Particle representations for stochastic partial differential equations

New material joint with Dan Crisan and Chris Janjigian. Earlier work with Peter Donnelly, Phil Protter, Jie Xiong, Yoonjung Lee, Peter Kotelenez,

McKean-Vlasov

For $1 \leq i \leq n$,

$$X_i^n(t) = X_i^n(0) + \int_0^t \sigma(X_i^n(s), V^n(s))dB_i(s) + \int_0^t b(X_i^n(s), V^n(s))ds + \int_0^t \alpha(X_i^n(s), V^n(s))dW(s)$$

where $V^n(t)$ is the normalized empirical measure $\frac{1}{n}\sum_{i=1}^n \delta_{X_i^n(t)}$.

As $n \to \infty$, X_i^n "should" converge to a solution of the infinte system

$$X_i(t) = X_i(0) + \int_0^t \sigma(X_i(s), V(s))dB_i(s) + \int_0^t b(X_i(s), V(s))ds + \int_0^t \alpha(X_i(s), V(s))dW(s)$$

Problem: Does V^n converge, and if so, to what?

Exchangeability and de Finetti's theorem

$X_1, X_2, \ldots \in S$ is *exchangeable* if

$$P\{X_1 \in \Gamma_1, \ldots, X_m \in \Gamma_m\} = P\{X_{s_1} \in \Gamma_1, \ldots, X_{s_m} \in \Gamma_m\}$$

$(s_1, \ldots, s_m)$ any permutation of $(1, \ldots, m)$.

Theorem 1 *(de Finetti) Let $X_1, X_2, \ldots$ be exchangeable. Then there exists a random probability measure Ξ such that for every bounded, measurable g,*

$$\lim_{n\to\infty} \frac{g(X_1) + \cdots + g(X_n)}{n} = \int g(x)\Xi(dx)$$

almost surely, and

$$E[\prod_{k=1}^{m} g_k(X_k)|\Xi] = \prod_{k=1}^{m} \int_S g_k d\Xi$$

Convergence of exchangeable systems Kotelenez and Kurtz (2010)

Lemma 2 *Let* $X^n = (X_1^n, \ldots, X_{N_n}^n)$ *be exchangeable families of* $D_E[0,\infty)$*-valued random variables such that* $N_n \Rightarrow \infty$ *and* $X^n \Rightarrow X$ *in* $D_{E^\infty}[0,\infty)$*. Define*

$$\Xi^n = \frac{1}{N_n}\sum_{i=1}^{N_n} \delta_{X_i^n} \in \mathcal{P}(D_E[0,\infty))$$

$$\Xi = \lim_{m\to\infty} \frac{1}{m}\sum_{i=}^{m} \delta_{X_i}$$

$$V^n(t) = \frac{1}{N_n}\sum_{i=1}^{N_n} \delta_{X_i^n(t)} \in \mathcal{P}(E)$$

$$V(t) = \lim_{m\to\infty} \frac{1}{m}\sum_{i=1}^{m} \delta_{X_i(t)}$$

Then $V^n \Rightarrow V$ *in* $D_{\mathcal{P}(E)}[0,\infty)$ *or more precisely,*

$$(V^n, X_1^n, X_2^n, \ldots) \Rightarrow (V, X_1, X_2, \ldots)$$

in $D_{\mathcal{P}(E)\times E^\infty}[0,\infty)$*. If* $X^n \to X$ *in probability in* $D_{E^\infty}[0,\infty)$*, then* $V^n \to V$ *in* $D_{\mathcal{P}(E)}[0,\infty)$ *in probability.*

McKean-Vlasov

$$X_i^n(t) = X_i^n(0) + \int_0^t \sigma(X_i^n(s), V^n(s))dB_i(s) + \int_0^t b(X_i^n(s), V^n(s))ds + \int_0^t \alpha(X_i^n(s), V^n(s))dW(s)$$

where $V^n(t)$ is the normalized empirical measure $\frac{1}{n}\sum_{i=1}^n \delta_{X_i^n(t)}$.

