university of groningen

Clustering-based model reduction of networked passive systems

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Acknowledgments

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The analysis and design of complex systems relies
on the use of accurate predictive models

Challenges

- Dynamics dependent on subsystems and interconnection
- Large-scale interconnection complicates analysis and synthesis

Introduction

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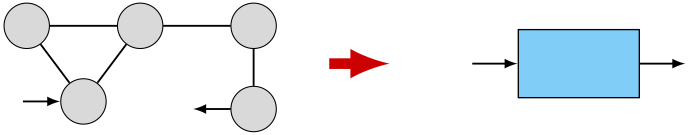
- Large-scale interconnection complicates analysis and synthesis

Goal: Model reduction of large-scale networked systems


Literature overview



- General methods, e.g., balancing, moment matching
  [Moore, Glover, Antoulas, Astolfi, ...]

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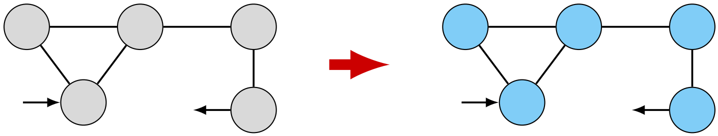

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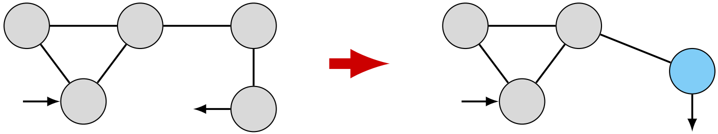

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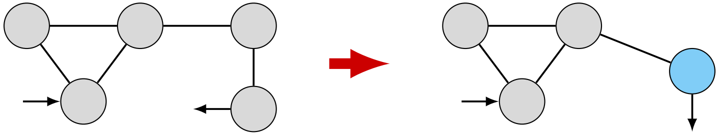

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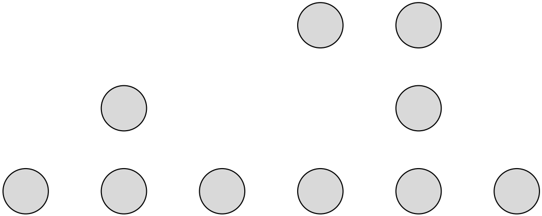
This presentation

- Subsystems with higher-order dynamics
- Controllability/observability-based cluster selection

Outline

- Problem setting
- Edge controllability and observability
- One-step clustering and multi-step clustering
- Example
- Conclusions & Future work

Problem setting



Networks of interconnected dynamical systems

1. Subsystem dynamics

$$\boldsymbol{\Sigma}_i : \; \dot{x}_i = Ax_i + Bv_i, \;\; z_i = Cx_i, \quad x_i \in \mathbb{R}^n, \, v_i, z_i \in \mathbb{R}^m$$

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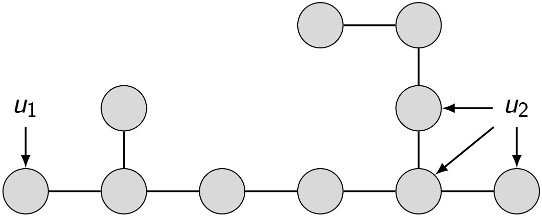

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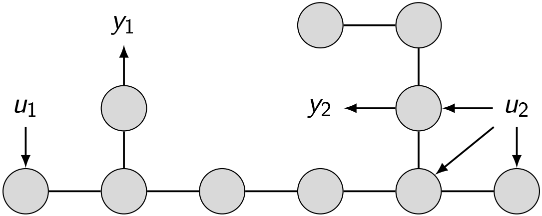
2. Interconnection topology with $w_{ij} \geq 0$

$$v_i = \textstyle\sum_{j=1, j\neq i}^{\bar{n}} w_{ij}(z_j - z_i) + \sum_{j=1}^{\bar{m}} g_{ij} u_j$$

Problem setting



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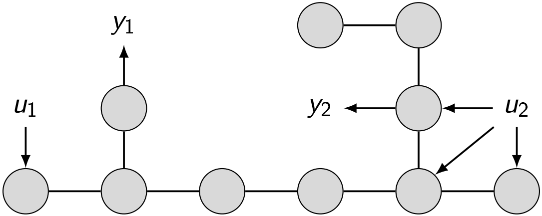
3. External outputs $y_i = \sum_{j=1}^{\bar{n}} h_{ij} z_j$

