Higher form gauge fields and membranes in D=4 supergravity

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Introduction and motivation

- 3-form gauge fields A_3 in D=4 do not carry local dynamical degrees of freedoms. On-shell values of their 4-form field strengths F_4=dA_3 are constant.
- A cosmologically attractive feature of these fields is two fold:
  - On the one hand, when coupled to gravity, F_4 induce *dynamically* a contribution to the cosmological constant *(Duff & van Nieuwenhuizen; Aurilia, Nicolai & Townsend '80, Hawking `84,…)*
  - On the other hand, when coupled to membranes, A_3 provide a mechanism for neutralizing the cosmological constant via membrane nucleation *(Brown & Teitelboim '87)*
- Some models of inflation involve A_3 coupled to an inflaton field *(e.g. Bousso & Polchinski '00, Kaloper & Sorbo '08,…)*
- A_3 provide a mechanism for resolution of strong CP violation problem (*Dvali '05*)
- In supersymmetric theories A_3 induce spontaneous supersymmetry breaking.
- Plenty of these fields appear in 4d upon type-II string compactifications on Calabi-Yau manifolds with fluxes and can realize the above physical scenarios.

Three-form gauge fields in 4D space-time

Rank-3 antisymmetric gauge fields:

$A_{[\mu\nu\rho]}(x)$, gauge transformations $\delta A_{\mu\nu\rho} = \partial_{[\mu}\varphi_{\nu\rho]}(x) \quad (\mu, \nu, \rho = 0,1,2,3)$

Guage-invariant field strength: $F_{\mu\nu\rho\sigma} = 4\partial_{[\mu}A_{\nu\rho\sigma]} = \varepsilon_{\mu\nu\rho\sigma}F(x)$

Three-form field Lagrangian coupled to gravity

$$L = \frac{1}{2k}\sqrt{-g}R - \tfrac{1}{2\cdot 4!}\sqrt{-g}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} \underbrace{- \tfrac{1}{3!}\partial_\mu\left(\sqrt{-g}A_{\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right)}_{\text{boundary term}}$$

$$k = 8\pi G c^{-4}$$

Equations of motion and cosmological constant generation

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = \frac{k}{3!} \left(F_{\mu\lambda\rho\sigma} F_{\nu}{}^{\lambda\rho\sigma} - \tfrac{1}{8} g_{\mu\nu} F_{\lambda\rho\sigma\delta} F^{\lambda\rho\sigma\delta} \right)$$

$$\partial_{\mu} \left(\sqrt{-g} F^{\mu\nu\rho\sigma} \right) = 0 \quad \Rightarrow \quad F^{\mu\nu\rho\sigma} = \tfrac{1}{\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} E \quad (E = constant)$$

Hence $R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\tfrac{k}{2} g_{\mu\nu} E^{2} \equiv -g_{\mu\nu} \Lambda$

$$\boxed{\Lambda = \tfrac{k}{2} E^{2}}$$

The cosmological constant has been generated ***dynamically*** by a classical constant value *E* of the gauge field strength

Membrane nucleation and diminishing of the cosmological constant (*Brown & Teitelboim '87*)

3-form gauge field minimally couples to 2d domain walls (membranes)

$$S_{M2} = -m\int \mathrm{d}^3\xi \sqrt{-\det\left(g_{\mu\nu}\partial_i z^\mu \partial_j z^\nu\right)} + q\int \mathrm{d}^3\xi A_{\mu\nu\rho}(z)\,\partial_i z^\mu \partial_j z^\nu \partial_k z^\rho\ \ \varepsilon^{ijk}$$

membrane mass (tension)