Along any convergent subsequence, X^n converges to a solution of the infinite system

$$X_i(t) = X_i(0) + \int_0^t \sigma(X_i(s), V(s))dB_i(s) + \int_0^t b(X_i(s), V(s))ds + \int_0^t \alpha(X_i(s), V(s))dW(s)$$

where V is the $\mathcal{P}(\mathbb{R}^d)$-valued process given by $V(t) = \lim_{k\to\infty} \frac{1}{k}\sum_{i=1}^k \delta_{X_i(t)}$.

Derivation of SPDE

Applying Itô's formula

$$\begin{aligned}\varphi(X_i(t)) &= \varphi(X_i(0)) + \int_0^t \nabla\varphi(X_i(s))^T \sigma(X_i(s), V(s))dB_i(s) \\ &+ \int_0^t L(V(s))\varphi(X_i(s))ds + \int_0^t \nabla\varphi(X_i(s))^T \alpha(X_i(s), V(s))dW(s)\end{aligned}$$

where for $a(x,\nu) = \sigma(x,\nu)\sigma(x,\nu)^T + \alpha(x,\nu)\alpha(x,\nu)^T$

$$L(\nu)\varphi(x) = \frac{1}{2}\sum_{i,j} a_{ij}(x,\nu)\partial_i\partial_j\varphi(x) + b(x,\nu)\cdot\nabla\varphi(x).$$

Averaging gives

$$\begin{aligned}\langle V(t),\varphi\rangle = \langle V(0),\varphi\rangle &+ \int_0^t \langle V(s), L(V(s))\varphi(\cdot)\rangle ds \\ &+ \int_0^t \langle V(s), \nabla\varphi(\cdot)^T\alpha(\cdot, V(s))dW(s)\end{aligned}$$

Uniqueness

$$X_i(t) = X_i(0) + \int_0^t \sigma(X_i(s), V(s))dB_i(s) + \int_0^t b(X_i(s), V(s))ds + \int_0^t \alpha(X_i(s), V(s))dW(s) \quad (1)$$

Let $\rho(\mu_1, \mu_2) = \sup_{\{f:|f(x)-f(y)|\leq|x-y|\}} |\int_{\mathbb{R}^d} f d\mu_1 - \int_{\mathbb{R}^d} f d\mu_2|$.

ρ defines a metric on $\mathcal{P}_1(\mathbb{R}^d) = \{\mu \in \mathcal{P}(\mathbb{R}^d) : \int |x|\mu(dx) < \infty\}$.

If $\{X_i\}$ and $\{\widetilde{X}_i\}$ are solutions of (1), then

$$\rho(V(t), \widetilde{V}(t)) \leq \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^n |X_i(t) - \widetilde{X}_i(t)|$$

and if

$$|\sigma(x_1, \mu_1) - \sigma(x_2, \mu_2)| + |b(x_1, \cdot\mu_1) - b(x_2, \cdot, \mu_2)| + |\alpha(x_1, \mu_1) - \alpha(x_2, \mu_2)| \leq C(|x_1 - x_2| + \rho(\mu_1, \mu_2)),$$

the solution of the infinite system is unique.

Propagation of chaos

Theorem 3 *If $\{X_i\}$ satisfies a system of equations of the form*

$$X_i = F(X_i, V, U_i),$$

where the U_i are iid, V is the de Finetti measure for $\{X_i\}$, and if the solution of the system is strongly unique, then the X_i are independent.

Uniqueness of SPDE

Theorem 4 *Uniqueness for the particle system implies uniqueness for the SPDE.*

Weighted particle representations

Kurtz and Xiong (1999)

Here we assume each particle has a weight $A_i(t)$ so that the measure-valued state is given by

$$V(t) = \lim_{m\to\infty} \frac{1}{m} \sum_{i=1}^{m} A_i(t)\delta_{X_i(t)}$$

that is $\langle V(t), \varphi\rangle = \lim_{m\to\infty} \frac{1}{m} \sum_{i=1}^{m} A_i(t)\varphi(X_i(t)), \quad \varphi \in B(E)$.