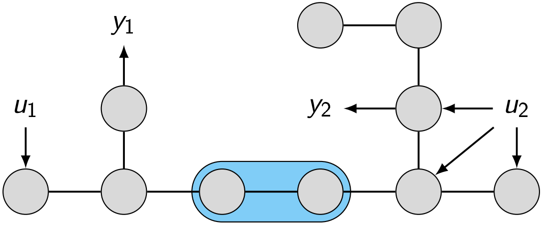
Approach



$$\Sigma : \begin{cases} \dot{x} = (I \otimes A - L \otimes BC)x + (G \otimes B)u \\ y = (H \otimes C)x \end{cases}$$

Goal. Approximate the input-output behavior of Σ by a clustering-based reduced-order system $\hat{\Sigma}$

Approach



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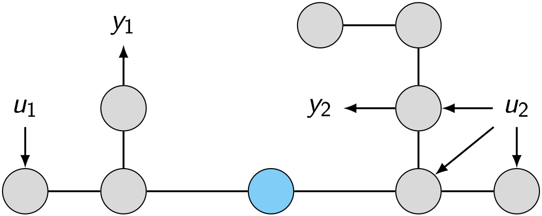

$$\boldsymbol{\Sigma} : \begin{cases} \dot{x} = (I \otimes A - L \otimes BC)x + (G \otimes B)u \\ y = (H \otimes C)x \end{cases}$$

Goal. Approximate the input-output behavior of $\boldsymbol{\Sigma}$ by a clustering-based reduced-order system $\hat{\boldsymbol{\Sigma}}$

$$\hat{\boldsymbol{\Sigma}} : \begin{cases} \dot{\xi} = (I \otimes A - \hat{L} \otimes BC)\xi + (\hat{G} \otimes B)u \\ \hat{y} = (\hat{H} \otimes C)\xi \end{cases}$$

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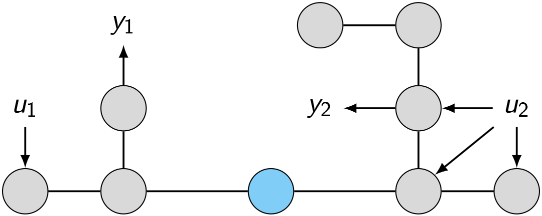

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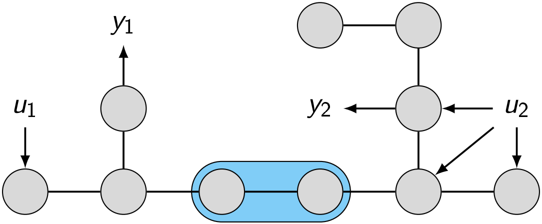
Objectives

1. Preservation of synchronization
2. A priori bound on the reduction error

Approach



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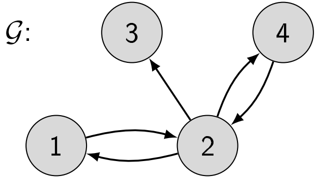
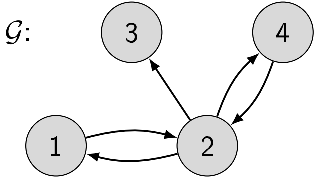
Approach. Find neighboring subsystems that are

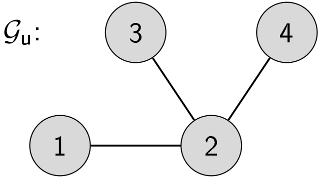
- hard to steer individually from the inputs
- hard to distinguish from the outputs



Laplacian matrix of $\mathcal{G}$

$$L = \begin{bmatrix} w_{12} & -w_{12} & 0 & 0 \\ -w_{21} & w_{21} + w_{24} & 0 & -w_{24} \\ 0 & -w_{32} & w_{32} & 0 \\ 0 & -w_{42} & 0 & w_{42} \end{bmatrix}, \qquad L\mathbb{1} = 0$$





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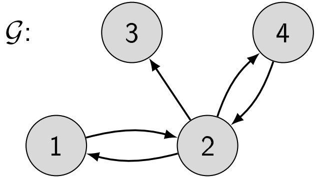
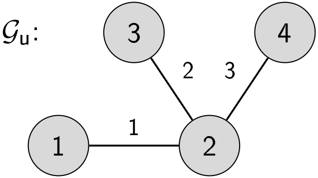
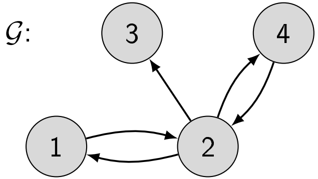
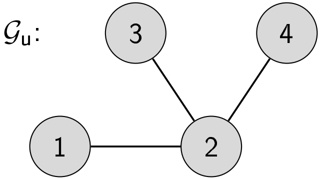




