

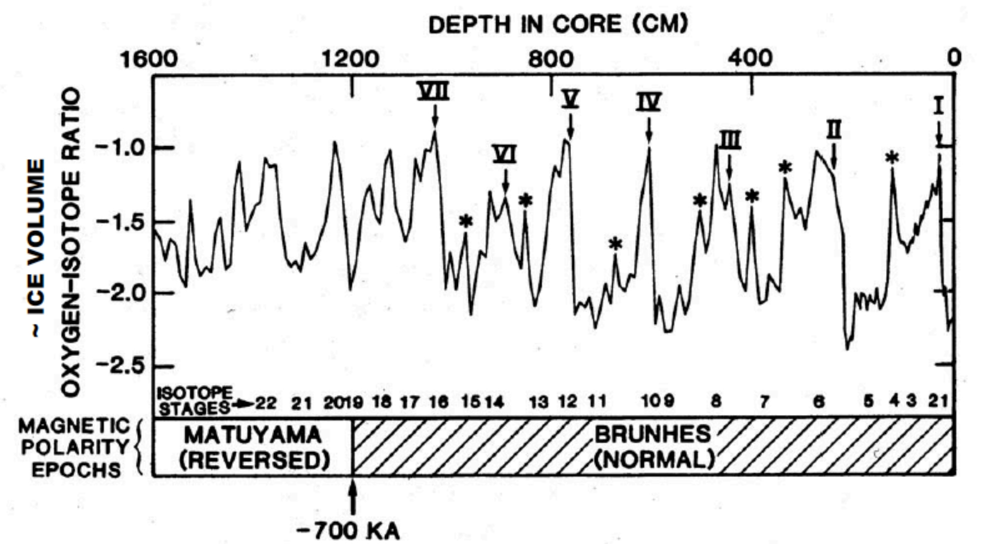
$$H(t, x, T) = \int_{-\tau}^{0} \beta(s, x)\, T(t + s, x)\, \mathrm{d}s,\ \ t > 0,\ \ x \in (0, 1),$$

with the non-negative kernel $\beta = \beta(s, x)$ that sums to unity, thus yielding the general EBMM:

$$c(x, H(t, x, T))\, \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\, \frac{\partial T}{\partial x}\right) + f(t, x, T, H(t, x, T));$$

here $f = R_i - R_o$ is the net radiation balance, affected by the past history.

The observational data come from proxy records of past temperatures and ice volume, with errors in both age-dating (abscissa = time axis) and "transfer function" (ordinate = climate variable).



Roques *et al.* (*Phil. Trans.*, 2011, submitted)

Parameter estimation for energy balance models with memory (EBMMs) – III

The initial data for this functional PDE are

$$T(s,x) = T_0(s,x),\ \ s \in [-\tau, 0],\ \ x \in [0,1]$$

and we use Neumann boundary conditions at the 2 poles (or pole and equator, by symmetry). This semi-empirical EBMM requires determining coefficients from the proxy records, e.g., the ratio $\alpha = \alpha(x)$ between R_i and R_o:

$$f = f_\alpha(t,x,T,H) = f_1(t,x,T,H) + \alpha(x)\, f_2(t,T,H),\ \ H = H(t,x,T).$$

Here $f = f[\alpha] = R_\mathrm{i} - R_\mathrm{o}$ is the reaction function in our reaction-diffusion model. Under reasonable assumptions on $f_1, f_2, H, c, k, \alpha$ and β, one can prove that — given exact initial data over $-\tau \le t \le 0$ and exact data on T and T_x at a single point $0 < x_0 < 1$ (i.e., for a single "core") over some interval $0 < t < t^*$ — the coefficient $\alpha(x)$ is determined uniquely!

But we are interested now in the more realistic situation in which a statistical model of the observation process is needed. We assume that $T(t, x) = T_0$ is the initial data with prior distribution π_1 and that the unknown coefficient α has prior distribution π_2. Data will be provided at three sites (cores) S_k, k =1, 2, 3, in the interval's right half.

