SimTech
Cluster of Excellence

LMS EPSRC Symposium
Durham 2017

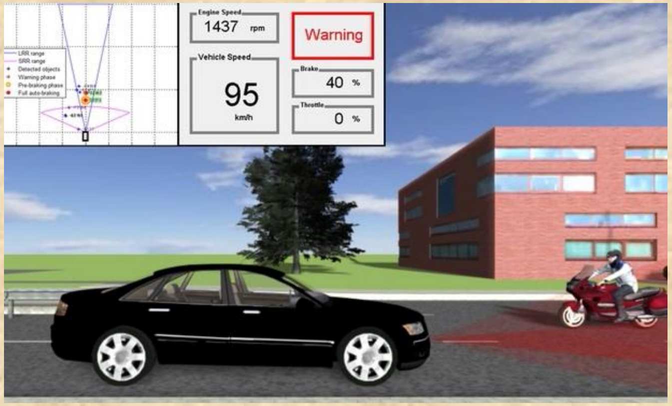
Error Estimation for the Simulation of Elastic Multibody Systems

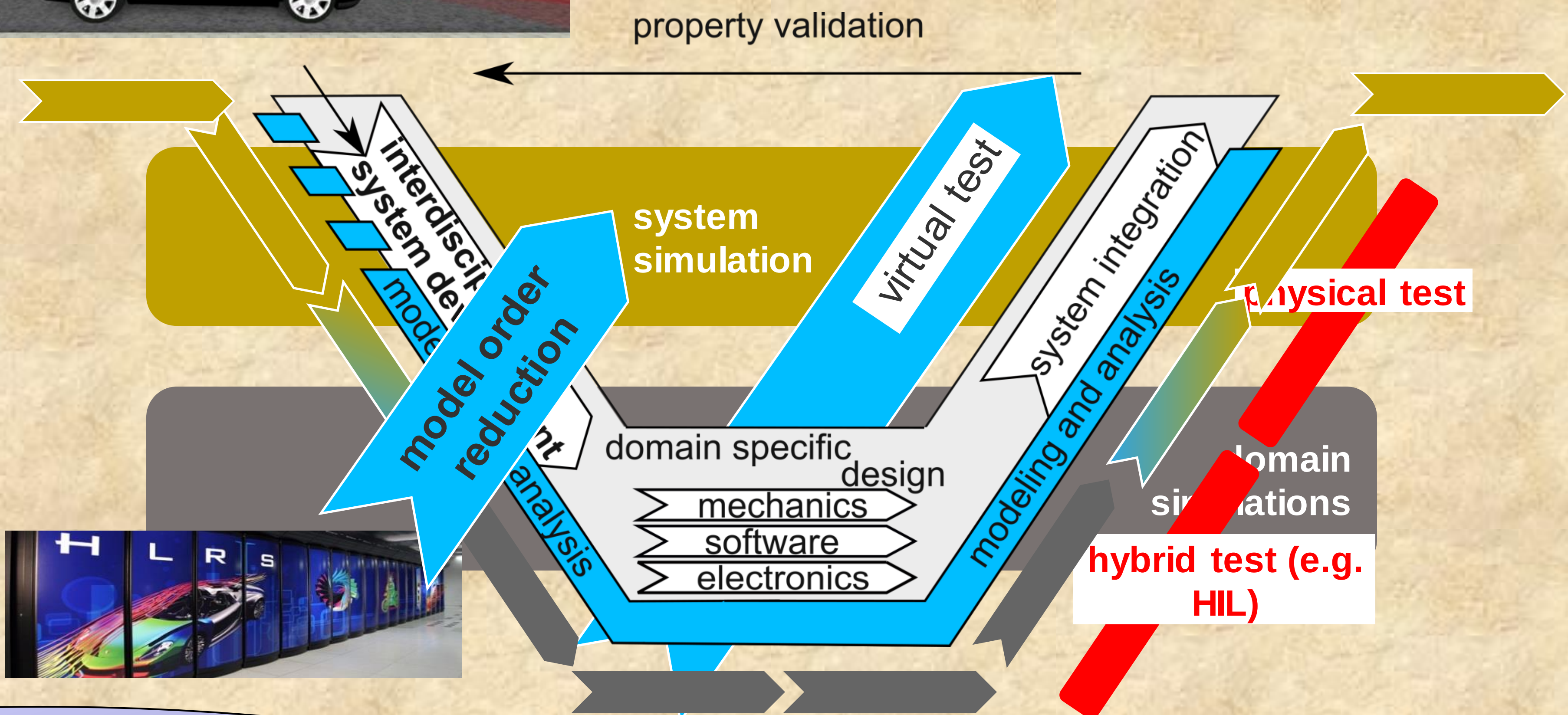
Jörg Fehr, Dennis Grunert,
Ashish Bhatt, Bernard Haasdonk



Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

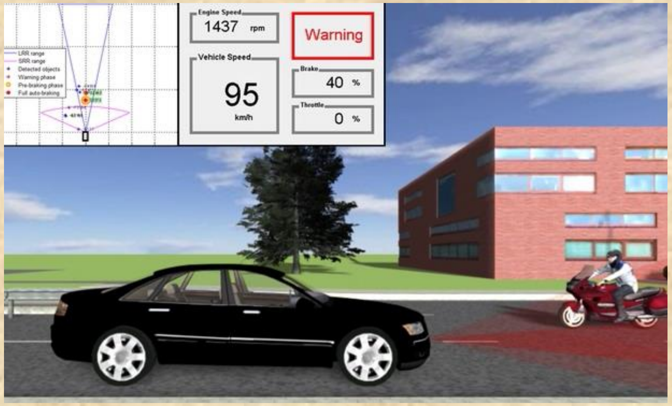
Advanced System Development Process



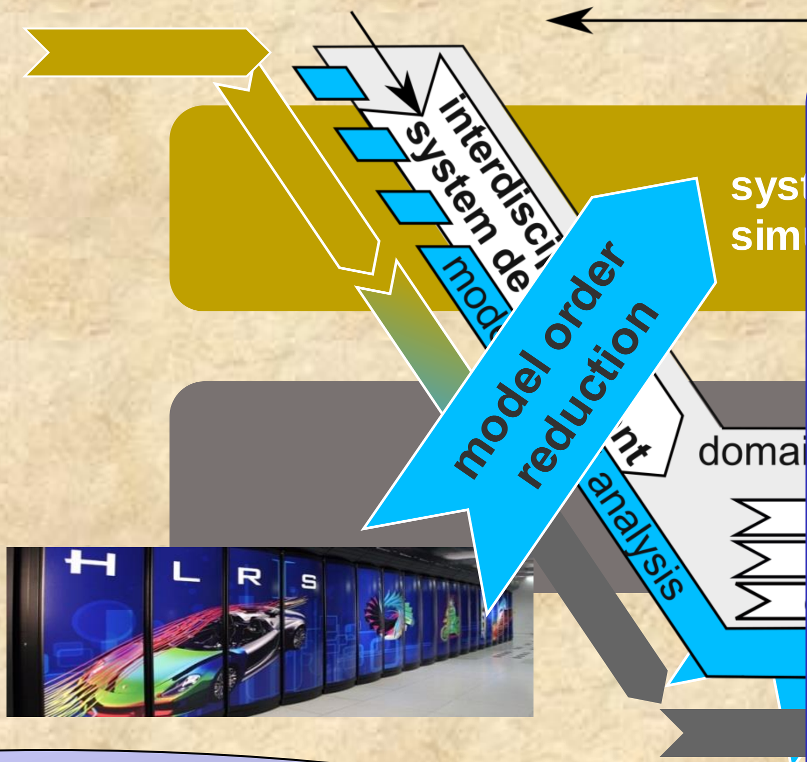


Institute of Engineering and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

Advanced System Development Process



property validation



error by reduction?

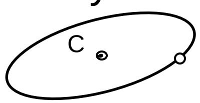
- without error quantification simulation cannot be trusted and potentially gives wrong results
- error known
  - simulation results are certified
  - robust simulation process
  - added value for the decision process

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

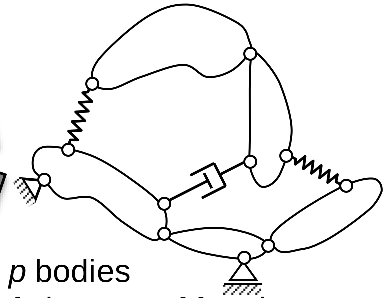
Elastic Multibody Systems

multibody system

rigid body



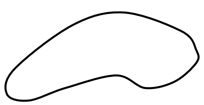
bearings and coupling elements



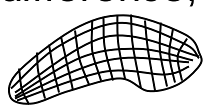
p bodies
f degrees of freedom
q reaction force

elastic body

continuum

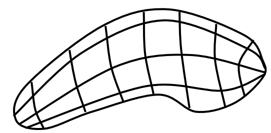


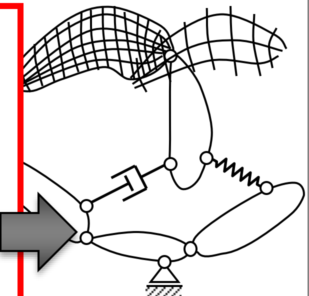
discretization
finite element,
finite difference,
...



elastic multibody system

reduction of the elastic degrees of freedom





- models are getting larger and more detailed
  - many degrees of freedom
  - FE-models have to be reduced
- with the floating frame of reference formulation linear model order reduction is possible

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

EMBS: The Floating Frame of Reference Approach

floating frame of reference
dividing the motion into

- nonlinear motion of reference frame K_i
- linear elastic deformation with respect to K_i

$$\mathbf{r}_k(t) = \mathbf{r}_i(t) + \mathbf{R}_{ik} + \mathbf{u}_k(t)$$

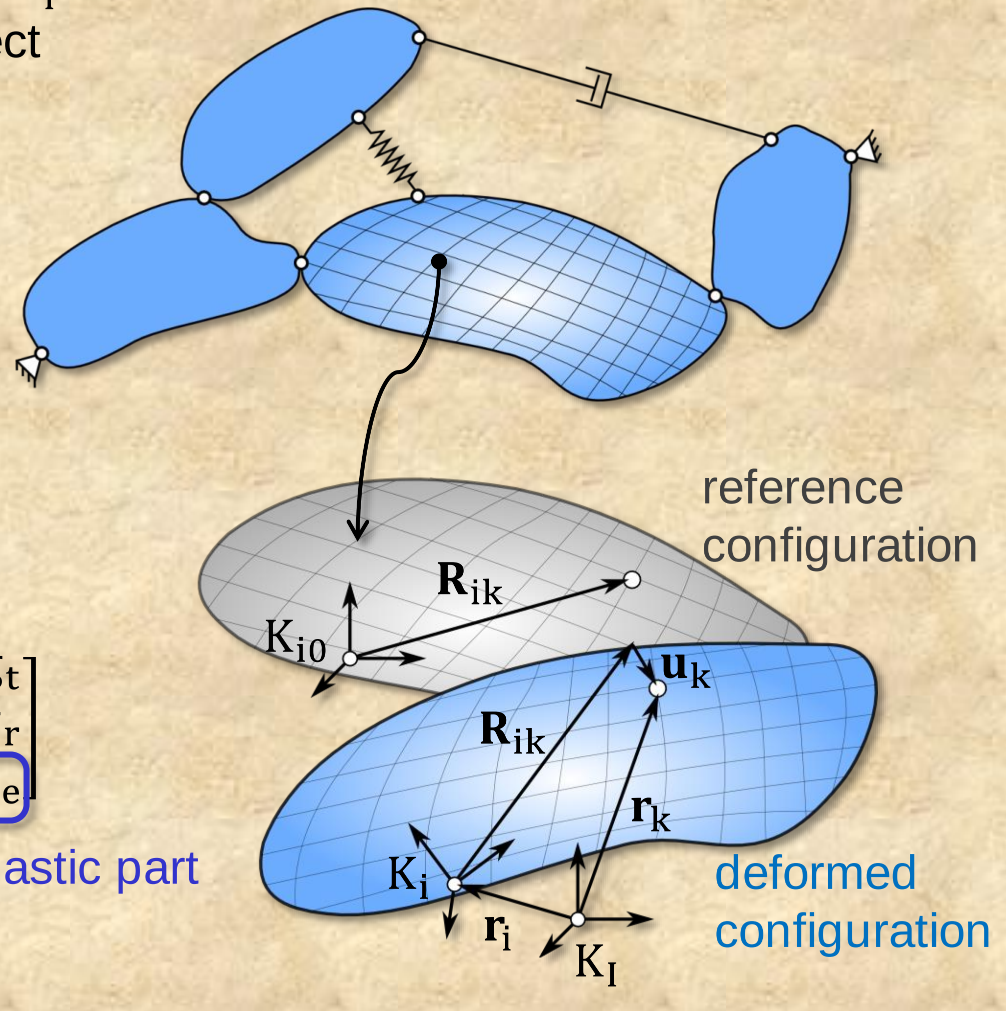
equation of motion of the elastic body
nonlinear equation describes the dynamics of the elastic body

$$\mathbf{M}(\mathbf{q}) \cdot \ddot{\mathbf{q}} + \mathbf{k}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t)$$

$$\begin{bmatrix} m\mathbf{I} & & \text{sym.} \\ m\tilde{\mathbf{c}}(q) & \mathbf{J}(q) & \\ \mathbf{C}_t(q) & \mathbf{C}_r(q) & \mathbf{M}_e \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{q}}_t \\ \ddot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_e \end{bmatrix} + \begin{bmatrix} \mathbf{k}_t \\ \mathbf{k}_r \\ \mathbf{k}_e \end{bmatrix} = \begin{bmatrix} \mathbf{g}_t \\ \mathbf{g}_r \\ \mathbf{g}_e \end{bmatrix}$$

coupling to reference frame motion

linear elastic part



Model Reduction by Projection

finite element model

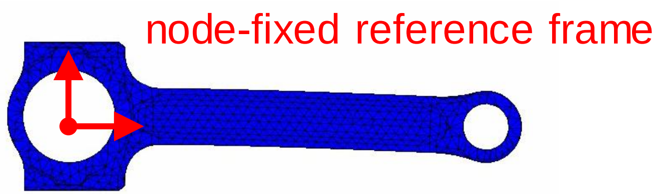
$$\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e + \mathbf{K}_e \cdot \mathbf{q}_e = \mathbf{h}_e$$

I/O aspect of forces and moments

- define input or control matrix $\mathbf{B}_e$
- define output/observation matrix $\mathbf{C}_e$
- consider EMBS specifica
  - boundary conditions of ref. frame
  - inertia terms coupling forces