Incidence matrix of $\mathcal{G}_{\mathsf{u}}$ (for a given orientation)

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





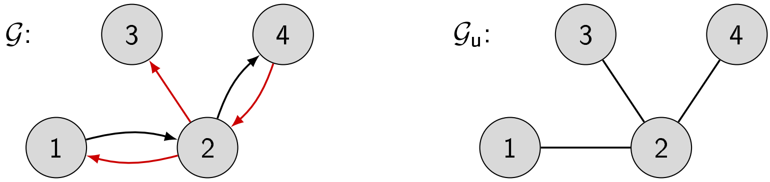
Lemma. Consider L and let E be an oriented incidence matrix of the underlying undirected graph. Then,

$$L = FE^{\mathsf{T}}$$

where F has the same structure as E, i.e.,

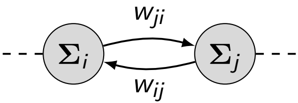
$$E = \begin{bmatrix} * & e_i - e_j & * \end{bmatrix}, \quad F = \begin{bmatrix} * & w_{ij}e_i - w_{ji}e_j & * \end{bmatrix}$$

Interconnection topology and Laplacian



Assumption A1. The graph $\mathcal{G}$ with graph Laplacian L is such that

a. The underlying undirected graph is a tree
b. $\mathcal{G}$ contains a directed rooted spanning tree

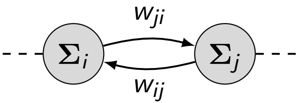


Lemma. Under **A1**, the edge Laplacian

$$L_e = E^\top F$$

has all eigenvalues in the open right-half complex plane

Edge Laplacian and edge dynamics



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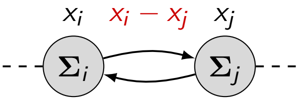

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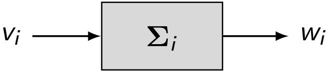
Edge system in coordinates $x_e = (E^{\top} \otimes I)x$

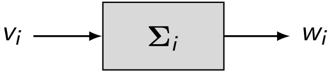
$$\Sigma_e : \quad \dot{x}_e = (I \otimes A - L_e \otimes BC)x_e + (E^{\top} G \otimes B)u, \quad y_e = (H_e \otimes C)x_e$$

Passivity and synchronization



Passivity [Willems]. A system Σ_i is passive if there exists a differentiable $V : \mathbb{R}^n \to \mathbb{R}$, $V(0) = 0$, $V \geq 0$ such that

$$\dot{V}(x_i) \leq v_i^{\mathsf{T}} w_i$$



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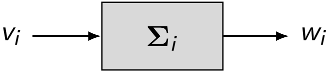
Assumption A2. The systems Σ_i are passive and $V(x_i) = \frac{1}{2} x_i^\top Q x_i$



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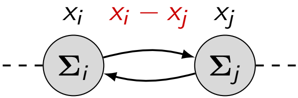
Theorem. Under **A1** and **A2**, the subsystems of Σ synchronize for $u = 0$, i.e.,

$$\lim_{t \to \infty} \left(x_i(t) - x_j(t) \right) = 0.$$

Equivalently, Σ_e is asymptotically stable

Edge controllability



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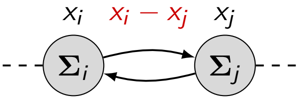

$\boldsymbol{\Sigma}_i$ and $\boldsymbol{\Sigma}_j$ are hard to steer individually ⟷ weakly controllable coordinate in $\boldsymbol{\Sigma}_\mathrm{e}$

Edge controllability gramian P_e characterizes controllability

$$x_\mathrm{e}^\mathsf{T} P_\mathrm{e}^{-1} x_\mathrm{e} = \inf \left\{ \int_{-\infty}^{0} |u(t)|^2 \, \mathrm{d}t \;\middle|\; u \in \mathcal{L}_2^m((-\infty, 0]) \text{ s.t. } 0 \rightsquigarrow x_\mathrm{e} \right\}$$

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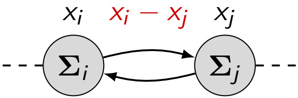

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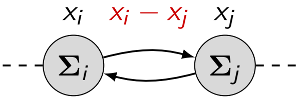
Challenges