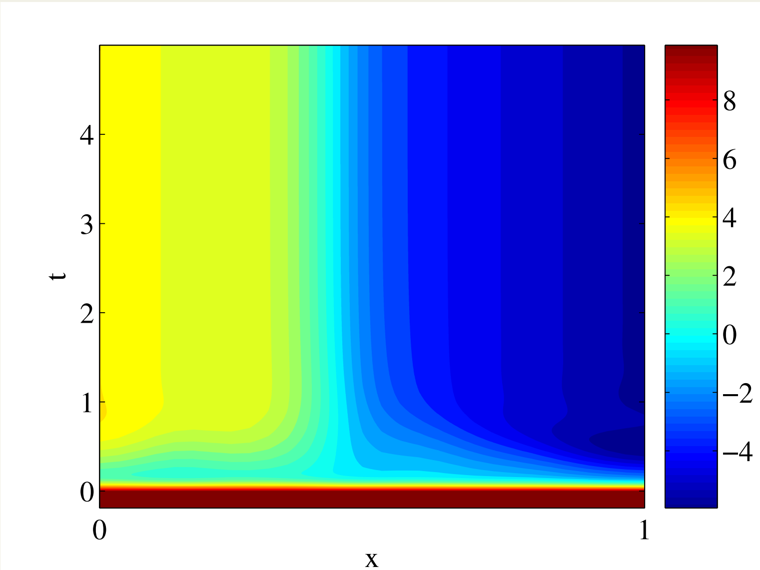
Roques *et al.* (*Phil. Trans.*, 2011, submitted)

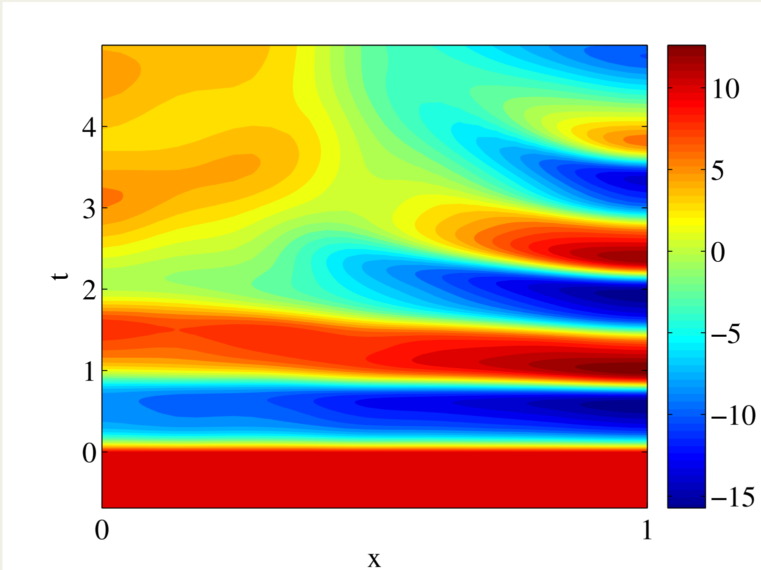
Parameter estimation for energy balance models with memory (EBMMs) – IV

The mechanistic-statistical model now includes the specific EBMM

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \alpha(x)\,(1 - a(T)) - q_0 - q_1\,T - \left(\frac{1}{\tau}\int_{-\tau}^{0} T(t+s)\,\mathrm{d}s\right)^3,$$

where the albedo $a(t)$ is a known, piecewise-linear ramp function (cf. Sellers, 1969, and Ghil, 1976), and we study numerically the two cases $\tau = 0.2$ and $\tau = 0.7\,\mathrm{ky}.$





As expected, the solution tends rapidly to stationarity for small lag and has a longer transient, with large amplitude, for the larger lag.

The proxy records at the three sites S_k, k =1, 2, 3 have 2 sources of uncertainty.