The limit will exist provided $\{(X_i(t), A_i(t))\}$ is exchangeable and

$$E[|A_i(t)|] < \infty.$$

If $V(t, dx) = v(x, t)\pi(dx)$, then

$$\begin{aligned} \lim_{m\to\infty} \sum_{i=1}^{m} A_i(t)G(v(X_i(t), t))\varphi(X_i(t)) &= \langle V(t), G(v(\cdot, t))\varphi\rangle \\ &= \int v(x, t)G(v(x, t))\varphi(x)\pi(dx) \end{aligned}$$

Stochastic Allen-Cahn equation

Consider a family of SPDEs of the form

$$
\begin{aligned}
dv &= \Delta v dt + F(v)dt + \text{noise}, \\
v(0,x) &= h(x), \quad x \in D, \\
v(t,x) &= g(x), \quad x \in \partial D, t > 0,
\end{aligned}
$$

where $F(v) = G(v)v$ and G is bounded above. For example,

$$F(v) = v - v^3 = (1 - v^2)v.$$

To be specific, in weak form the equation is

$$
\begin{aligned}
\langle V(t), \varphi \rangle &= \langle V(0), \varphi \rangle + \int_0^t \langle V(s), \Delta\varphi \rangle ds + \int_0^t \langle V(s), \varphi G(v(s,\cdot)) \rangle ds \\
&\quad + \int_{\mathbb{U}\times[0,t]} \int_D \varphi(x)\rho(x,u) dx W(du \times ds),
\end{aligned}
$$

for $\varphi \in C_c^2(D)$.

cf. Bertini, Brassesco, and Buttà (2009)

Constructing a particle representation

Crisan, Janjigian, and Kurtz (2017)

Assume D is bounded and $\{X_i\}$ are independent, stationary, reflecting diffusions in D. To be specific, take the X_i to satisfy

$$X_i(t) = X_i(0) + \int_0^t \sigma(X_i(s))dB_i(s) + \int_0^t c(X_i(s))ds + \int_0^t \eta(X_i(s))dL_i(s), \tag{2}$$

where $\eta(x)$ is a vector field defined on the boundary ∂D and L_i is a local time on ∂D for X_i, that is, L_i is a nondecreasing process that increases only when X_i is in ∂D.

$$a(x) = \sigma(x)\sigma^T(x) \qquad \text{nondegenerate.}$$

Itô's formula

For $\varphi \in C_b^2(D)$, let

$$\mathbb{L}\varphi(x) = \frac{1}{2}\sum_{i,j} a_{ij}(x)\partial^2_{x_i x_j}\varphi(x) + \sum_i c_i(x)\partial_{x_i}\varphi(x), \tag{3}$$

Then

$$\varphi(X_i(t)) = \varphi(X_i(0)) + \int_0^t \nabla\varphi(X_i(s))\sigma(X_i(s))dB_i(s) + \int_0^t \mathbb{L}\varphi(X_i(s))ds + \int_0^t \nabla\varphi(X_i(s))\eta(X_i(s))dL_i(s)$$

In (3), $a(x) = \sigma(x)\sigma(x)^T$, where σ^T is the transpose of σ.

Particle weights

$$dA_i(t) = G(v(t, X_i(t)))A_i(t)dt + \int_{\mathbb{U}} \rho(X_i(t), u)W(du \times dt)$$

$A_i(0) = h(X_i(0))$

If X_i hits the boundary at time t, $A_i(t)$ is reset to $g(X_i(t))$.

For $V(t) = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} A_i(t)\delta_{X_i(t)}$,

$$\langle V(t), \varphi\rangle = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} A_i(t)\varphi(X_i(t))$$

we have

$$\langle V(t), \varphi\rangle = \int_D \varphi(x)v(t, x)\pi(dx)$$

where π is the stationary distribution for X_i (normalized Lebesgue measure on D for normally reflecting Brownian motion).