- P_e dependent on subsystems and interconnection topology
- Role of individual edges not apparent from P_e

Edge controllability



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Theorem. The edge controllability Gramian P_e can be bounded as

$$P_\mathrm{e} \preccurlyeq \Pi^\mathrm{c} \otimes Q^{-1}$$

if there exists $\Pi^\mathrm{c} = \mathrm{diag}\{\pi_1^\mathrm{c}, \ldots, \pi_{\bar{n}-1}^\mathrm{c}\} \succcurlyeq 0$ such that

$$L_\mathrm{e}\Pi^\mathrm{c} + \Pi^\mathrm{c} L_\mathrm{e}^\mathsf{T} - E^\mathsf{T} G G^\mathsf{T} E \succcurlyeq 0$$

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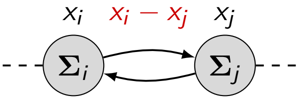

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Lemma. $\Pi^\mathrm{c} \succcurlyeq 0$ exists if $w_{ij} > 0 \Leftrightarrow w_{ji} > 0$

Properties

$$\Pi^c = \text{diag}\{\pi^c_1, \dots, \pi^c_{n-1}\}, \quad L_e \Pi^c + \Pi^c L_e^\top - E^\top G G^\top E \succcurlyeq 0$$

Properties

- Gramian can be defined as Σ_e is asymptotically stable
- Π^c only dependent on interconnection properties
- Measure of controllability for each individual edge

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Edge observability follows similarly, i.e.,

$$\Pi^o = \text{diag}\{\pi_1^o, \ldots, \pi_{n-1}^o\}, \quad L_e^\top \Pi^o + \Pi^o L_e - F^\top H^\top H F \succcurlyeq 0$$

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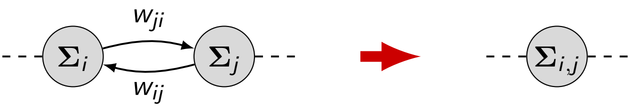
Assume ordering

$$(L_e^{-1})_{ii}^2 \pi_i^c \pi_i^o \geq (L_e^{-1})_{i+1,i+1}^2 \pi_{i+1}^c \pi_{i+1}^o \geq 0, \quad i \in \{1, \ldots, \bar{n}_e - 1\}$$

One-step clustering



$$V = \begin{bmatrix} I & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad W = \begin{bmatrix} I & 0 \\ 0 & \frac{w_{ji}}{w_{ij}+w_{ji}} \\ 0 & \frac{w_{ij}}{w_{ij}+w_{ji}} \end{bmatrix}$$

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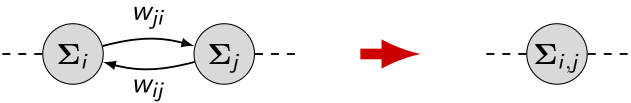

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Reduced-order system through projection with $(V \otimes I)(W \otimes I)^\mathsf{T}$

$$\hat{\Sigma}_{\bar{n}-1} : \ \dot{\xi} = (I \otimes A - \hat{L} \otimes BC)\xi + (\hat{G} \otimes B)u, \ \ \hat{y} = (\hat{H} \otimes C)\xi$$

$$\text{with} \quad \hat{L} = W^\mathsf{T} L V, \quad \hat{G} = W^\mathsf{T} G, \quad \hat{H} = HV$$

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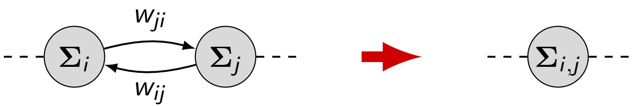

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$$\hat{\Sigma}_{\bar{n}-1} : \ \dot{\xi} = (I \otimes A - \hat{L} \otimes BC)\xi + (\hat{G} \otimes B)u, \ \ \hat{y} = (\hat{H} \otimes C)\xi$$

$$\text{with} \quad \hat{L} = W^\mathsf{T} L V, \quad \hat{G} = W^\mathsf{T} G, \quad \hat{H} = HV$$

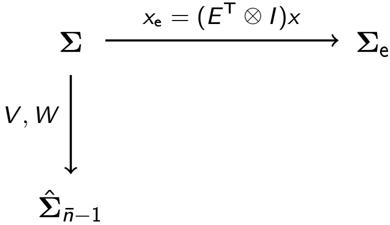
Lemma. Consider the reduced-order system $\hat{\Sigma}_{\bar{n}-1}$. Then,