Parameter estimation for energy balance models with memory (EBMMs) – V

The statistical model for these uncertainties is as follows:

(i) $Y_k(t_i)$ is the measurement of temperature T at time t_i and location S_k,

$$Y_k(t_i) \mid s(t_i) \sim \text{indep. } \mathcal{N}\left\{T(s(t_i), S_k), \sigma^2\right\},$$

where σ^2 is the variance of the temperature measurement noise; and

(ii) the date of t_i is in fact $s(t_i)$, with

$$s(t_i) = \theta - \sum_{j=1}^{i} \eta_j \text{ with } \eta_j \sim \text{indep. } \Gamma\left(\frac{t_{j-1} - t_j}{\kappa^2}, \kappa^2\right),$$

where Γ is the gamma distribution, $\kappa^2 > 0$ is a shape parameter, and $t_0 = \theta.$

This model is order-preserving, i.e.

$$t_i > t_j \text{ implies } s(t_i) > s(t_j),$$

$E(s(t)) = t,$ and its variance increases as we "sink" further into the past.

Sobrino *et al.* (*Boreas*, 2008): Age-depth models for 3 pollen cores In NW Iberia.

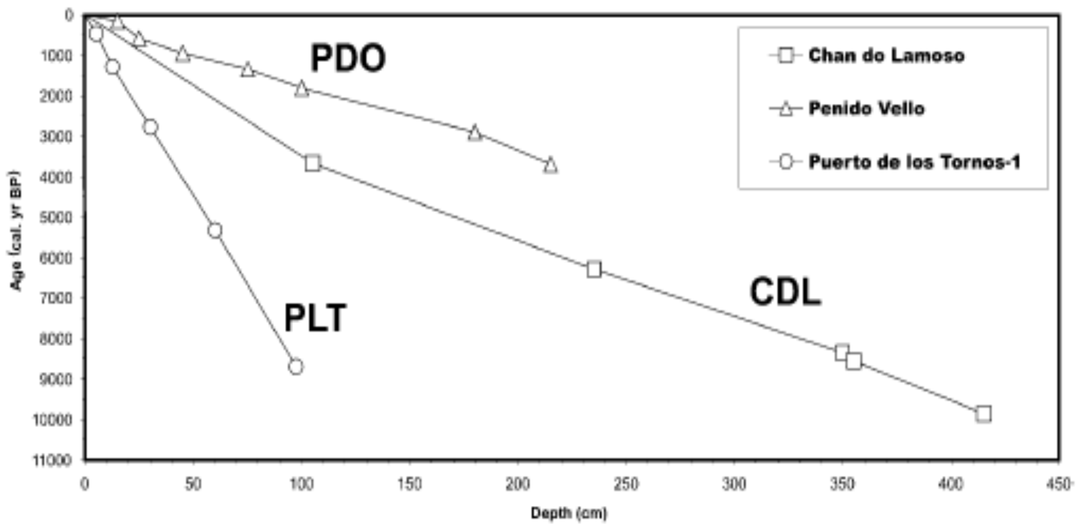
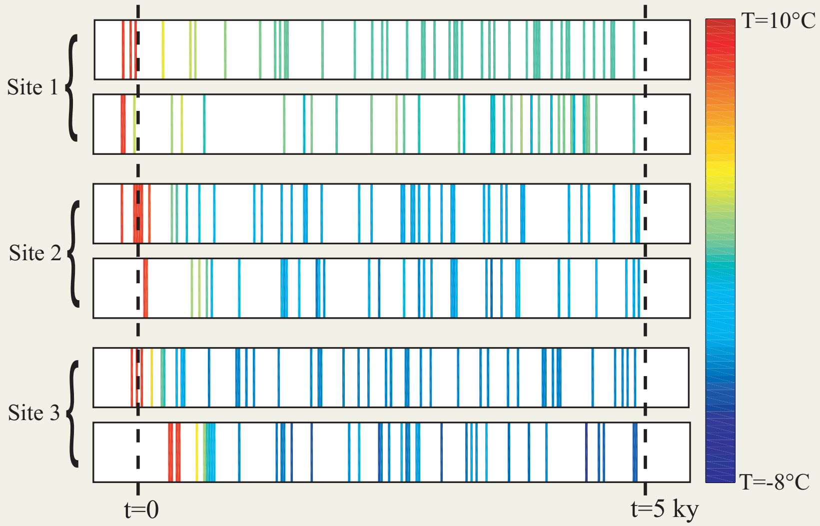
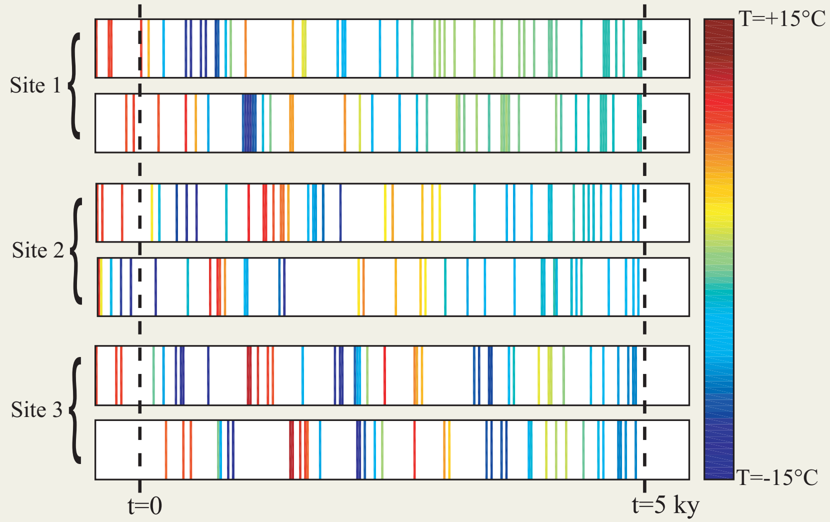