Particle representation

Let $\tau_i(t) = 0 \vee \sup\{s < t : X_i(s) \in \partial D$, and

$$
\begin{aligned}
A_i(t) \;=\; & g(X_i(\tau_i(t)))\mathbf{1}_{\{\tau_i(t)>0\}} + h(X_i(0))\mathbf{1}_{\{\tau_i(t)=0\}} \\
& + \int_{\tau_i(t)}^t G(v(s, X_i(s)), X_i(s))A_i(s)ds + \int_{\tau_i(t)}^t b(X_i(s))ds \\
& + \int_{\mathbb{U}\times(\tau_i(t),t]} \rho(X_i(s), u)W(du \times ds),
\end{aligned} \tag{4}
$$

where

$$
\langle V(t), \varphi\rangle = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^n \varphi(X_i(t))A_i(t) = \int \varphi(x)v(t,x)\pi(dx).
$$

Note that V will be absolutely continuous with respect to π.

Corresponding SPDE

For $\varphi \in C_c^2(D)$, define $M_{\varphi,i}(t) = \varphi(X_i(t)) - \int_0^t \mathbb{L}\varphi(X_i(s))ds$.

$$\begin{aligned}
\varphi(X_i(t))A_i(t) &= \varphi(X_i(0))A_i(0) + \int_0^t \varphi(X_i(s))dA_i(s) \\
&\quad + \int_0^t A_i(s)dM_{\varphi,i}(s) + \int_0^t \mathbb{L}\varphi(X_i(s))A_i(s)ds \\
&= \varphi(X_i(0))A_i(0) + \int_0^t \varphi(X_i(s))G(v(s,X_i(s)),X_i(s))A_i(s)ds \\
&\quad + \int_0^t \varphi(X_i(s))b(X_i(s))ds \\
&\quad + \int_{\mathbb{U}\times[0,t]} \varphi(X_i(s))\rho(X_i(s),u)W(du\times ds) \\
&\quad + \int_0^t A_i(s)dM_{\varphi,i}(s) + \int_0^t \mathbb{L}\varphi(X_i(s))A_i(s)ds
\end{aligned}$$

Averaging

$$\begin{aligned}\langle V(t),\varphi\rangle &= \langle V(0),\varphi\rangle + \int_0^t \langle V(s),\varphi G(v(s,\cdot),\cdot)\rangle ds + \int_0^t \int b\varphi d\pi ds \\ &\quad + \int_{\mathbb{U}\times[0,t]} \int_D \varphi(x)\rho(x,u)\pi(dx)W(du\times ds) + \int_0^t \langle V(s),\mathbb{L}\varphi\rangle ds\end{aligned}$$

which is the weak form of

$$\begin{aligned}v(t,x) = v(0,x) &+ \int_0^t (G(v(s,x),x)v(s,x)+b(x))ds \\ &+ \int_{\mathbb{U}\times[0,t]} \rho(x,u)W(du\times ds) + \int_0^t \mathbb{L}^* v(x,s)ds,\end{aligned}$$

where $\mathbb{L}^*$ is the adjoint determined by

$$\int g\mathbb{L}f d\pi = \int f\mathbb{L}^* g d\pi.$$

Boundary behavior

By the Riesz representation theorem that there exists a measure β on ∂D which satisfies

$$\varphi \mapsto \frac{1}{t}\mathbb{E}\left[\int_0^t \varphi(X_i(s))dL_i(s)\right] = \int_{\partial D} \varphi(x)\beta(dx). \tag{5}$$

For sufficiently regular space-time functions φ, we have

$$\int_0^t \int_{\partial D} \varphi(x,s)\beta(dx)ds = \mathbb{E}\left[\int_0^t \varphi(X_i(s),s)dL_i(s)\right]. \tag{6}$$

Denote partial derivatives with respect to time by ∂. Then

$$\int_0^t \int_D (\partial + \mathbb{L})\,\varphi(x,s)\pi(dx)ds \;=\; \int_0^t \int_{\partial D} \nabla\varphi(x,s)\cdot\eta(x)\beta(dx)ds.$$

Boundary value identity

Theorem 5 *Under mild regularity conditions, almost surely, for dL_i almost every t, $A_i(t) = A_i(t-) = g(X_i(t))$ and therefore*

$$\begin{aligned}
&\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}\int_0^t A_i(s-)\eta(X_i(s))\cdot\nabla\varphi(X_i(s),s)dL_i(s)\\
= \;& \mathbb{E}\left[\int_0^t A_i(s-)\eta(X_i(s))\cdot\nabla\varphi(X_i(s),s)dL_i(s)|\sigma(W)\right]\\
= \;& \int_0^t\int_{\partial D} g(x)\eta(x)\cdot\nabla\varphi(x,s)\beta(dx)ds.
\end{aligned}$$