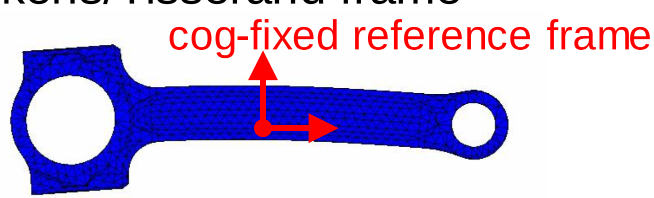
node-fixed

- tangent frame, chord frame



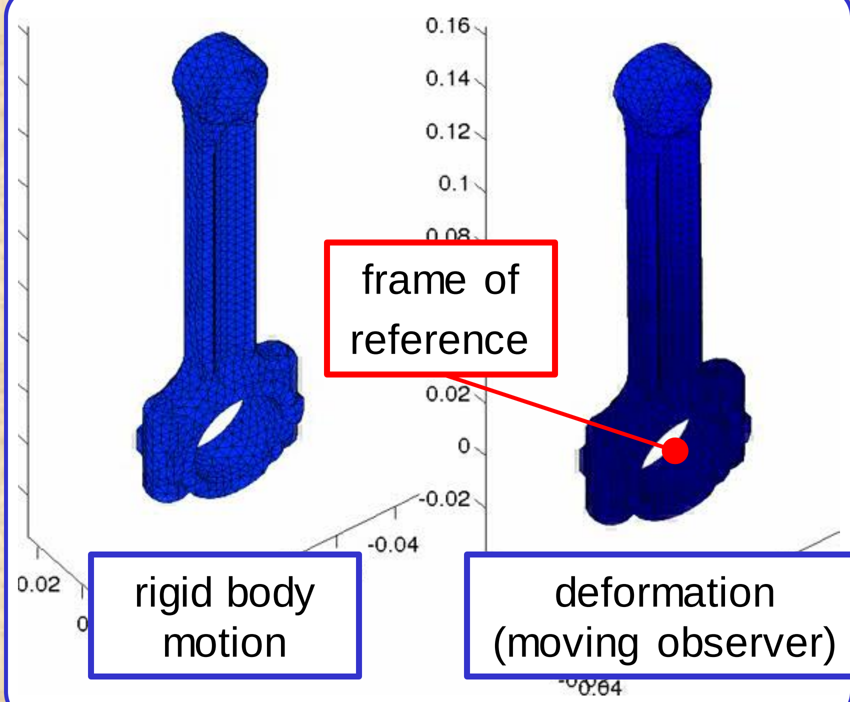
fixed to the center of gravity

- Buckens/Tisserand frame



inertia terms introduce coupling forces

- acceleration of reference frame K_i leads to elastic deformation



$$\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e + \mathbf{K}_e \cdot \mathbf{q}_e = \mathbf{B}_e \cdot \mathbf{u}_e$$
$$\mathbf{y} = \mathbf{C}_e \cdot \mathbf{q}_e$$

Model Reduction by Projection

finite element model

$$\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e + \mathbf{K}_e \cdot \mathbf{q}_e = \mathbf{h}_e$$

I/O aspect of forces and moments

- define input or control matrix $\mathbf{B}_e$
- define output/observation matrix $\mathbf{C}_e$
- consider EMBS specifica
  - boundary conditions of ref. frame
  - inertia terms coupling forces

$$\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e + \mathbf{K}_e \cdot \mathbf{q}_e = \mathbf{B}_e \cdot \mathbf{u}_e$$

$$\mathbf{y} = \mathbf{C}_e \cdot \mathbf{q}_e$$

linear model order reduction

- reduced FE equation of motion with $\dim(\bar{\mathbf{q}}_e) \ll \dim(\mathbf{q}_e)$, $\mathbf{q}_e \approx \mathbf{V} \cdot \bar{\mathbf{q}}_e$

$$\bar{\mathbf{M}}_e \cdot \ddot{\bar{\mathbf{q}}}_e + \bar{\mathbf{D}}_e \cdot \dot{\bar{\mathbf{q}}}_e + \bar{\mathbf{K}}_e \cdot \bar{\mathbf{q}}_e = \bar{\mathbf{h}}_e$$

$$\bar{\mathbf{M}}_e = \mathbf{V}^T \cdot \mathbf{M}_e \cdot \mathbf{V}, \ldots$$

$$\bar{\mathbf{h}}_e = \mathbf{V}^T \cdot \mathbf{B}_e \cdot \mathbf{u}_e$$

projection matrix $\mathbf{V} \in \mathbb{R}^{N \times n}$

reduction algorithms

- modal truncation
- CMS methods
- input-output based methods: Krylov, Balanced Truncation
  - focus on transfer behavior of the system
  - 'local' properties

- use linear projection space in nonlinear FFR formulation

$$\bar{\mathbf{M}}(\mathbf{q}) \cdot \ddot{\mathbf{q}} + \bar{\mathbf{k}}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t)$$

$$\begin{bmatrix} m\mathbf{I} & & \text{sym.} \\ m\tilde{\mathbf{c}}(\bar{\mathbf{q}}) & \mathbf{J}(\bar{\mathbf{q}}) & \\ \bar{\mathbf{C}}_t(\bar{\mathbf{q}}) & \bar{\mathbf{C}}_r(\bar{\mathbf{q}}) & \bar{\mathbf{M}}_e \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{q}}_t \\ \ddot{\mathbf{q}}_r \\ \ddot{\bar{\mathbf{q}}}_e \end{bmatrix} + \begin{bmatrix} \mathbf{k}_t \\ \mathbf{k}_r \\ \bar{\mathbf{K}} \cdot \bar{\mathbf{q}}_e + \bar{\mathbf{D}} \cdot \dot{\bar{\mathbf{q}}}_e \end{bmatrix} = \begin{bmatrix} \mathbf{g}_t \\ \mathbf{g}_r \\ \mathbf{g}_e \end{bmatrix}$$

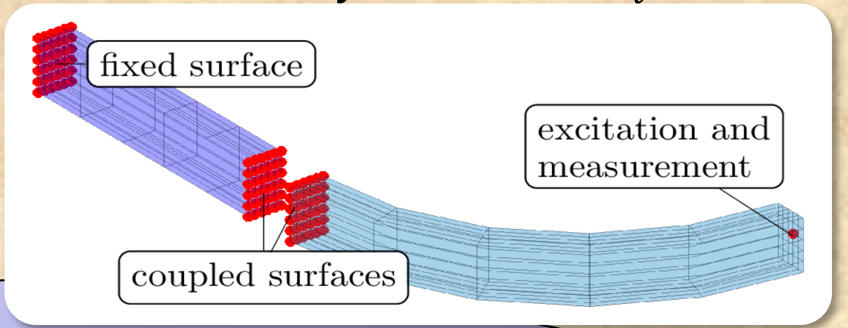
Model Hierarchies

- H1: single linear FE body expressed as a linear ODE system
  $\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e + \mathbf{K}_e \cdot \mathbf{q}_e = \mathbf{h}_e$
- H2: single elastic body in the FFR formulation expressed as a nonlinear ODE system

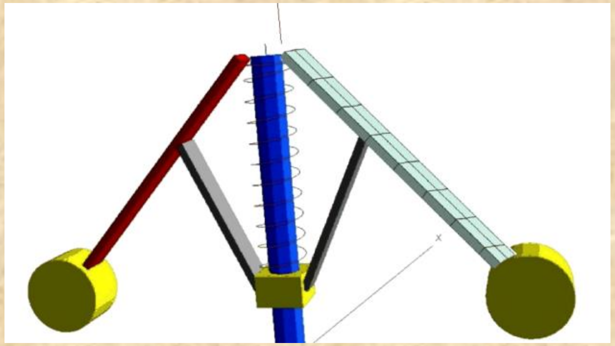
$$\begin{bmatrix} \mathbf{M}_r & \mathbf{M}_{er}^T \\ \mathbf{M}_{er} & \mathbf{M}_e \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_e \end{bmatrix} + \begin{bmatrix} \mathbf{k}_r \\ \mathbf{k}_e \end{bmatrix} = \begin{bmatrix} \mathbf{g}_r \\ \mathbf{g}_e \end{bmatrix}$$

linear elastic part

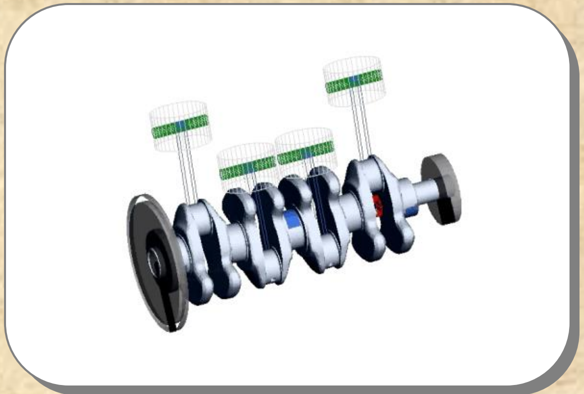
- H3: Multiple FE bodies
  linear ODE systems with N_i DOF



- H4: multiple rigid and elastic bodies are nonlinearly coupled with each other in the FFR formulation resulting in an EMBS



- H5: EMBS simulates mechanical part of a multiphysics environment

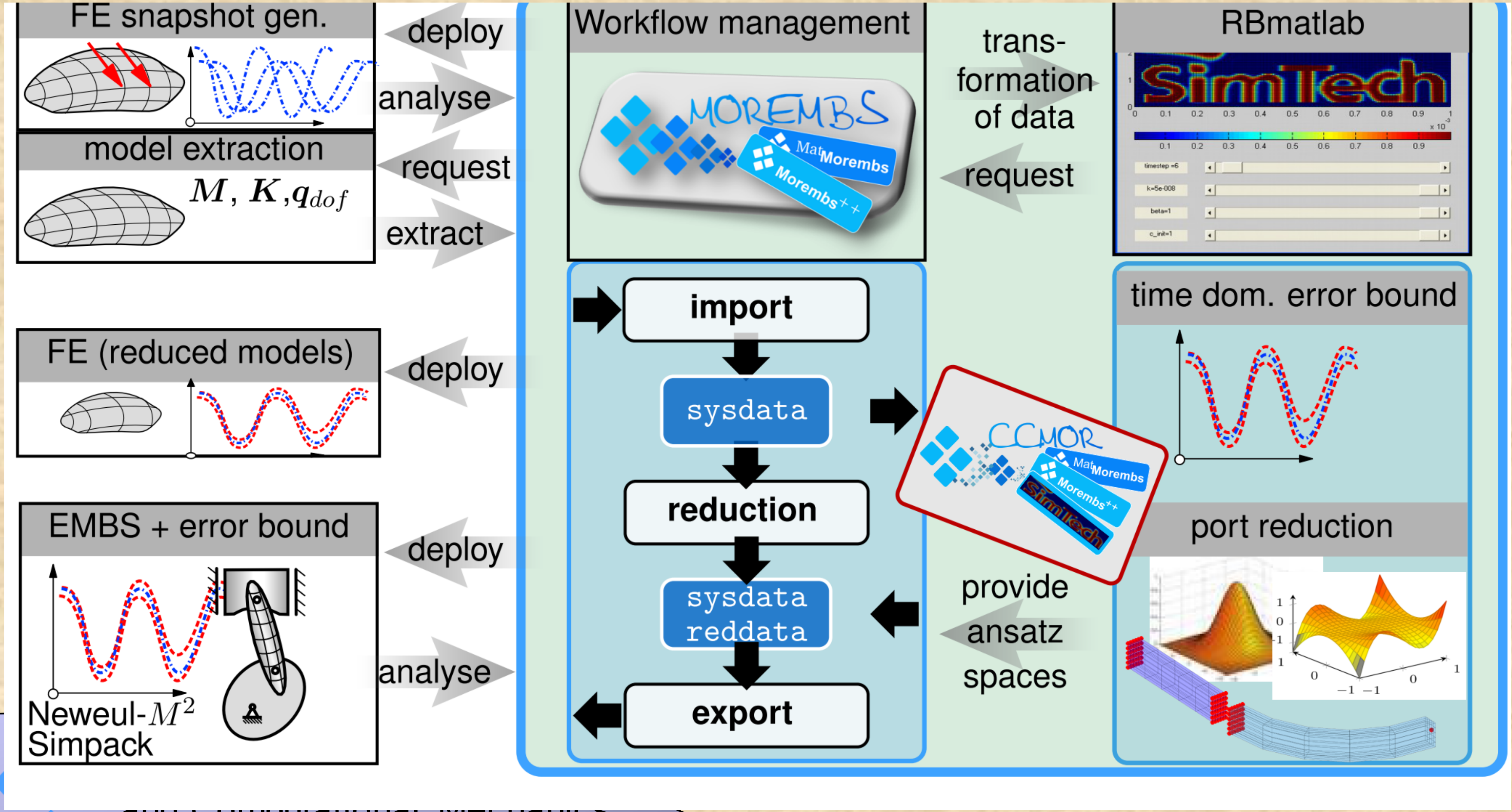


Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

usage of commercial software

Workflow for Engineers

- $\{\mathbf{M}_e, \mathbf{D}_e, \mathbf{K}_e\}$ e.g. from Ansys
- implementation of error estimator in third party code



Workflow for Engineers

- automated workflow
- standard FE programs
  - to describe elasticity

model reduction → multibody dynamics

error bound

preprocessing

simulation

- MOR process in Morembs
  - workhorse for {linear, parametric} model reduction at ITM [FehrEtAl17]

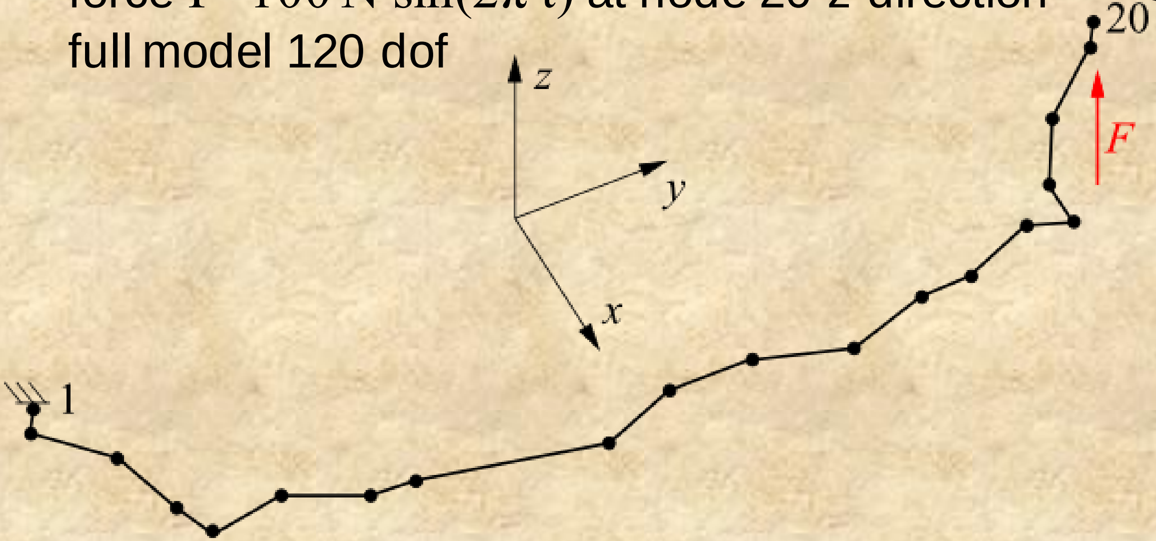
- in-house EMBS cods
- combines the benefits of numerical computation (Matlab) and computer algebra (Maple/MuPAD)
- equation of motion derived in symobolic form