Fig. 6. Age-depth curves for the three cores analysed.

Parameter estimation for energy balance models with memory (EBMMs) – VI





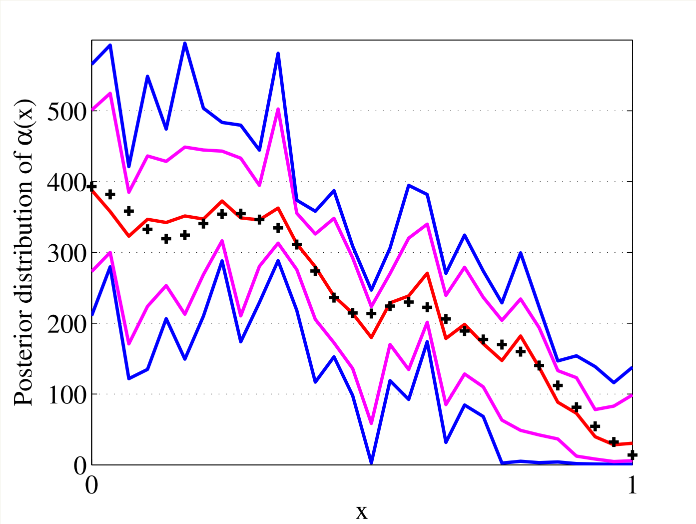
Actual temperatures vs. measured temps, at the 3 sites. At each of them, the upper row corresponds to the actual T's at the actual times, while the lower row corresponds to the measured T's at the estimated times. (a) $\tau = 0.2$ ky, and (b) $\tau = 0.7$ ky. Clearly the errors in both the estimates of T's and times are larger for the larger delay, which resulted in the more irregular solution.