SPDE for test functions in $C_0^2(D)$

$\varphi(x,s)$ twice continuously differentiable in x, continuously differentiable in s, and zero on $\partial D \times [0,\infty)$. Applying Itô's formula to $\varphi(X_i(s),s)$ and averaging,

$$\begin{aligned}\langle \varphi(\cdot,t), V(t)\rangle &= \langle \varphi(\cdot,0), V(0)\rangle + \int_0^t \langle \varphi(\cdot,s) G(v(s,\cdot),\cdot), V(s)\rangle ds \\ &\quad + \int_0^t \int_D \varphi(x,s) b(x) \pi(dx) ds \\ &\quad + \int_{\mathbb{U}\times[0,t]} \int_D \varphi(x,s)\rho(x,u)\pi(dx) W(du\times ds) \\ &\quad + \int_0^t \langle \mathbb{L}\varphi(\cdot,s) + \partial\varphi(\cdot,s), V(s)\rangle ds \\ &\quad + \int_0^t \int_{\partial D} g(x)\eta(x)\cdot\nabla\varphi(x,s)\beta(dx) ds,\end{aligned} \tag{7}$$

Linearized systems

Let ψ be an $L^1(\pi)$-valued stochastic process that is compatible with W, and assume (W,ψ) is independent of $\{X_i\}$. Define A_i^ψ to be the solution of

$$\begin{aligned} A_i^\psi(t) &= g(X_i(\tau_i(t)))\mathbf{1}_{\{\tau_i(t)>0\}} + h(X_i(0))\mathbf{1}_{\{\tau_i(t)=0\}} \\ &\quad + \int_{\tau_i(t)}^t G(\psi(s,X_i(s)),X_i(s))A_i^\psi(s)ds + \int_{\tau_i(t)}^t b(X_i(s))ds \\ &\quad + \int_{\mathbb{U}\times(\tau_i(t),t]} \rho(X_i(s),u)W(du\times ds). \end{aligned}$$

The $\{A_i^\psi\}$ will be exchangeable, so we can define $\Phi\psi(t,x)$ to be the density of the signed measure determined by

$$\langle \Phi\Psi(t),\varphi\rangle \equiv \int_D \varphi(x)\Phi\psi(t,x)\pi(dx) = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^n A_i^\psi(t)\varphi(X_i(t)).$$

Apriori bounds

Assume

$$
\begin{aligned}
K_1 &\equiv \sup_{x,D} |b(x)| < \infty \\
K_2 &\equiv \sup_{x\in D} \int \rho(x,u)^2\mu(du) < \infty \\
K_3 &\equiv \sup_{v\in\mathbb{R},x\in D} G(v,x) < \infty.
\end{aligned}
$$

Lemma 6 *Let*

$$H_i(t) = \int_{\mathbb{U}\times[0,t]} \rho(X_i(s),u)W(du\times ds) = B_i(\int_0^t \int_{\mathbb{U}} \rho(X_i(s),u)^2\mu(du)ds).$$

Then

$$
\begin{aligned}
|A_i^\psi(t)| &\leq (\|g\| \vee \|h\| + K_1(t-\tau_i(t)) + \sup_{\tau_i(t)\leq r\leq t} |H_i(t)-H_i(r)|)e^{K_3(t-\tau_i(t))} \\
&\leq (\|g\| \vee \|h\| + K_1 t + \sup_{0\leq s\leq t} |H_i(t)-H_i(s)|)e^{K_3 t} \equiv \Gamma_i(t).
\end{aligned}
$$