1. $\hat{L} = \hat{F}\hat{E}^\mathsf{T}$ with $\hat{E}$ an oriented incidence matrix
2. Assumptions **A1** and **A2** hold for $\hat{\Sigma}_{\bar{n}-1}$

Multi-step clustering

$$\Sigma \xrightarrow{\quad x_e = (E^\top \otimes I)x \quad} \Sigma_e$$

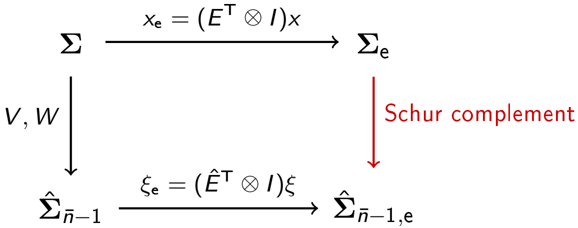
Multi-step clustering



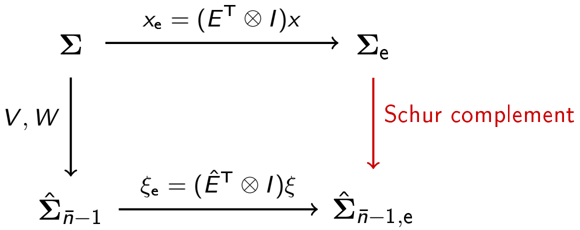
Multi-step clustering

$$
\begin{array}{ccc}
\boldsymbol{\Sigma} & \xrightarrow{\;x_{\mathrm{e}} = (E^{\mathsf{T}} \otimes I)x\;} & \boldsymbol{\Sigma}_{\mathrm{e}} \\
\Big\downarrow {\scriptstyle V,\,W} & & \\
\hat{\boldsymbol{\Sigma}}_{\bar{n}-1} & \xrightarrow{\;\xi_{\mathrm{e}} = (\hat{E}^{\mathsf{T}} \otimes I)\xi\;} & \hat{\boldsymbol{\Sigma}}_{\bar{n}-1,\mathrm{e}}
\end{array}
$$

Multi-step clustering

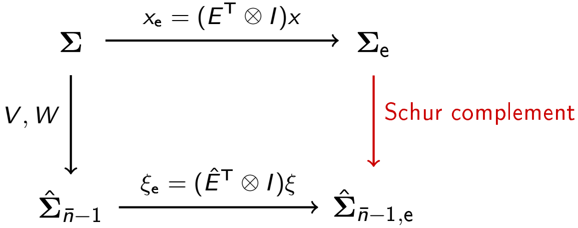


Multi-step clustering



Theorem. Consider Σ and the one-step clustered $\hat{\Sigma}_{\bar{n}-1}$. Then,

1. The edge controllability Gramian of $\hat{\Sigma}_{\bar{n}-1}$ satisfies
$$\hat{P}_e \preccurlyeq \hat{\Pi}^c \otimes Q^{-1}, \quad \hat{\Pi}^c = \mathrm{diag}\{\pi_1^c, \ldots, \pi_{\bar{n}-2}^c\}$$
2. The edge observability Gramian of $\hat{\Sigma}_{\bar{n}-1}$ satisfies
$$\hat{Q}_e \preccurlyeq \hat{\Pi}^o \otimes Q, \quad \hat{\Pi}^o = \mathrm{diag}\{\pi_1^o, \ldots, \pi_{\bar{n}-2}^o\}$$



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Allows for repeated one-step clusterings

Theorem. The subsystems of $\hat{\boldsymbol{\Sigma}}_{\bar{k}}$ synchronize for $u = 0$, i.e.,

$$\lim_{t \to \infty} \big(\xi_i(t) - \xi_j(t)\big) = 0, \qquad (i,j) \in \hat{\mathcal{V}} \times \hat{\mathcal{V}}$$

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Theorem. For trajectories $x(\cdot)$ of $\boldsymbol{\Sigma}$ and $\xi(\cdot)$ of $\hat{\boldsymbol{\Sigma}}_{\bar{k}}$ for the same input $u(\cdot)$ and $x(0) = 0$, $\xi(0) = 0$, the output error is bounded as

$$\|y - \hat{y}\|_2 \leq 2 \left(\sum_{l=\bar{k}}^{\bar{n}-1} \left(L_{\mathrm{e}}^{-1}\right)_{ll} \sqrt{\pi_l^{\mathrm{c}} \pi_l^{\mathrm{o}}} \right) \|u\|_2$$