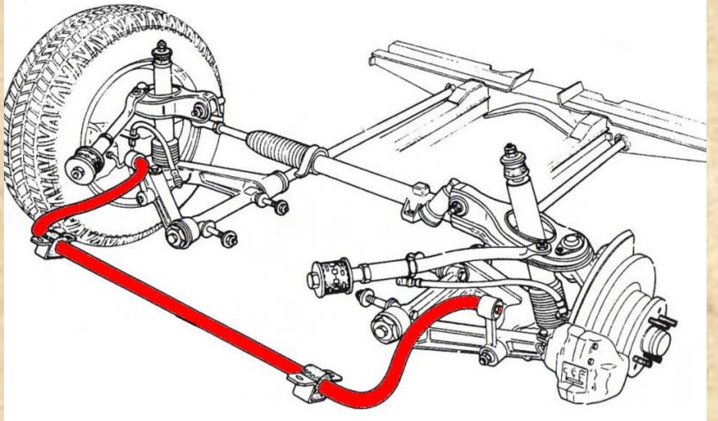
$$\begin{bmatrix} \mathbf{M}_\mathrm{r} & \mathbf{M}_\mathrm{er}^\mathrm{T} \\ \mathbf{M}_\mathrm{er} & \mathbf{M}_\mathrm{e} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{q}}_\mathrm{r} \\ \ddot{\mathbf{q}}_\mathrm{e} \end{bmatrix} + \begin{bmatrix} \mathbf{k}_\mathrm{r} \\ \mathbf{k}_\mathrm{e} \end{bmatrix} = \begin{bmatrix} \mathbf{g}_\mathrm{r} \\ \mathbf{g}_\mathrm{e} \end{bmatrix}$$

Illustrative Example
Anti-Roll bar

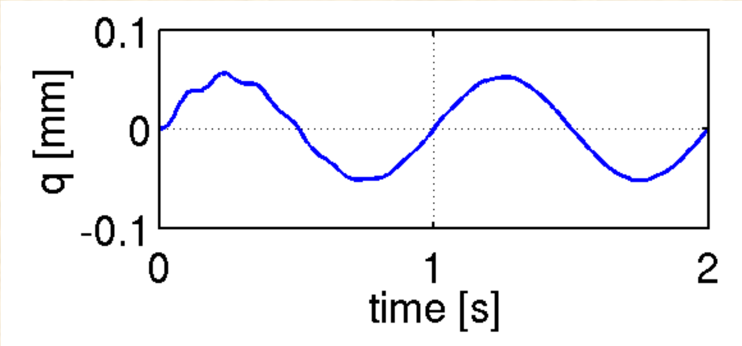
FEM-Model (Wallrapp anti-roll bar)

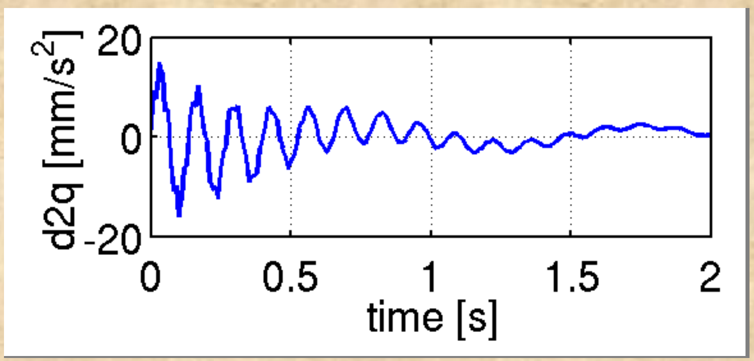
anti-roll bar fixed at node 1
force $F = 100\,\mathrm{N}\,\sin(2\pi\,t)$ at node 20 z-direction
full model 120 dof





http://en.wikipedia.org/wiki/Anti-roll_bar#mediaviewer/File:Alfetta_front_suspension_antiroll.jpg





Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Prof. Dr.-Ing. Prof. E.h. Peter Eberhard

MOR Techniques / Error Estimators

$$\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e + \mathbf{K}_e \cdot \mathbf{q}_e = \mathbf{B}_e \cdot \mathbf{u}_e$$
$$\mathbf{y} = \mathbf{C}_e \cdot \mathbf{q}_e$$

Laplace Transform

$$\mathbf{H}(s) = \mathbf{C}_e(s^2\mathbf{M}_e + s\mathbf{D}_e + \mathbf{K}_e)^{-1}\mathbf{B}_e$$

- error measured in the frequency domain or in a specific system norm [Panzer14]

Krylov-Based/CMS

- find Hermite rational interpolant $\overline{\mathbf{H}}$, s.t. moments match in specified order at specified points

$$\overline{\mathrm{H}}_{ij}(s) = \frac{\sum_{l=0}^{n-2} a_{ij,l} s^l}{1+\sum_{k=1}^{n} b_{ij,k} s^k} \text{ s.t.}$$
$$\overline{\mathbf{H}}(s_\mathbf{k}) = \mathbf{H}(s_k)$$
$$\overline{\mathbf{H}}'(s_\mathbf{k}) = \mathbf{H}'(s_k)$$
$$\overline{\mathbf{H}}''(s_\mathbf{k}) = \mathbf{H}''(s_k)$$

- $\mathcal{H}_2$-optimal MOR IRKA [GugercinAntoulasBeattie08] $\max_{t>0}|y(t) - \bar{y}\,(t)| \leq \|\boldsymbol{H} - \overline{\boldsymbol{H}}\|_{\mathcal{H}_2}$

balanced truncation / Gramian-matrix based reduction

- representation where a specific importance can be identified for each state
- second-order Gramian matrix on position level

$$\mathbf{P}_p^\omega = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} \mathbf{L}^{-1}(\omega)\mathbf{B}\mathbf{B}^T\mathbf{L}^{-H}(\omega)\mathrm{d}\omega$$

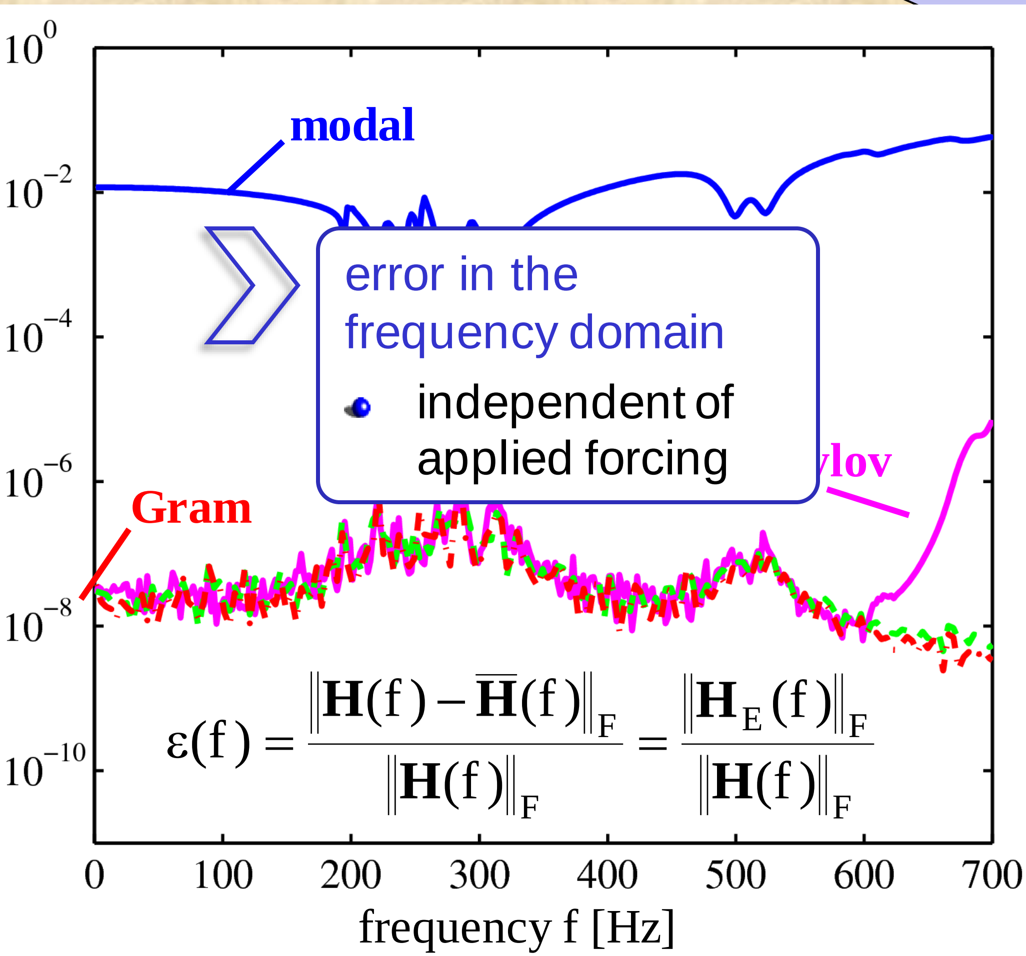
with $\mathbf{L}(\omega) = -\omega^2\mathbf{M}_e + i\omega\mathbf{D}_e + \mathbf{K}_e$

- solve Eigenproblem

$$(\zeta_i\mathbf{I} - \mathbf{P}_p)\boldsymbol{\varphi}_i = \mathbf{0}$$

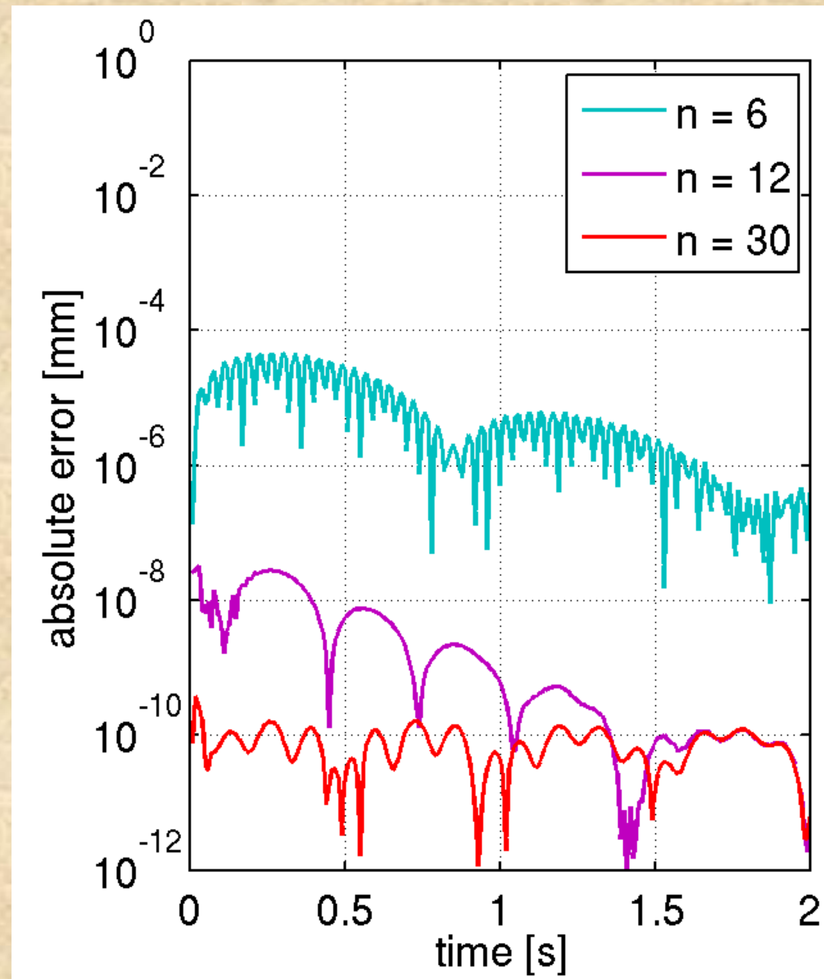
- large generalized Hankel singular values ζ_i $i = 1 \dots n$ remain in reduced system
- $\|\boldsymbol{H} - \overline{\boldsymbol{H}}\|_{\mathcal{H}_\infty} \leq 2\sum_{i=n+1}^{N} \zeta_i$

H1: Single FE Body Advantage of MOR



error of different methods with the same model size

error in the time domain



Error Estimators / Time Domain

- L2-error estimates in state and frequency space are connected by Parseval type equalities
- POD
  - a-priori time-domain error bounds for the state-space error [Volkwein13]
- a-priori error bounds
  - worst case behavior bounds
  - ensure **good approximation independent of setting**
  - individual simulation could be much better than worst case
  - largely overestimating the actual error

- certified RB methods
- a posteriori error control
  - each special input signal, loading case, parameter, etc.
  - reduced model give additional error information
- ingredients
  - norm of the residual
  - efficiently computed by suitable offline/online decomposition
- provable upper bounds
  - rigorosity / reliability
- not overestimate the true error
  - effectivity / efficiency

Error Estimators / Time Domain

efficient a-posteriorri error estimation

- first order state space system
  $\dot{\mathbf{x}}(t) = \mathbf{A}_s \cdot \mathbf{x}(t) + \mathbf{B}_s \cdot \mathbf{u}$
  $\mathbf{y}(t) = \mathbf{C}_s \cdot \mathbf{x}(t)$
- reduction by two bi-orthonormal projection matrices $\mathbf{V}_s$ and $\mathbf{W}_s$
- error $\boldsymbol{e}_s(t) = \mathbf{x}(t) - \mathbf{V}_s \bar{\mathbf{x}}$ (t)
- residual $\mathbf{R}_s = \mathbf{A}_s \cdot \mathbf{V}_s \cdot \bar{\mathbf{x}} + \mathbf{B}_s \cdot \mathbf{u} - \mathbf{V}_s \cdot \dot{\bar{\mathbf{x}}}$
- error equation

$$\mathbf{e}_s = \mathbf{\Phi}(t) \cdot \mathbf{e}_{s,0} + \int_0^t \mathbf{\Phi}(t - \tau) \cdot \mathbf{R}_s(t) d\tau$$

- fundamental matrix of the system
  $\mathbf{\Phi}(\mathrm{t}) = \mathrm{e}^{\mathbf{A}_\mathrm{s}(\mathrm{t})}$