membrane charge

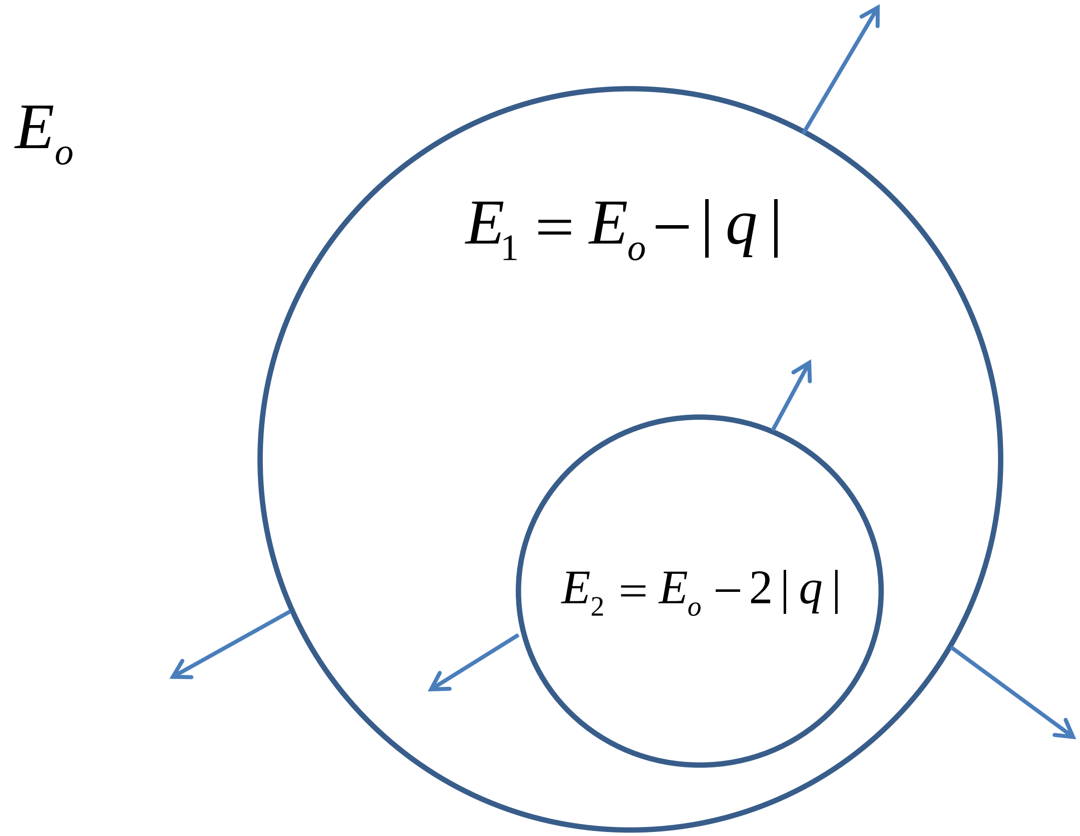
$\xi^i \ \ (i = 0,1,2)$ parametrizes membrane worldvolume $\quad z^\mu \equiv x^\mu(\xi)$

3-form field equation:

$$\partial_\mu\left(\sqrt{-g}\,F^{\mu\nu\rho\sigma}\right) = -q\ J^{\nu\rho\sigma}(x)$$

$$J^{\nu\rho\sigma}(x) = \int d^3\xi\ \partial_i z^\nu \partial_j z^\rho \partial_k z^\sigma\ \ \varepsilon^{ijk}\delta(x - z(\xi))$$ - membrane current

Membrane nucleation and neutrilization of the cosmological constant (*Brown & Teitelboim '87*)



Membrane creation probability $P \sim \exp\left(- \frac{m^2}{\hbar \,|\, q(E_o - \frac{|q|}{2}) \,|} \right)$

Process stops at $E_n = \frac{|q|}{2}$

Issues and rectification of the mechanism

- In this simple scenario, to reach the experimentally observed value of the cosmological constant the membranes should be heavy (large m) and weakly coupled (small q)
- with this characteristics the membranes are nucleated too slowly and time of transition from an initial (Plank) value of the cosmological constant to its present value is much greater than the age of the Universe.
- Improvments (Bousso & Polchinski, '00 and others)
  - multiple 3-form gauge fields are required
  - their coupling to the inflaton and membrane nucleation produce regions in the Universe with different cosmological constants. We leave in a region in which its value is small.
- Recent inflationary models intensively exploit these ideas.