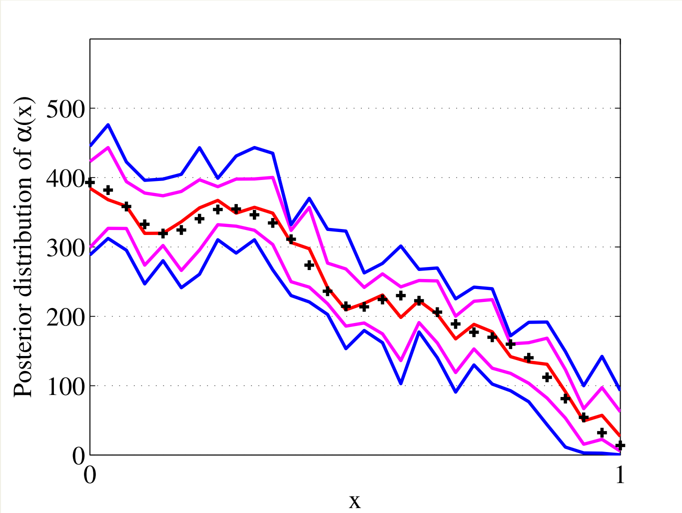
We seek the coefficient $\alpha(x)$ by a Bayesian approach, assume uniform prior distributions for T_0 and for $\alpha(x)$, and draw a sample from the joint posterior distribution of $(T_0, \alpha(x))$ by Markov chain Monte Carlo (MCMC).

Roques *et al.* (2011, submitted)

Parameter estimation for energy balance models with memory (EBMMs) – VII

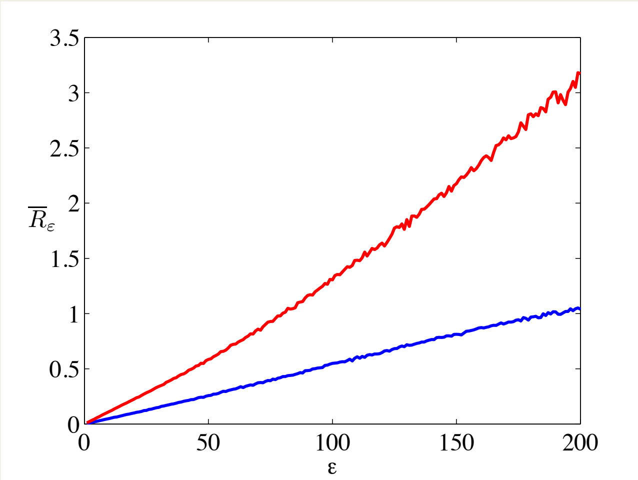
The results are shown below:





Estimates of the coefficient α(x): posterior median (red), first and last deciles (magenta), first and last percentiles (blue); the true values are the + signs. (a) τ = 0.2 ky, and (b) τ = 0.7 ky. Clearly the estimates are better in (b).

The last figure shows the average L_2-response $\overline{R}_\varepsilon$ of our EBMM model to random perturbations α'(x) in α(x) drawn from a random field A with std. dev. ε, over the interval $0 < t < 5$ ky: blue for τ = 0.2 ky, and red for τ = 0.7 ky. Both curves show linear response, but the red one has double the slope.



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Concluding remarks

We've come a long way in 30 years — some advances are laborious and incremental (e.g., sequential vs. control-theoretical methods), but others are fresh and exciting.

The latter include new areas of application
– biology, paleoclimate, space physics, …;
as well as novel methodological challenges
– multi-scale and multi-model problems
– inverse problems for evolution equations, …

Technological advances both pose new problems (more data, higher resolution, …) and help solve them.

Overall, it's a brave new world, in which data and models actively speak to each other, and we do so to both: enjoy!

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Concluding remarks

We've come a long way in 30 years — some advances are laborious and incremental (e.g., sequential vs. control-theoretical methods), but others are fresh and exciting.

The latter include new areas of application
– biology, paleoclimate, space physics, …;
as well as novel methodological challenges
– multi-scale and multi-model problems
– inverse problems for evolution equations, …

Technological advances both pose new problems (more data, higher resolution, …) and help solve them.

Overall, it's a brave new world, in which data and models actively speak to each other, and we do so to both: enjoy!

General references

Bengtsson, L., 1975: *Four-Dimensional Data Assimilation of Meteorological Observations*, GARP Publ. Ser., No. 15, WMO-ICSU, Geneva, Switzerland, 76 pp.