- error bound $\mathbf{\Delta}_\mathrm{x}(\mathrm{t})$ [HaasdonkOhlberger11]
  $\|\mathbf{e}_s(t)\|_{\mathbf{G}_s} \leq \Delta_x(t)$
  $= C_1 \left\|\mathbf{e}_{s,0}\right\|_{\mathbf{G}_s} + C_1 \int_0^t \|\mathbf{R}_s(\tau)\|_{\mathbf{G}_s} d\tau$
- with $C_1 \geq \max_t \|\mathbf{\Phi}(t)\|_{\mathbf{G}_s}$
- use scaled matrix norm $\|\cdot\|_{\mathbf{G}_s}$ induced norm with scaled inner product $<\mathbf{a},\mathbf{b}>=\mathbf{b}^\mathrm{T} \cdot \mathbf{G} \cdot \mathbf{a}$
- because $\mathbf{x}$ consists of $\mathbf{q}_\mathrm{i}, \boldsymbol{\varphi}_i, \mathbf{v}_\mathrm{i}, \boldsymbol{\omega}_i$
- apply this error estimator to second order systems

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

Application to Second Order Systems

- transformation to first order system

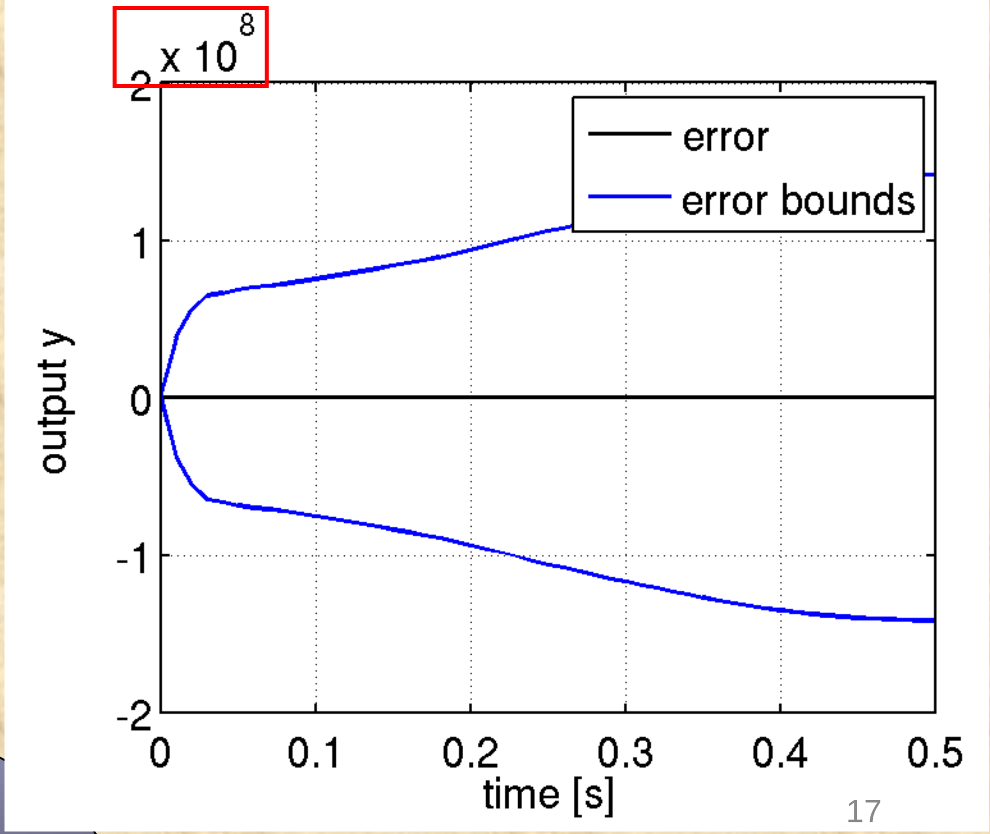
$$\underbrace{\begin{bmatrix}\dot{\mathbf{q}}_e\\ \ddot{\mathbf{q}}_e\end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix}\mathbf{0} & \boldsymbol{I}\\ -\mathbf{M}_e^{-1}\cdot\mathbf{K_e} & -\mathbf{M}_e^{-1}\cdot\mathbf{D_e}\end{bmatrix}}_{\mathbf{A}_s}\cdot\underbrace{\begin{bmatrix}\mathbf{q}_e\\ \dot{\mathbf{q}}_e\end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix}\mathbf{0}\\ \mathbf{B}_e\end{bmatrix}}_{\mathbf{B_s}}\cdot\boldsymbol{u}(t)$$

$$\mathbf{y} = \underbrace{[\mathbf{C}_e \quad \mathbf{0}]}_{\mathbf{C}_s}\cdot\underbrace{\begin{bmatrix}\mathbf{q}_e\\ \dot{\mathbf{q}}_e\end{bmatrix}}_{x(t)}$$

- $\boldsymbol{e_s}(t) = \begin{bmatrix}\boldsymbol{e_m}(t)\\ \dot{\boldsymbol{e}}_{\boldsymbol{m}}(t)\end{bmatrix} = \begin{bmatrix}\mathbf{q}_e(t) - \mathbf{V}\cdot\bar{\mathbf{q}}_e\\ \dot{\mathbf{q}}_e(t) - \mathbf{V}\cdot\dot{\bar{\mathbf{q}}}_e\end{bmatrix}$

- $\boldsymbol{R_s}(t) = \begin{bmatrix}\mathbf{0}\\ \widetilde{\boldsymbol{R}}_{\boldsymbol{m}}(t)\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \mathbf{M}_e^{-1}\cdot\mathbf{R}_m(t)\end{bmatrix}$

- large over prediction of error



Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

Over prediction / Hump Phenomenon [FehrEtAl14]

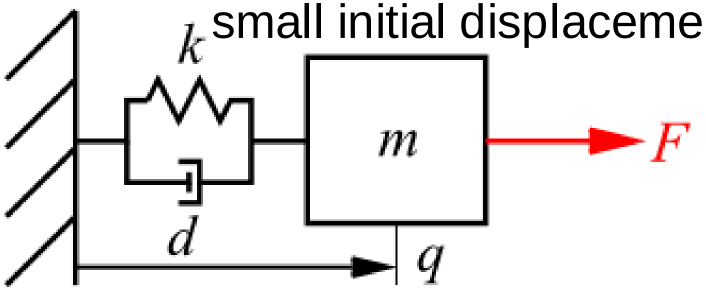
$$\Delta_x(t) = C_1 \left\|\mathbf{e}_{s,0}\right\|_{\mathbf{G}_s} + C_1 \int_0^t \|\mathbf{R}_s(\tau)\|_{\mathbf{G}_s} d\tau$$

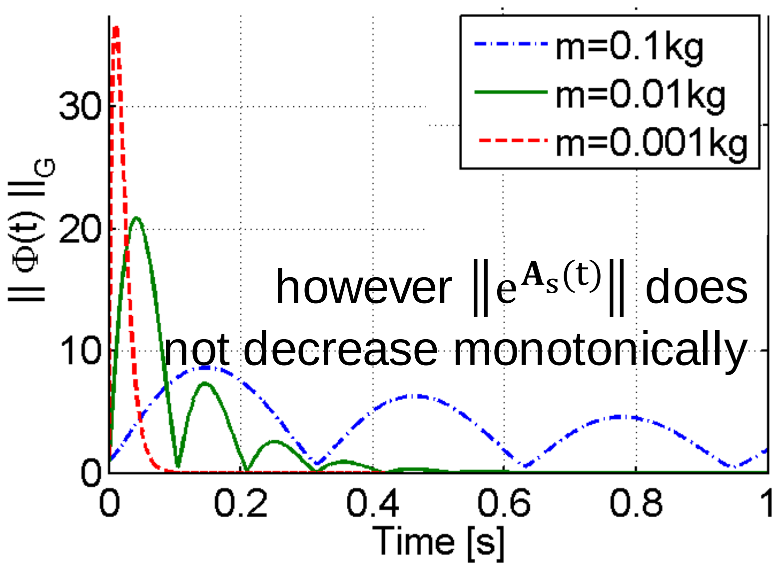
- extreme value of constant

$$C_1 \geq \max_t \left\|e^{\mathbf{A}_s(t)}\right\|_{\mathbf{G}_s}$$

simple mass spring damper system

small initial displacement

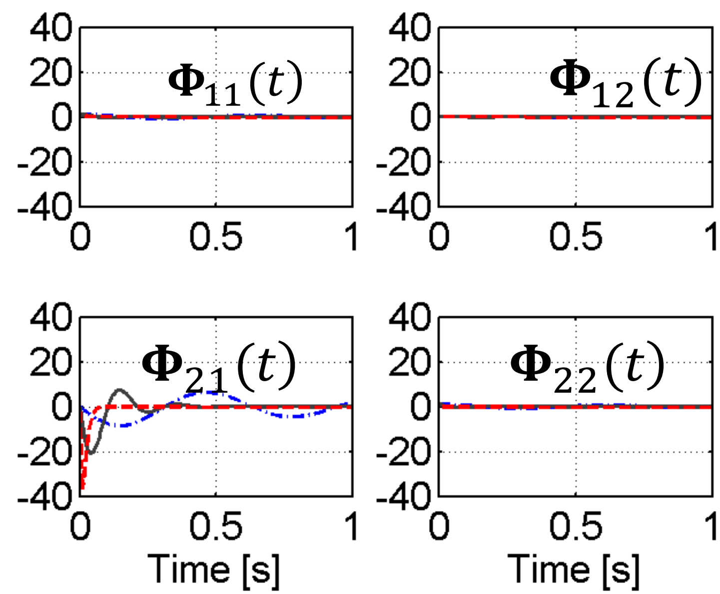




however $\left\|e^{\mathbf{A}_s(t)}\right\|$ does not decrease monotonically

- split fundamental matrix

$$\boldsymbol{x}(t) = \begin{bmatrix} \boldsymbol{\Phi}_{11}(t) & \boldsymbol{\Phi}_{12}(t) \\ \boldsymbol{\Phi}_{21}(t) & \boldsymbol{\Phi}_{22}(t) \end{bmatrix} \cdot \mathbf{x}_0$$



- $\boldsymbol{\Phi}_{21}(t)$ represent connection initial displacement to velocity
- single error bound for both state variables, $\mathbf{q}_\mathrm{e}$ and $\dot{\mathbf{q}}_\mathrm{e}$

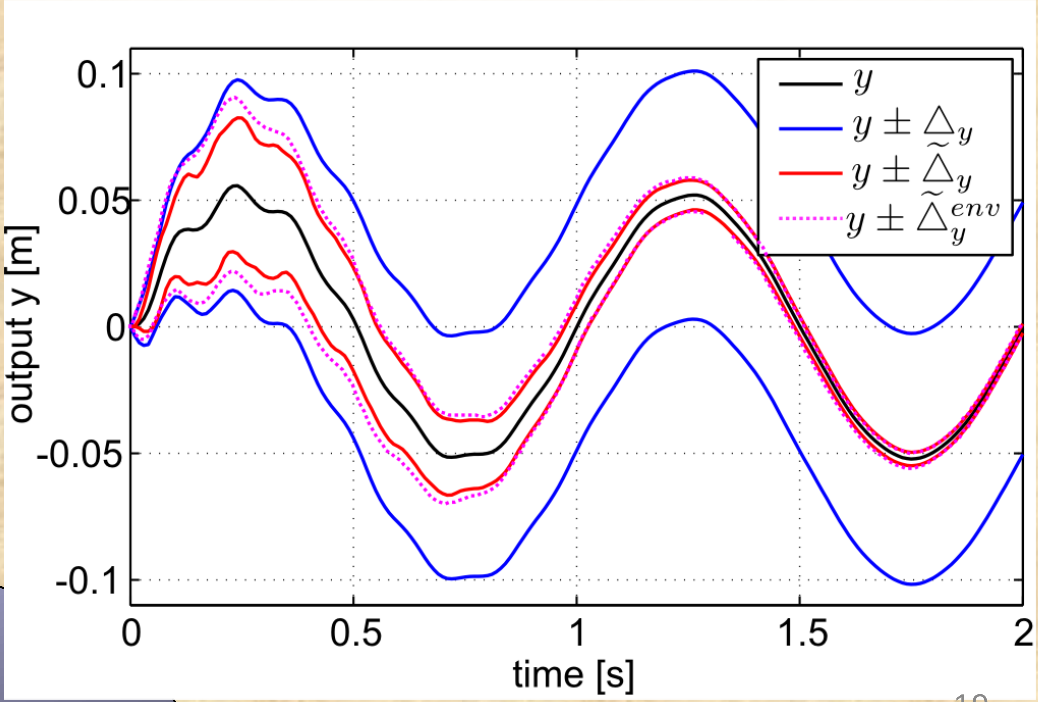
Error Estimation / Second Order Systems

- error estimator delivers impractical results for EMBS
- modified error estimator [FehrEtAl14]

$$\begin{bmatrix} \boldsymbol{e_m}(t) \\ \dot{\boldsymbol{e}}_{\boldsymbol{m}}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{11}(t) & \boldsymbol{\Phi}_{12}(t) \\ \boldsymbol{\Phi}_{21}(t) & \boldsymbol{\Phi}_{22}(t) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{e}_{\boldsymbol{m},0} \\ \dot{\boldsymbol{e}}_{\boldsymbol{m},0} \end{bmatrix} + \int_0^t \begin{bmatrix} \boldsymbol{\Phi}_{11}(t-\tau) & \boldsymbol{\Phi}_{12}(t-\tau) \\ \boldsymbol{\Phi}_{21}(t-\tau) & \boldsymbol{\Phi}_{22}(t-\tau) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{0} \\ \widetilde{\boldsymbol{R}}_{\boldsymbol{m}}(t) \end{bmatrix} \boldsymbol{d}\tau$$

- relevant $\boldsymbol{e_m}(t) = \boldsymbol{\Phi}_{11}(t) \cdot \boldsymbol{e}_{\boldsymbol{m},0} + \boldsymbol{\Phi}_{12}(t) \cdot \dot{\boldsymbol{e}}_{\boldsymbol{m},0} + \int_0^t \boldsymbol{\Phi}_{12}(t-\tau) \cdot \widetilde{\boldsymbol{R}}_{\boldsymbol{m}}(t)\, \boldsymbol{d}\tau$
- term $\boldsymbol{\Phi}_{21}(t)$, which causes large hump no longer required
- three new error estimators $\Delta_q(t) = C_{11}\|\boldsymbol{e}_{\boldsymbol{m},0}\|_{\mathrm{G_M}} + C_{12}\|\dot{\boldsymbol{e}}_{\boldsymbol{m},0}\|_{\mathrm{G_M}}$