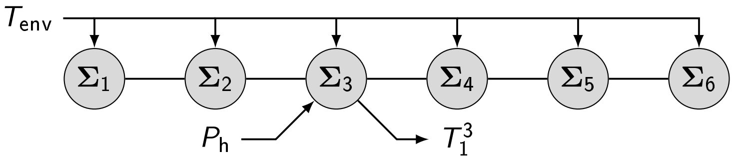
with $\|\cdot\|_2$ the $\mathcal{L}_2$ signal norm

Example

Thermal model of a corridor of six rooms

- Subsystems: thermal dynamics within a room

$$C_1 \dot{T}_1^i = R_{\text{int}}^{-1}(T_2^i - T_1^i) - R_{\text{out}}^{-1} T_1^i + P_i$$
$$C_2 \dot{T}_2^i = R_{\text{int}}^{-1}(T_1^i - T_2^i)$$



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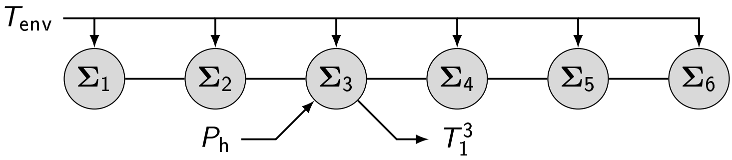
- Edges: thermal resistances of walls, $u_j = [\, P_{\text{h}} \;\; T_{\text{env}} \,]^{\mathsf{T}}$

$$P_i = \sum_{j=1, j\neq i}^{\bar{n}} R_{\text{wall}}^{-1}(T_1^j - T_1^i) + \sum_{j=1}^{\bar{m}} g_{ij} u_j$$



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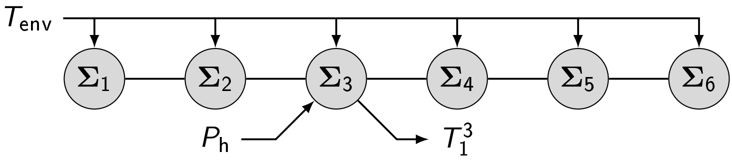
- Reduction from $\bar{n} = 6$ to $\bar{k} = 3$



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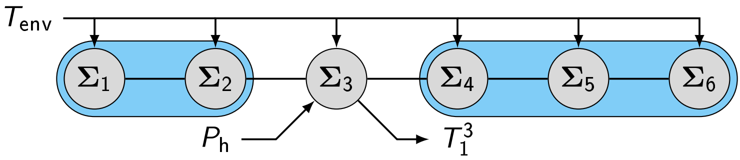

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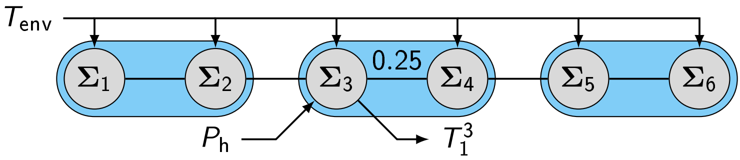
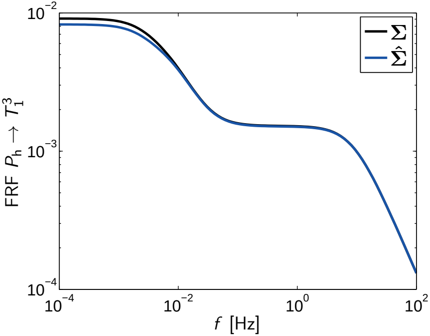


Example: Results

Frequency response function from input P_{h} to output T_1^3



Error bound: $2\sum_{l=3}^{5}\left(L_{\mathrm{e}}^{-1}\right)_{ll}\sqrt{\pi_l^{\mathrm{c}}\pi_l^{\mathrm{o}}} = 11.4 \cdot 10^{-3}$

Conclusions & Future work

Conclusions

- Clustering-based reduction procedure
- Edge controllability and observability properties
- Preservation of synchronization and error bound

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- B. Besselink, H. Sandberg, K.H. Johansson, J.-I. Imura. Controllability of a class of networked passive linear systems. In *Proceedings of the 52nd IEEE Conference on Decision and Control, Florence, Italy*, 4901–4906, 2013.
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Potential approach: exploit theory of monotone systems

Groningen Autumn School on MOR

Invited speakers

- Serkan Gugercin (Virginia Tech)
- Paolo Rapisarda (University of Southampton)

Topics. Model reduction for design and optimization, data-based model reduction, and model reduction of networks

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