Bengtsson, L., M. Ghil and E. Källén (Eds.), 1981. *Dynamic Meteorology: Data Assimilation Methods*, Springer-Verlag, 330 pp.

Daley, R., 1991. *Atmospheric Data Analysis*. Cambridge Univ. Press, Cambridge, U.K., 460 pp.

Ghil, M., and P. Malanotte-Rizzoli, 1991. Data assimilation in meteorology and oceanography. *Adv. Geophys*., **33**, 141–266.

Bennett, A. F., 1992. *Inverse Methods in Physical Oceanography*. Cambridge Univ. Press, 346 pp.

Malanotte-Rizzoli, P. (Ed.), 1996. *Modern Approaches to Data Assimilation in Ocean Modeling*. Elsevier, Amsterdam, 455 pp.

Wunsch, C., 1996. *The Ocean Circulation Inverse Problem*. Cambridge Univ. Press, 442 pp.

Ghil, M., K. Ide, A. F. Bennett, P. Courtier, M. Kimoto, and N. Sato (Eds.), 1997. *Data Assimilation in Meteorology and Oceanography: Theory and Practice*, Meteorological Society of Japan and Universal Academy Press, Tokyo, 496 pp.

Perec, G., 1969: *La Disparition*, Gallimard, Paris.

Van Leeuwen, P.-J., 2009: Particle filtering in geophysical systems, in *Special Math. Advances DA Collection*, *Mon. Wea. Rev*., **137**, 4089–4114.

Bocquet, M., C. Pires, and L. Wu, 2010: Beyond Gaussian statistical modeling in geophysical data assimilation, in *Special 4D-Var←→EnKF Collection*, *Mon. Wea. Rev*., **138**, 2997–3023.

More references

Chin, T. M., M. J. Turmon, J. B. Jewell, and M. Ghil, 2007: An ensemble-based smoother with retrospectively updated weights for highly nonlinear systems, *Mon. Wea. Rev.*, **135** (1), 186–202.

Ghil, M., S. Cohn, J. Tavantzis, K. Bube, and E. Isaacson, 1981: Applications of estimation theory to numerical weather prediction, *Dynamic Meteorology: Data Assimilation Methods*, L. Bengtsson, M. Ghil and E. Källén (Eds.), Springer Verlag, pp. 139–224.

Ghil, M., K. Ide, A. F. Bennett, P. Courtier, M. Kimoto, and N. Sato (Eds.), 1997. *Data Assimilation in Meteorology and Oceanography: Theory and Practice*, Meteorological Society of Japan and Universal Academy Press, Tokyo, 496 pp.

Ide, K., P. Courtier, M. Ghil, and A. Lorenc, 1997: Unified notation for data assimilation: Operational, sequential and variational. *J. Meteor. Soc. Japan,* **75**, 181–189.

Kalnay, E., 2003. *Atmospheric Modeling, Data Assimilation and Predictability*.Cambridge Univ. Press, Cambridge/London, UK, 341 pp.

Kondrashov, D., Y. Shprits, M. Ghil, and R. Thorne, 2007: Estimation of relativistic electron lifetimes in the outer radiation belt: A Kalman filtering approach, *J. Geophys. Res.-Space Phys*., **112**, A10227, doi: 10.1029/2007JA012583.

Kondrashov, D., C.-j. Sun, and M. Ghil, 2008: Data assimilation for a coupled ocean-atmosphere model. Part II: Parameter estimation, *Mon. Wea. Rev.*, **136**, 5062–5076, doi: 10.1175/2008MWR2544.1.

Kondrashov, D., Y. Shprits, and M. Ghil, 2011: Log-normal Kalman filter for assimilating phase-space density data in the radiation belts, *Space Weather*, submitted.

Roques, L., M. D. Chekroun, M. Cristofol, S. Soubeyrand and M. Ghil, 2011: Parameter estimation for energy balance models with memory, *Phil. Trans. R. Soc. (London) A*, submitted.

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