  $+ C_{12} \int_0^t \| \widetilde{\boldsymbol{R}}_{\boldsymbol{m}}(\tau)\|_{\mathrm{G_M}} \boldsymbol{d}\tau$
- computation time is saved significantly with approximation of fundamental matrix
  56.25 s vs. 0.077 s
- offline/online decomposition for calculation of residual



Institute of Engineering and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

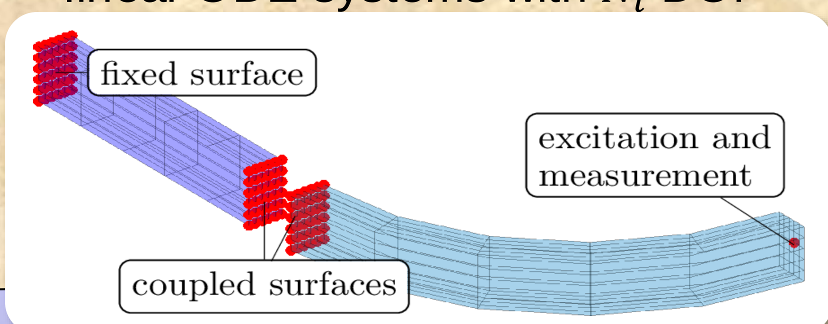
Model Hierarchies

- H1: single linear FE body expressed as a linear ODE system
  $$\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e + \mathbf{K}_e \cdot \mathbf{q}_e = \mathbf{h}_e$$
- H2: single elastic body in the FFR formulation expressed as a nonlinear ODE system

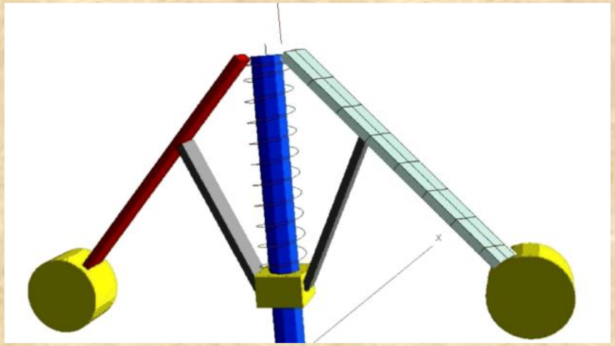
$$\begin{bmatrix} \mathbf{M}_r & \mathbf{M}_{er}^T \\ \mathbf{M}_{er} & \mathbf{M}_e \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_e \end{bmatrix} + \begin{bmatrix} \mathbf{k}_r \\ \mathbf{k}_e \end{bmatrix} = \begin{bmatrix} \mathbf{g}_r \\ \mathbf{g}_e \end{bmatrix}$$

linear elastic part

- H3: Multiple FE bodies linear ODE systems with N_i DOF



- H4: multiple rigid and elastic bodies are nonlinearly coupled with each other in the FFR formulation resulting in an EMBS

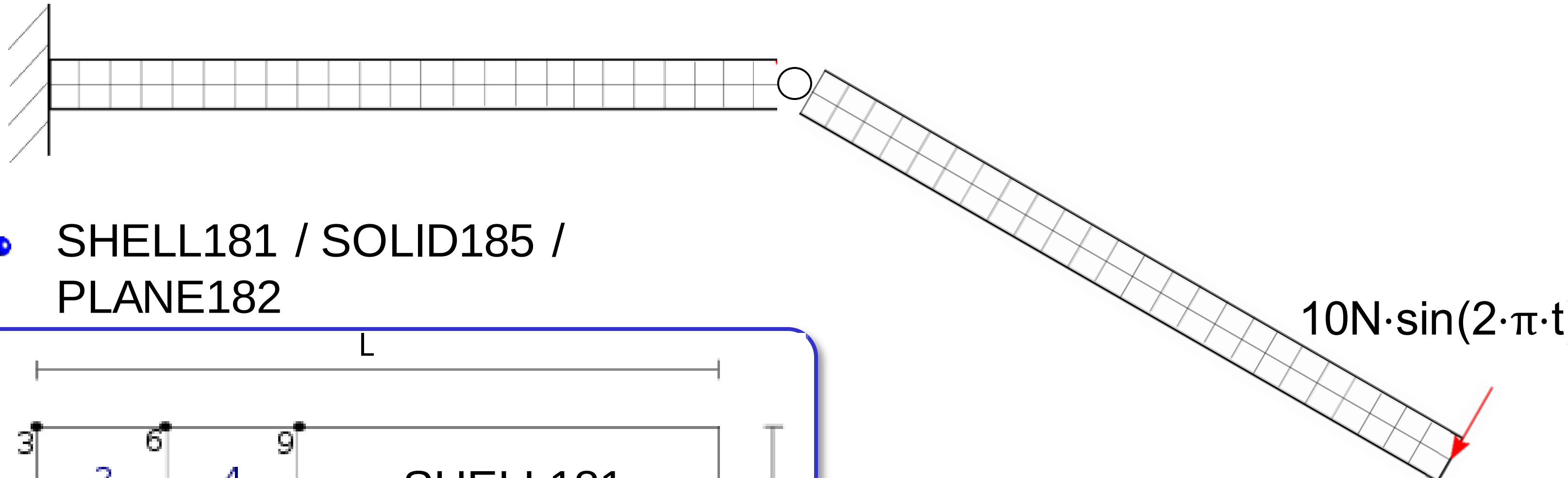


- H5: EMBS simulates mechanical part of a multiphysics environment

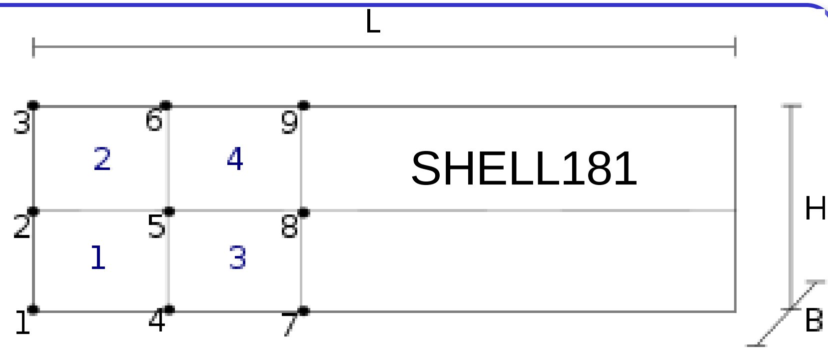
Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

Two Link Flexible Arm

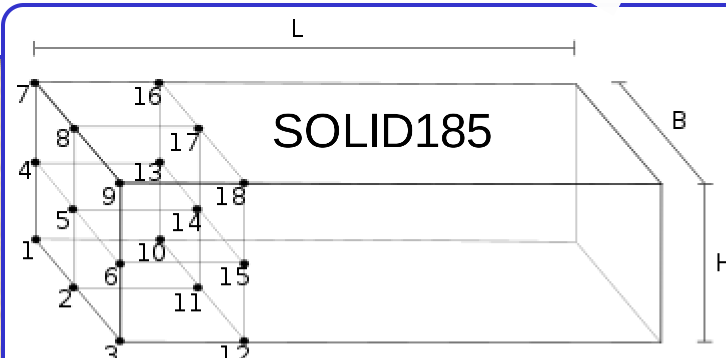
- very slender beam

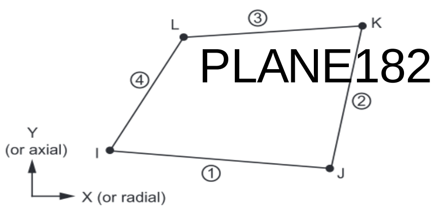


- SHELL181 / SOLID185 / PLANE182



- finite strain shell
- penalty method relate independent rotational degrees of freedom with in-plane components





- 2D-modeling of solids
- plane element or
- axisymmetric element
- 33 nodes per body, 20 elements

Workflow for Engineers

- automated workflow
- standard FE programs
  - to describe elasticity

model reduction → multibody dynamics

error bound

preprocessing | simulation

- MOR process in Morembs
  - workhorse for {linear, parametric} model reduction at ITM [FehrEtAl17]

- in-house EMBS cods
- combines the benefits of numerical computation (Matlab) and computer algebra (Maple/MuPAD)
- equation of motion derived in symobolic form

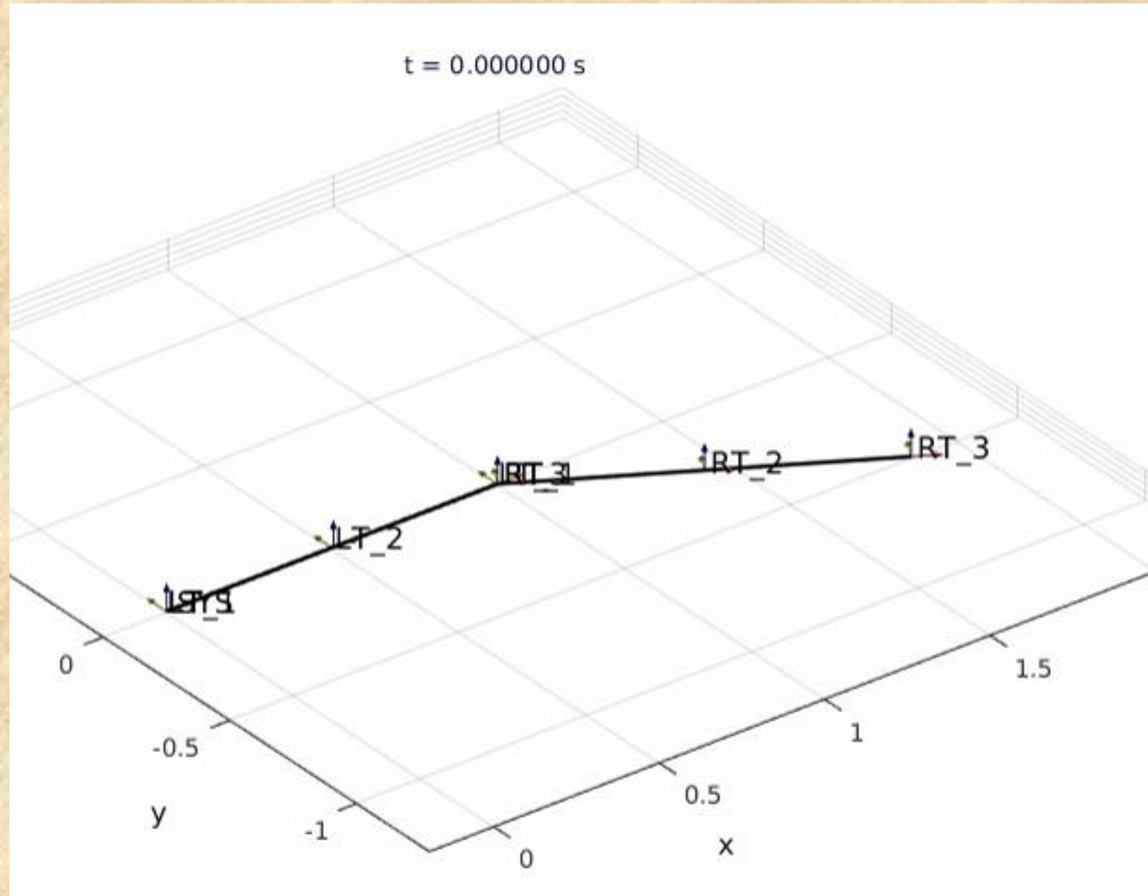
$$\begin{bmatrix} \mathbf{M}_r & \mathbf{M}_{er}^T \\ \mathbf{M}_{er} & \mathbf{M}_e \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_e \end{bmatrix} + \begin{bmatrix} \mathbf{k}_r \\ \mathbf{k}_e \end{bmatrix} = \begin{bmatrix} \mathbf{g}_r \\ \mathbf{g}_e \end{bmatrix}$$

radau5Mex integration $t = [0, 2]s$

- implicit Runge-Kutta method of order 5 (Radau IIA) for problems of the form $\boldsymbol{M}\boldsymbol{y}' = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})$ with possibly singular matrix **M**

sim. time (Intel Xeon E3-1245 3.30 GHz, RAM: 8 GB DDR3-1333)

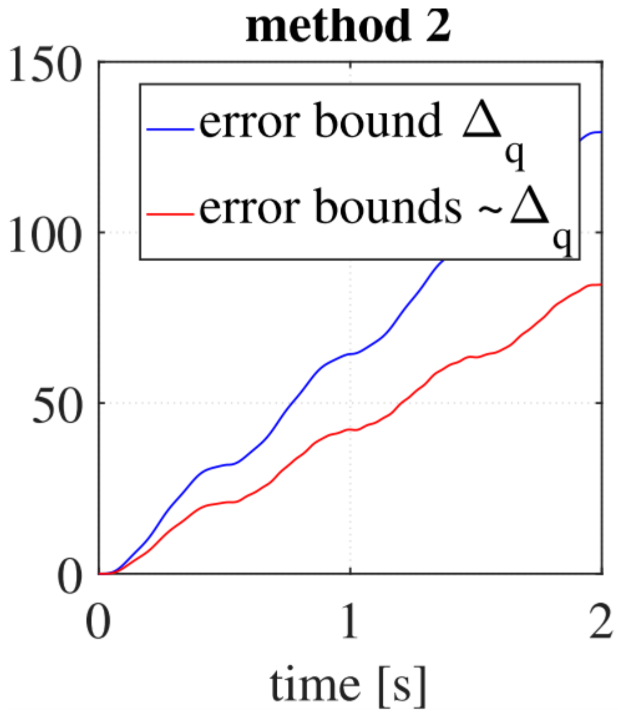
- fulll system : ~ 20 min
  - PLANE182 model
- red. system: ~ 37 s
  - 10 Rational Krylov modes per beam



Institute of Engineering and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