Weights and solution values

Lemma 7 *Suppose that* (W,ψ) *is independent of* $\{X_i\}$. *Then* $\Phi\psi$ *is* $\{\mathcal{F}_t^{W,\psi}\}$*-adapted and for each* i*,*

$$E[A_i^{\psi}(t)|W,\psi,X_i(t)] = \Phi\psi(t,X_i(t))$$

so

$$\Phi\psi(t,X_i(t)) \leq E[\Gamma_i(t)|W,\psi,X_i(t)]$$

Uniqueness

$L_1 \equiv \sup_{v,x\in D} \frac{|G(v,x)|}{1+|v|^2} < \infty.$

$L_2 \equiv \sup_{v_1,v_2,x\in D} \frac{|G(v_1,x)-G(v_2,x)|}{|v_1-v_2|(|v_1|+|v_2|)} < \infty.$

$$
\begin{aligned}
|A_i^{v_1}(t) - A_i^{v_2}(t)| &\leq \int_{\tau_i(t)}^t |G(v_1(s,X_i(s)),X_i(s))A_i^{v_1}(s) - G(v_2(s,X_i(s)),X_i(s))A_i^{v_2}(s)|ds \\
&\leq \int_{\tau_i(t)}^t L_1(1+E[\Gamma_i(s)|W,X_i(s)]^2)|A_i^{v_1}(s) - A_i^{v_2}(s)|ds \\
&\quad + \int_{\tau_i(t)}^t 2L_2 E[\Gamma_i(s)|W,X_i(s)]\Gamma_i(s)|v_1(s,X_i(s)) - v_2(s,X_i(s))|ds \\
&\leq \int_0^t L_1(1+C^2)|A_i^{v_1}(s) - A_i^{v_2}(s)|ds \\
&\quad + \int_0^t 2L_2C^2|v_1(s,X_i(s)) - v_2(s,X_i(s))|ds \\
&\quad + \int_0^t \mathbf{1}_{\{\Gamma_i(s)>C\}\cup\{E[\Gamma_i(s)|W,X_i(s)]>C\}}\Gamma_i(s)L_3(1+E[\Gamma_i(s)|W,X_i(s)]^2)ds
\end{aligned}
$$

Uniqueness for nonlinear SPDE

Theorem 8 *Uniqueness for the linear infinite system and the nonlinear infinite system and uniqueness for the linear SPDE*

$$\begin{aligned}\langle\varphi(\cdot,t),V^{\psi}(t)\rangle &= \langle\varphi(\cdot,0),V(0)\rangle+\int_0^t\langle\varphi(\cdot,s)G(\psi(s,\cdot),\cdot),V^{\psi}(s)\rangle ds \\ &\quad+\int_0^t\int_D\varphi(x,s)b(x)\pi(dx)ds \\ &\quad+\int_{\mathbb{U}\times[0,t]}\int_D\varphi(x,s)\rho(x,u)\pi(dx)W(du\times ds) \\ &\quad+\int_0^t\langle\mathbb{L}\varphi(\cdot,s)+\partial\varphi(\cdot,s),V^{\psi}(s)\rangle ds \\ &\quad+\int_0^t\int_{\partial D}g(x)\eta(x)\cdot\nabla\varphi(x,s)\beta(dx)ds,\end{aligned} \tag{8}$$

implies uniqueness for the nonlinear SPDE.

Proof. Suppose ψ is a solution of the nonlinear SPDE. Use ψ as the input into the linear infinite system. Uniqueness of the linear infinite system implies $\Phi\psi$ is a solution of the linear SPDE, but ψ is also a solution of the linear SPDE, so $\psi = \Phi\psi$ and uniqueness of the nonlinear infinite system implies there is only one such ψ. (See Section 3 of Kurtz and Xiong (1999).) □

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Thomas G. Kurtz and Jie Xiong. Particle representations for a class of nonlinear SPDEs. *Stochastic Process. Appl.*, 83(1):103–126, 1999. ISSN 0304-4149.

Abstract

Particle representations for stochastic partial differential equations

Stochastic partial differential equations arise naturally as limits of finite systems of weighted interacting particles. For a variety of purposes, it is useful to keep the particles in the limit obtaining an infinite exchangeable system of stochastic differential equations for the particle locations and weights. The corresponding de Finetti measure then gives the solution of the SPDE. These representations frequently simplify existence, uniqueness and convergence results. Beginning with the classical McKean-Vlasov limit, the basic results on exchangeable systems along with several examples will be discussed.