Sensitivity of Error Estimation

- SHELL 181 results



- large overestimation

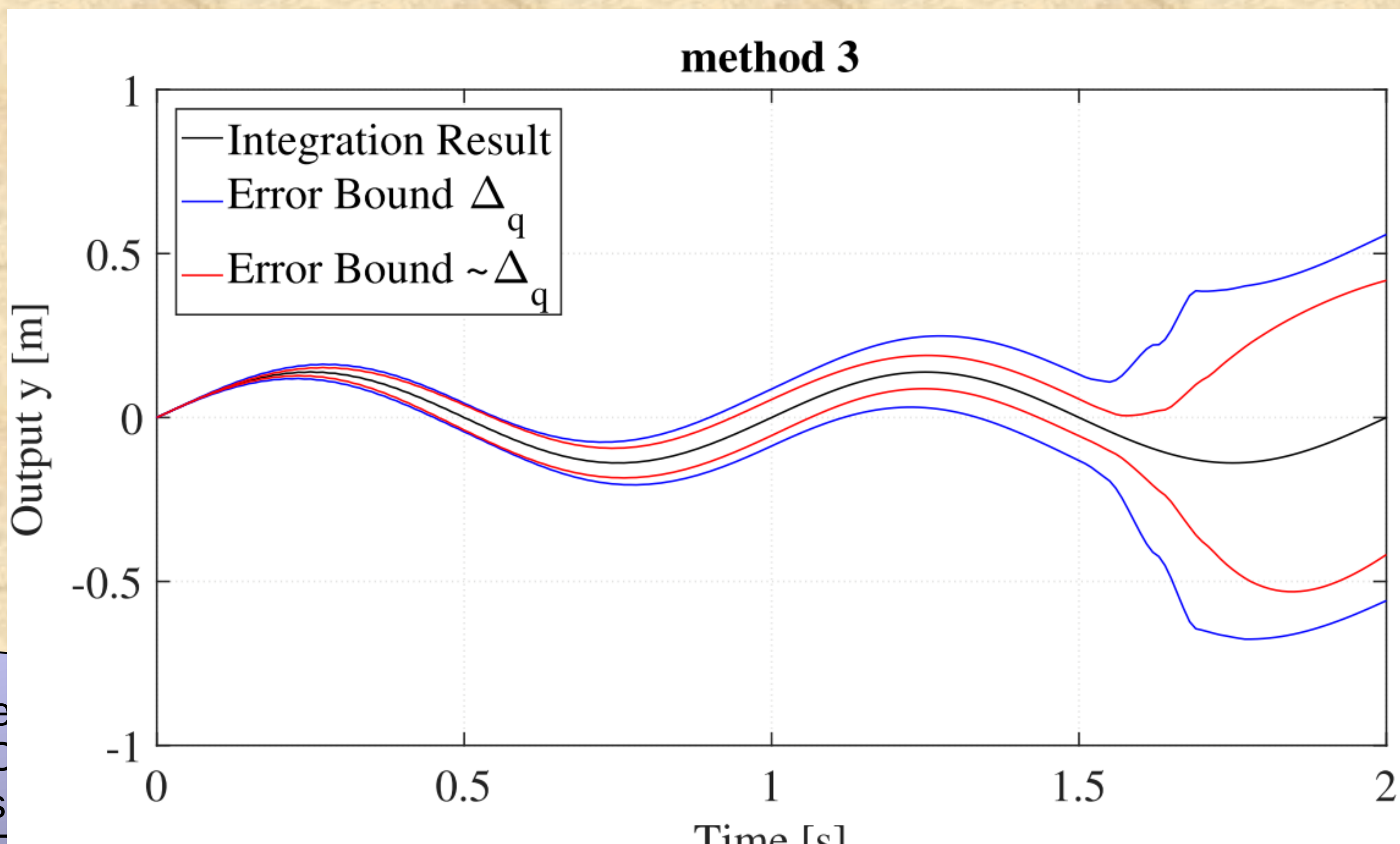
$$\boldsymbol{e_m}(t) = \boldsymbol{\Phi}_{11}\,(t)\,\cdot \boldsymbol{e_{m,0}} + \boldsymbol{\Phi}_{12}(t)\cdot \dot{\boldsymbol{e}}_{\boldsymbol{m},\boldsymbol{0}} + \int_{\boldsymbol{0}}^{\boldsymbol{t}} \boldsymbol{\Phi}_{12}(t-\tau)\cdot \underbrace{\boldsymbol{M_e}^{-1}\cdot \boldsymbol{R_m}}_{\widetilde{\boldsymbol{R}}_{\boldsymbol{m}}}(t)\, d\tau$$

- $\boldsymbol{R_m}(t)$ small $\rightarrow$ multiplication with $\boldsymbol{M_e}^{-1} \rightarrow \widetilde{\boldsymbol{R}}_{\boldsymbol{m}}(t)$ large

- condition of mass matrix $\boldsymbol{M_e}$
  - shells: $10^{14} - 10^{18}$
  - solids: ~ 100
  - depends on material, geometry, meshing
- scaling $\mathbf{G}_{\mathrm{M}} = \boldsymbol{M_e^2}$ and modal transformation improves results
- SHELL 181
  - incorrect modeling approach
- problem was not well formulated
- bad input -> bad output
- error estimator
  - detects wrong results

Error Estimation SOLID Elements

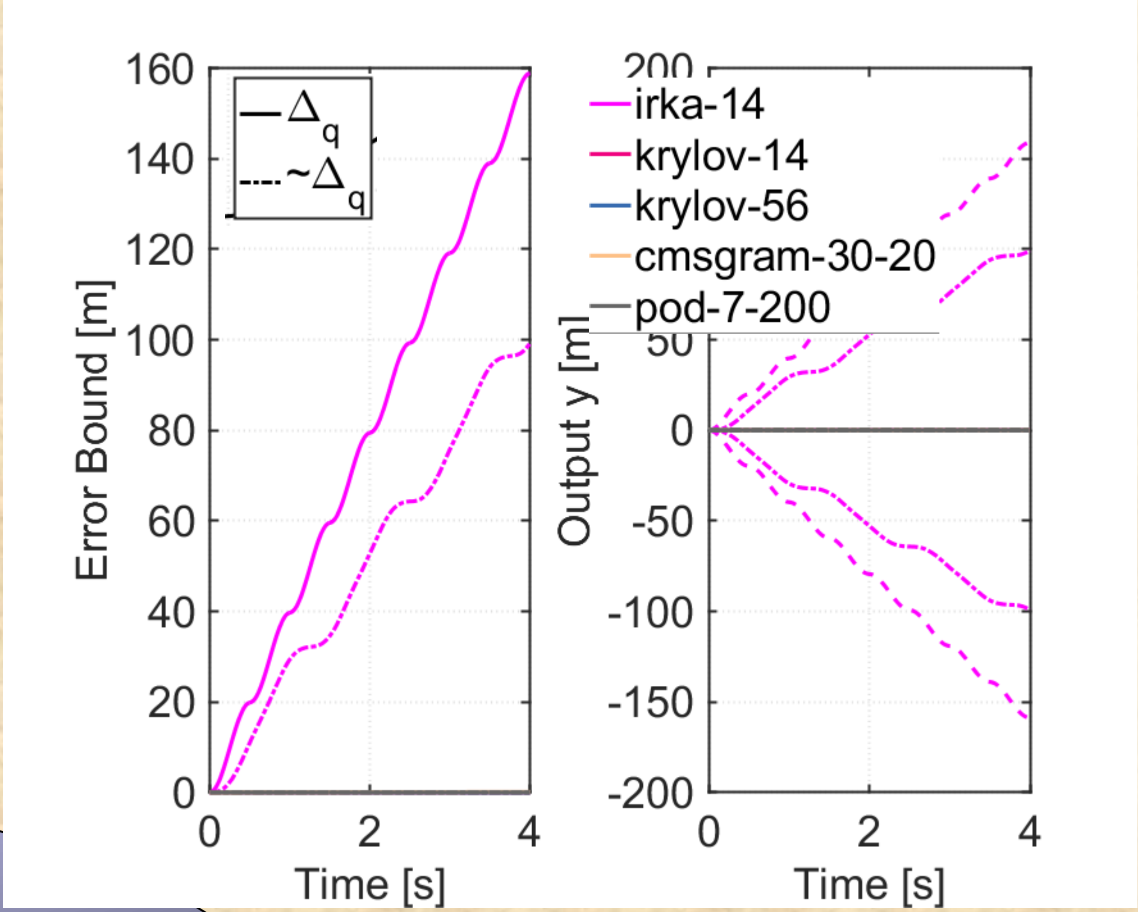
- SOLID 185 element
- reduction on dominant eigenspace of second order Gramian matrix $\mathbf{P}_\mathrm{p}$ (7 modes)
- error bounds are larger than exact error
- conservative estimation !



Institute
and C
Univers
Profs. E

Sensitivity of Error Estimator

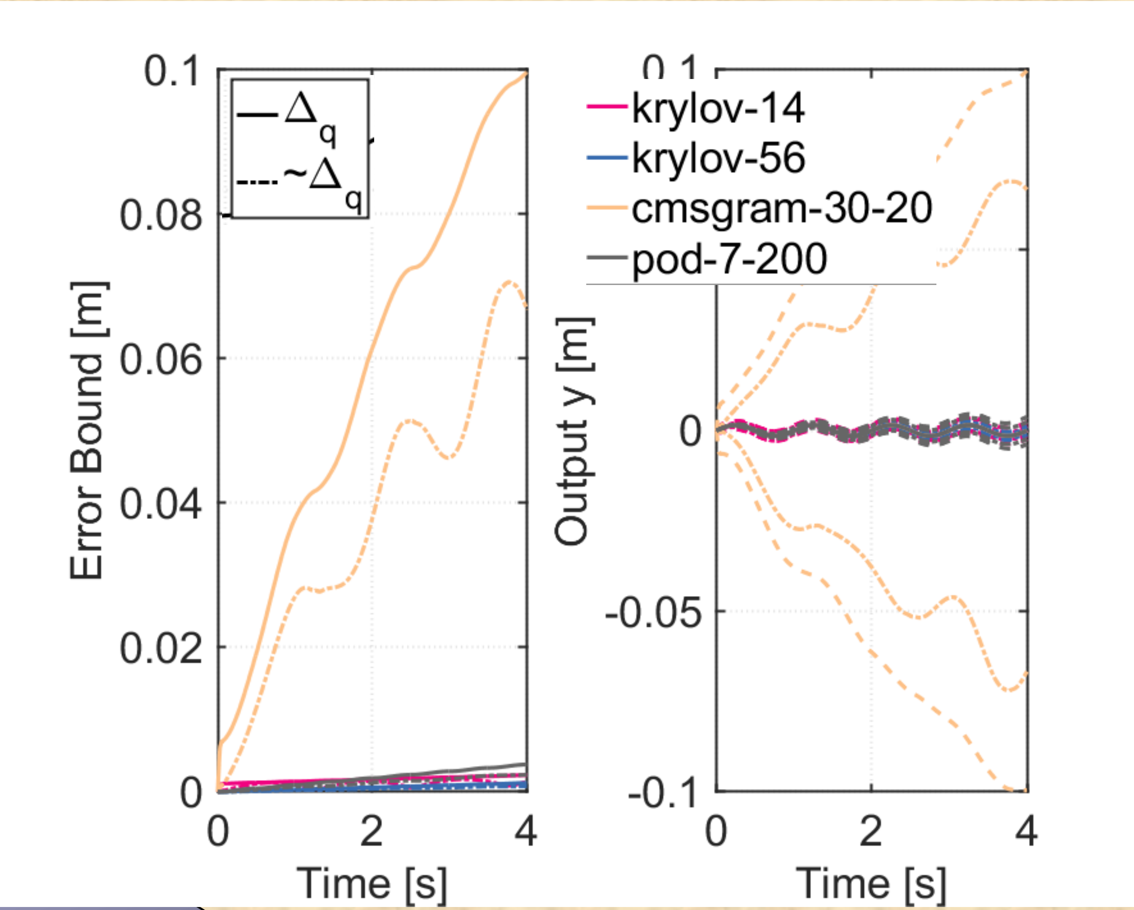
- error estimator can be used with any MOR technique
- IRKA algorithm
  - local H2-optimality
    - global problem
    - no inclusion of pre-knowledge
  - expansion points distributed over a wide range



Institute of Engineering and Computational Mechan
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

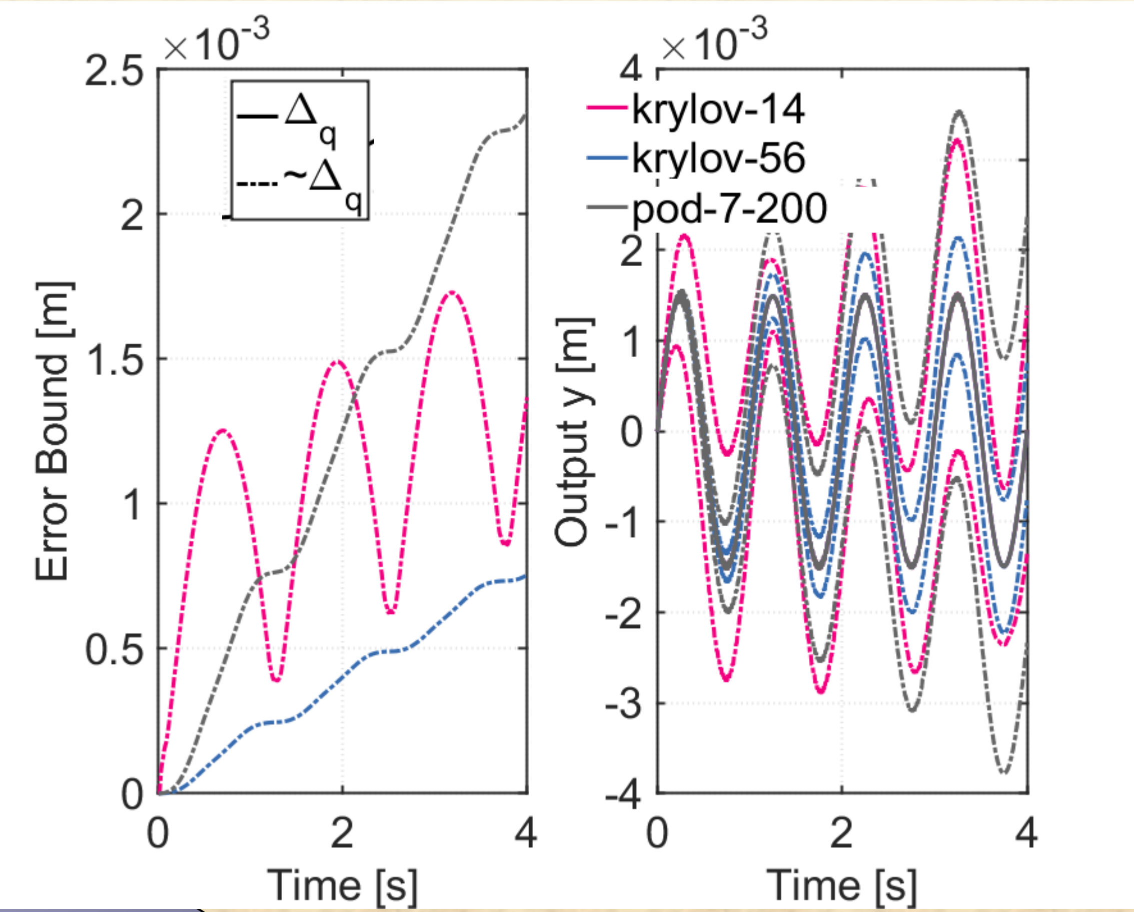
Sensitivity of Error Estimator

- error estimator can be used with any MOR technique
- CMS-Gram [HolzwarthEberhard15]
  - component mode synthesis
  - Gramian based approximation of inner degree of freedoms



Sensitivity of Error Estimator

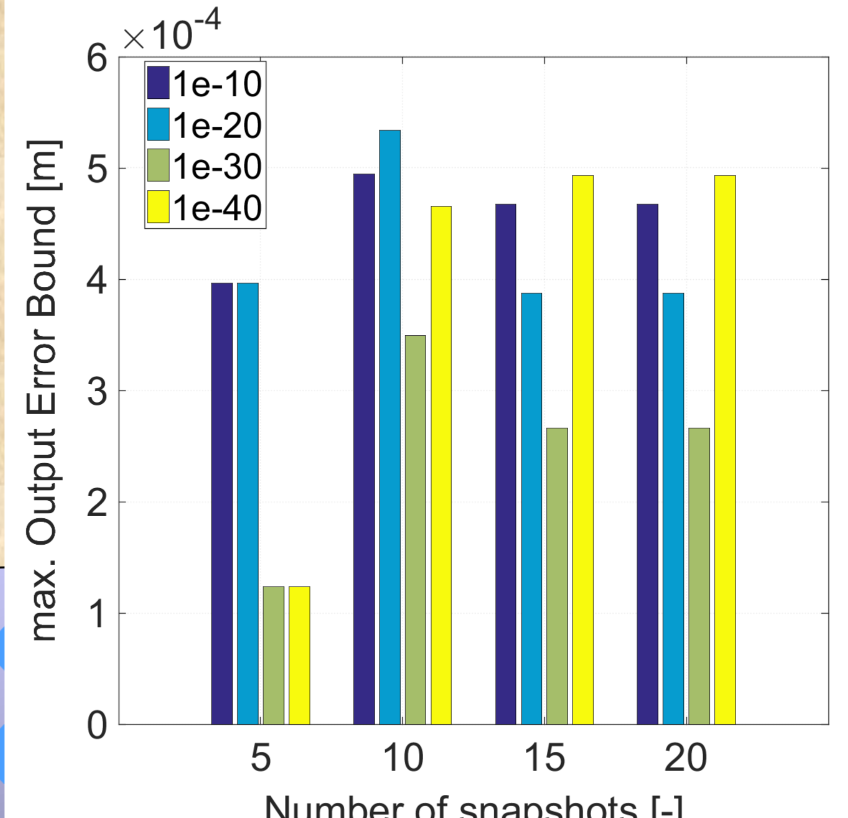
- error estimator can be used with any MOR technique
- rational Krylov (Hermite based reduction)
  - $s_k = 0 + i * 0{:}1{:}14 * 35/13$
- $\mathbf{P}_\mathrm{p}^{\omega}$ by a POD-Greedy approach [FehrEtAl12]
  - $\omega = [0, 30\ \mathrm{Hz}]$
- nice results



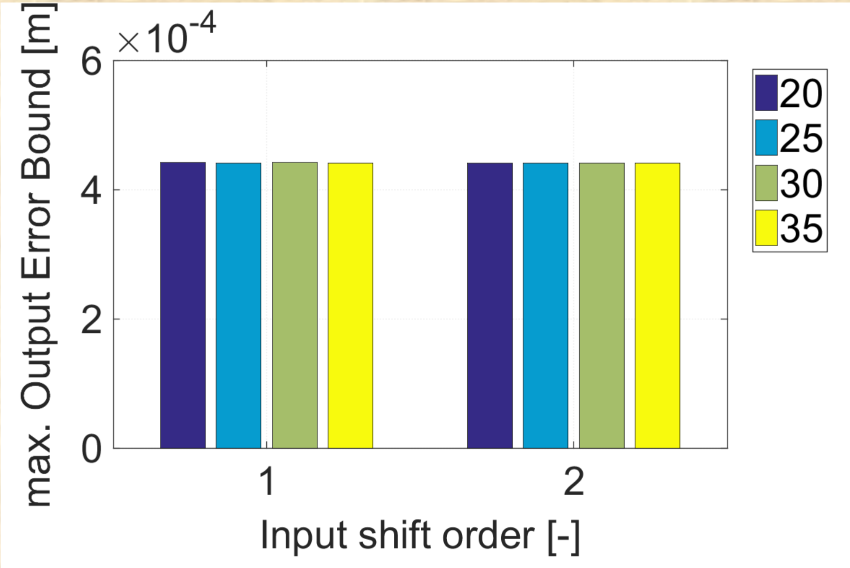
Institute of Engineering and Computational Mechan
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

Sensitivity Analysis with PLANE 182 [Meral17]

- PLANE 182
  - consistentence with linear elasticity in RBMatlab
- error bounds for reduction sizes around 10 modes
- POD $\mathbf{P}_{\mathrm{p}}^{\omega}$ $\omega = [1, 1500\ \mathrm{Hz}]$
  - standardized settings
  - deflation tolerance



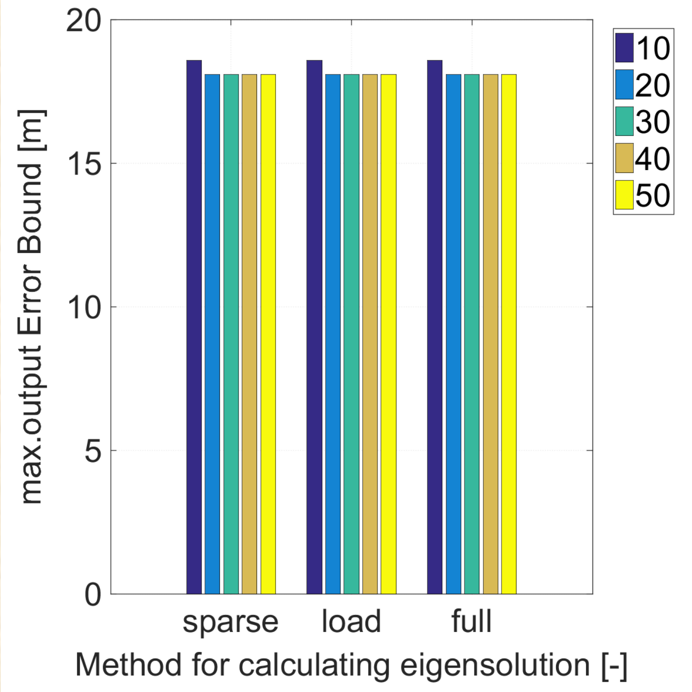
- Krylov approach
  - distance between shifts



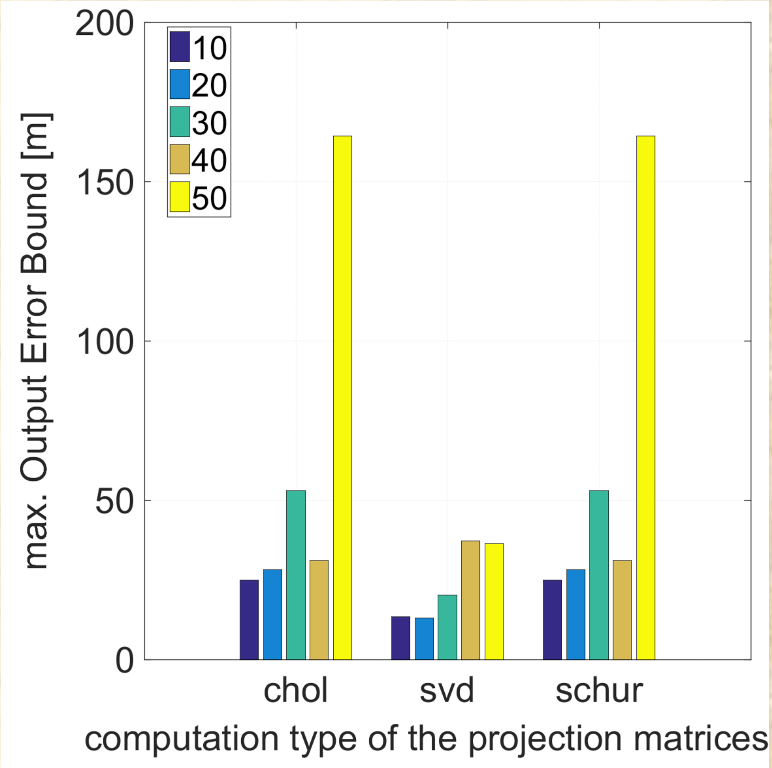
- Krylov and POD reduction methods provide the best results
- small size of the system is needed to receive small error bounds

Sensitivity Analysis with PLANE 182 [Meral17]

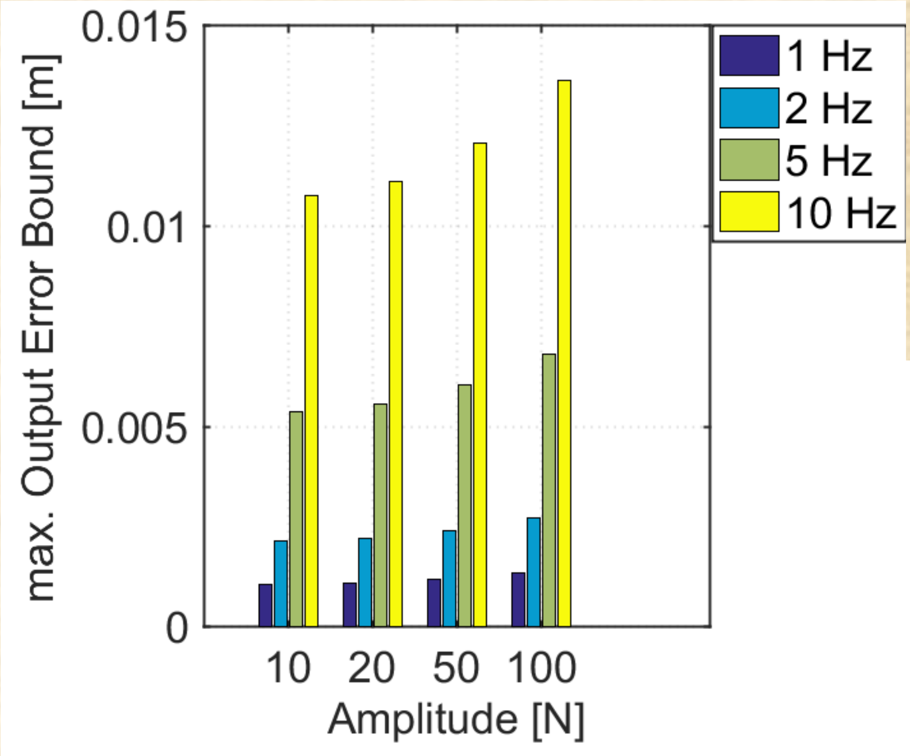
- modal reduction



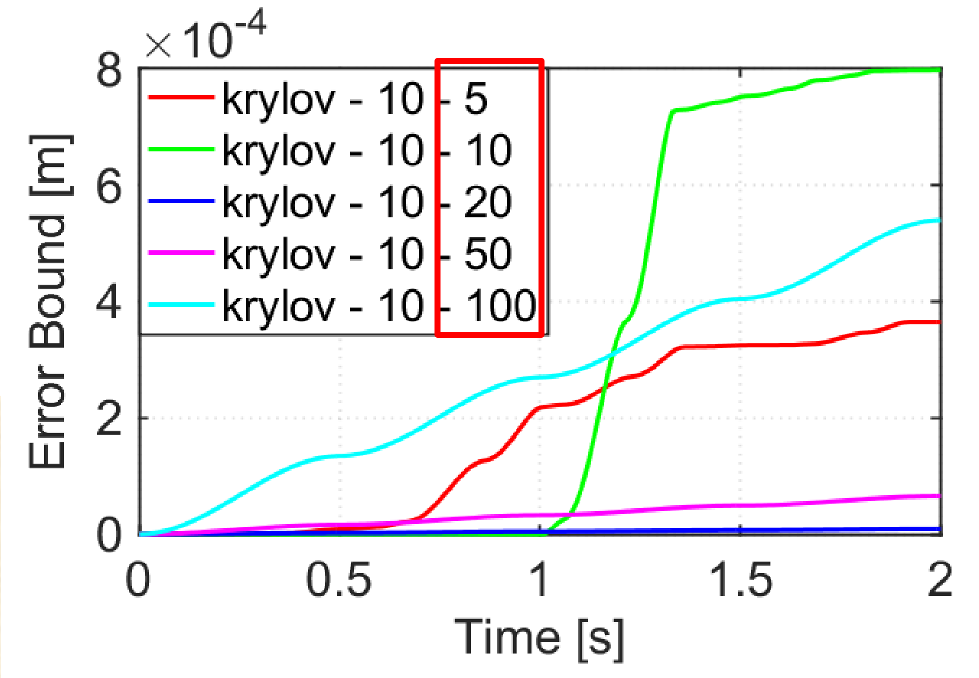
- balanced reduction



- influence of input force



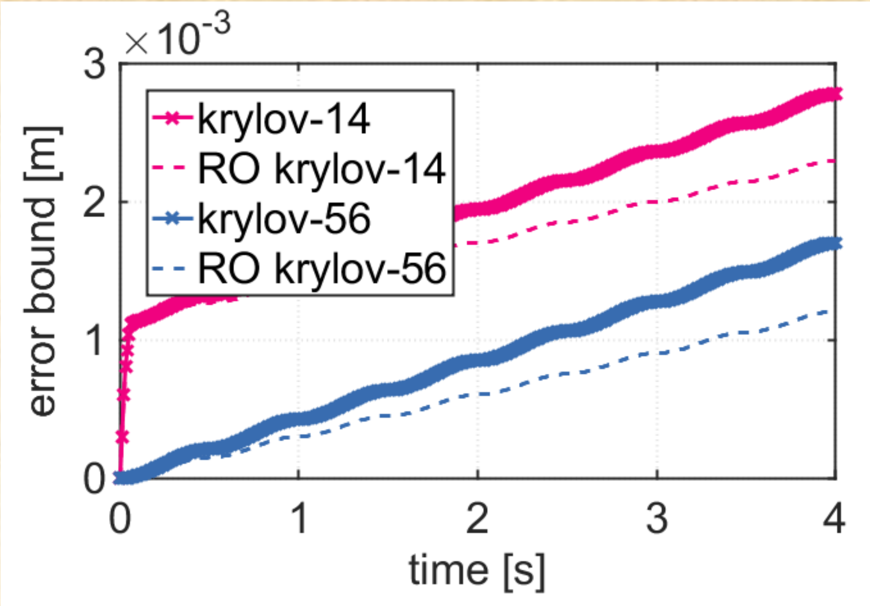
- influence of slender ration length/ height of beam



- output error bounds increase with raising frequency and raising amplitude

Improvements

- improved behavior due to reorthogonalization of bases [BuhrEtAl14]



- speedup of error estimator
  - small reduced system
    - error estimation takes as long as simulation

Dimension	Integration	Fehlerschätzung		
		$\triangle_y$	$\widetilde{\triangle}_y$	$\widetilde{C}_{11}(t), \widetilde{C}_{12}(t), C_{11}, C_{12}$
90 (full)	6.16 s	0.070 s	0.078 s	
80	5.79 s	0.068 s	0.072 s	
70	4.86 s	0.066 s	0.068 s	
60	3.96 s	0.063 s	0.065 s	~ 1.75 s
50	3.57 s	0.060 s	0.063 s	
40	2.63 s	0.059 s	0.060 s	

- approx. norm of matrix exponential

$$\|\mathbf{\Phi}\|_{\mathbf{G}} = \max_{\mathbf{z}\neq\mathbf{0}} \frac{\|\mathbf{\Phi z}\|_{\mathbf{G}}}{\|\mathbf{z}\|_{\mathbf{G}}} = \max_{\mathbf{z}\neq\mathbf{0}} \frac{\|\mathbf{G}^{\frac{1}{2}}\mathbf{\Phi z}\|_2}{\|\mathbf{G}^{\frac{1}{2}}\mathbf{z}\|_2}$$

$$\overset{\mathbf{w}=\mathbf{G}^{\frac{1}{2}}\mathbf{z}}{=} \max_{\mathbf{w}\neq\mathbf{0}} \frac{\|\mathbf{G}^{\frac{1}{2}}\mathbf{\Phi}\mathbf{G}^{-\frac{1}{2}}\mathbf{w}\|_2}{\|\mathbf{w}\|_2} = \|\mathbf{G}^{\frac{1}{2}}\mathbf{\Phi}\mathbf{G}^{-\frac{1}{2}}\|_2$$

$$\leq \|\mathbf{G}^{\frac{1}{2}}\mathbf{\Phi}\mathbf{G}^{-\frac{1}{2}}\|_{\mathrm{Fro}} \text{ and}$$

$$\leq \sqrt{\|\mathbf{G}^{\frac{1}{2}}\mathbf{\Phi}\mathbf{G}^{-\frac{1}{2}}\|_1 \|\mathbf{G}^{\frac{1}{2}}\mathbf{\Phi}\mathbf{G}^{-\frac{1}{2}}\|_\infty}$$

Implementation Strategy

- error estimator
  $\Delta_q(t) = C_{11}\|\boldsymbol{e}_{\boldsymbol{m},0}\|_{\mathrm{G_M}} +$
  $C_{12}\|\dot{\boldsymbol{e}}_{\boldsymbol{m},0}\|_{\mathrm{G_M}} + C_{12}\int_0^{\boldsymbol{t}}\|\widetilde{\boldsymbol{R}}_{\boldsymbol{m}}(\tau)\|_{\mathrm{G_M}}\,\boldsymbol{d}\tau$
- written as differential equation
  $\dot{\Delta}_q(t) = C_{12}\|\widetilde{\boldsymbol{R}}_{\boldsymbol{m}}(\tau)\|_{\mathrm{G_M}}$
  $\Delta_q(t_0) = C_{11}\|\boldsymbol{e}_{\boldsymbol{m},0}\|_{\mathrm{G_M}} +$
  $C_{12}\|\dot{\boldsymbol{e}}_{\boldsymbol{m},0}\|_{\mathrm{G_M}}$
- $\widetilde{\boldsymbol{R}}_{\boldsymbol{m}}(\tau)$ depends on $\bar{\mathbf{x}}_\mathrm{e}$
- add the on $\bar{\mathbf{x}}_\mathrm{e}$ dependending ODE to Neweul-M²
- possible eqm_nonlin_ss.m is given in symbolic form
- intrusive approach

- calculating error estimator after solver finished with a time step
  - hook OUTPUTFCN of Matlab ODESET
- minor modification to Neweul-M² core
- hook allows solver to stop if error estimator too high
- user needs to supply all time steps
  - allow optimal preallocation of variables
- blue print to other software packages

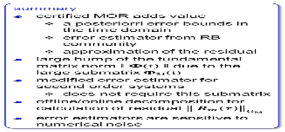
Summary

summary

- certified MOR adds value
  - a posteriorri error bounds in the time domain
  - error estimator from RB community
  - approximation of the residual
- large hump of the fundamental matrix norm $\| \mathbf{\Phi}(t) \|$ due to the large submatrix $\mathbf{\Phi}_{21}(t)$
- modified error estimator for second order systems
  - does not require this submatrix
- offline/online decomposition for calculation of residual $\| \tilde{\boldsymbol{R}}_m(\tau) \|_{G_M}$
- error estimators are sensitive to numerical noise

outlook

- application to multiphysics system
  - coupled system
- improvement of workflow,
  - automatic implementation

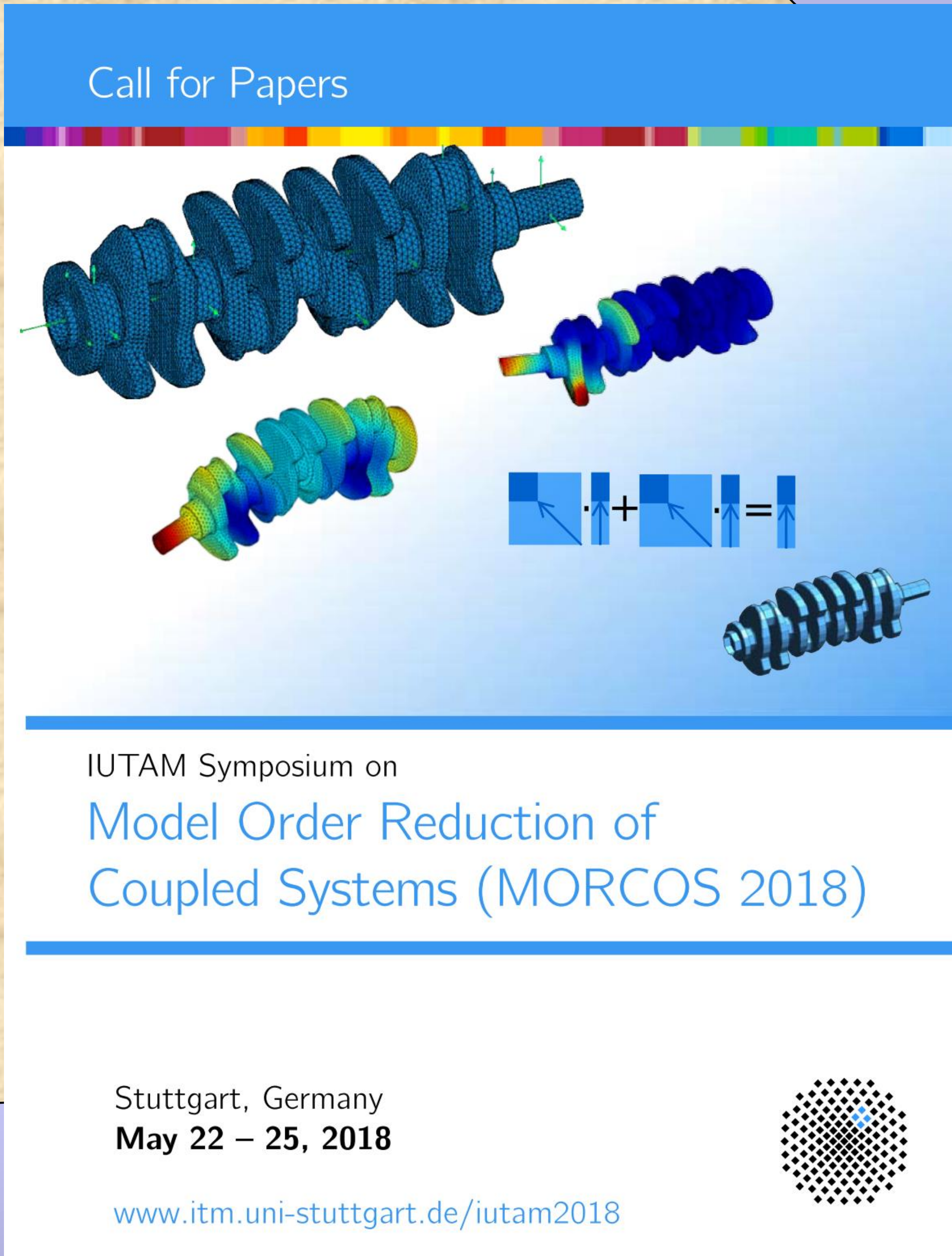


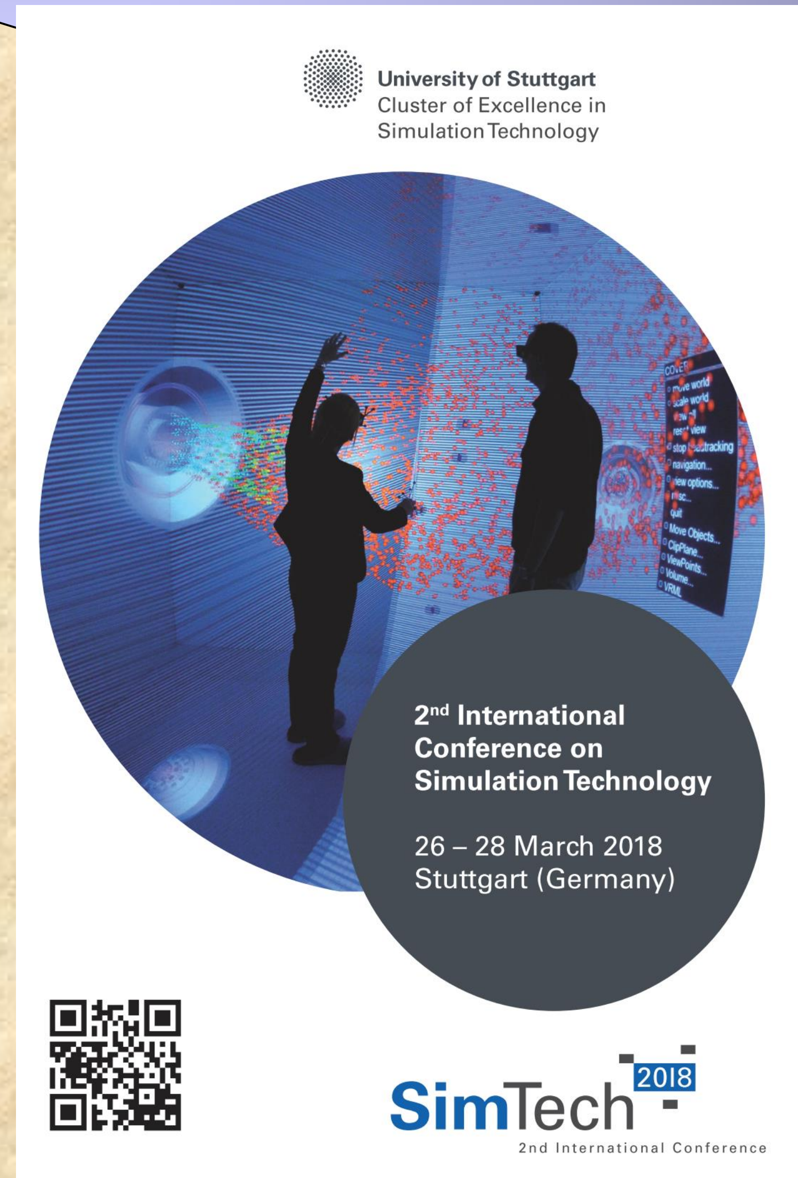
  - snapshot based reduction
  - search for refined error estimators

Deutsche Forschungsgemeinschaft

Institute of Engineering
and Computational Mechanics
University of Stuttgart, Germany
Profs. Eberhard / Hanss / Fehr

Commercials





References

- [BuhrEtAl14] Buhr, A.; Engwer, C.; Ohlberger, M.; Rave, S.: A Numberically Stable a Posteriori Error Estimator for Reduced Basis Approximation of Elliptic Equations. In E. Onate, X.O.; Huerta, A. (Eds.): 11th World Congress on Computational Mechanics, WCCM 2014, 5th European Conference on Computational Mechanics, ECCM 2014 and 6th European Conference on Computational Fluid Dynamics, ECFD 2014, pp. 4094–4102, CIMNE, Barcelona, 2014.
- [FehrEtAl12] Fehr, J.; Fischer, M.; Haasdonk, B.; Eberhard, P.: Greedy-based Approximation of Frequency-weighted Gramian Matrices for Model Reduction in Multibody Dynamics. Zeitschrift für angewandte Mathematik und Mechanik, Vol. 93, No. 8, pp. 501–519, 2012.
- [FehrEtAl14] Fehr, J.; Ruiner, T.; Haasdonk, B.; Eberhard, P.: A-Posteriori Error Estimation for Second Order Mechanical Systems. In Proceedings of the 8th EUROMECH Nonlinear Dynamics Conference (ENOC), Vienna, Austria, July 06 – 11, 2014. (6 pages).
- [FehrEtAl17] Fehr, J.; Grunert, D.; Holzwarth, P.; Fröhlich, B.; Walker, N.; Eberhard, P.: In Morembs – a Model Order Reduction Package for Elastic Multibody Systems and Beyond: KoMSO Challenge Workshop on “Reduced-Order Modeling for Simulation and Optimization”. Springer, 2017. (accepted for publication, 25 pages).
- [GugercinAntoulasBeattie08] Gugercin, S.; Antoulas, A.C.; Beattie, C.A.: H_2 Model Reduction for Large-Scale Linear Dynamical Systems. SIAM Journal on Matrix Analysis and Applications, Vol. 30, No. 2, pp. 609–638, 2008.

References

- [HaasdonkOhlberger11] Haasdonk, B.; Ohlberger, M.: Efficient Reduced Models and A-Posteriori Error Estimation for Parametrized Dynamical Systems by Offline/Online Decomposition. Mathematical and Computer Modelling of Dynamical Systems, Vol. 17, No. 2, pp. 145–161, 2011.
- [Holzwarth17] Holzwarth, P.: Modellordnungsreduktion für substrukturierte mechanische Systeme. No. 51 in Dissertation, Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart,. Aachen: Shaker Verlag, 2017.
- [Panzer14] Panzer, H.K.F.: Model Order Reduction by Krylov Subspace Methods with Global Error Bounds and Automatic Choice of Parameters. Dissertation, Technische Universität München. München: Verlag Dr. Hut, 2014.
- [RuinerEtAl12] Ruiner, T.; Fehr, J.; Haasdonk, B.; Eberhard, P.: A-posteriori Error Estimation for Second Order Mechanical Systems. Acta Mechanica Sinica, Vol. 28, No. 3, pp. 854–862, 2012.
- [Volkwein13] Volkwein, S.: Proper Orthogonal Decomposition: Theory and Reduced-Order Modelling. http://www.math.uni-konstanz.de/numerik/personen/-volkwein/teaching/POD-Book.pdf, 2013. Accessed 26. October 2015.