Effective field theories of type II string and M-theory compactifications with fluxes

Gukov, Vafa, Witten; Taylor, Vafa '99; Luis & Miku '02; Blumenhagen et.al. '03, Grimm, Luis '04 …
…Bielleman, Ibanez & Valenzuela '15,…

D=10 Type IIA bosonic field content

NS-NS sector: $g_{MN},\ \phi,\ \ H_3 = dB_2$

RR sector: $A_p \quad p = \text{"}-1\text{"},1,3,5,7,9 \qquad F_{p+1} = *_{10} F_{9-p}$

$$\mathbf{F} = (d\mathbf{A} + F_0\ ) \wedge e^{B_2}$$

$$F_0 = m = *F_{10} \quad - \quad \text{Romans mass}$$

$$F_2 = dA_1 + mB_2 = *F_8$$

$$F_4 = dA_3 + dA_1 \wedge B_2 + \tfrac{1}{2} mB_2 \wedge B_2 = *F_6$$

……..

Effective N=1, D=4 theories of type IIA string compactifications with RR fluxes (*Bielleman et.al '15*)

Compactification on $M_4 \times CY_3$ and F_4 fluxes on M_4, $\mathbf{F} = (d\mathbf{A} + F_0\,) \wedge e^{B_2}$

$$F_0 = m^0 = *(\widetilde{F}_{40}\,|_{M_4} \wedge \omega_6) + \cdots$$

dots stand for contributions of B_2

CY volume form

$$F_2\,|_{CY} = m^i \omega_{2i} = *(\widetilde{F}_{4i} \wedge \widetilde{\omega}_4^i) + \cdots$$

$$F_4\,|_{CY} = e_i \widetilde{\omega}_4^i = *(F_4^i \wedge \omega_{2i}) + \cdots$$

i=1,…,*n* dimension of $H^2(CY), H^4(CY)$

$$F_4\,|_{M_4} = *(e_0 \omega_6) + \cdots$$

$\omega_{2i}, \quad \widetilde{\omega}_4^i$ basis of closed forms

$$e_I = (e_0, e_i), \qquad m^I = (m^0, m^i)$$

2n+2 internal quantized constant fluxes

$$F_4^I = (F_4^0, F_4^i), \qquad \tilde{F}_{4\mathrm{I}} = (\tilde{F}_{40}, \tilde{F}_{4\mathrm{i}})$$

2n+2 F_4 field strengths on M_4

Relevant 4D scalar fields: $J_{(2)} = v^i(x)\,\omega_{2i}, \quad B_2 = b^i(x)\,\omega_{2i}, \quad \varphi^i = v^i + \mathrm{i} b^i$

4D on-shell relations: $*F_4^I = g^I(e, m, \varphi^i, \overline{\varphi}^i), \quad *\tilde{F}_{4\mathrm{I}} = \tilde{g}_I(e, m, \varphi^i, \overline{\varphi}^i)$

Effective N=1, D=4 SUGRA Lagrangian

(*F. Farakos, S. Lanza, L. Martucci, D.S.* '17)

The scalar moduli, the 4-form fluxes and their fermionic superpartners form special chiral superfields

$$S^I = \varphi^I + \theta^\alpha \psi^I_\alpha + \theta^2 f^I_S, \qquad I = 0, 1, \ldots, n = (0, i)$$

$$f^I_S = M^{IJ}(\varphi, \bar\varphi)\Big({}^*\tilde F_{4J} + G_{JK}(\varphi)\,{}^*F^K_4\Big) + \mathcal{O}(\psi)$$

4d Hodge-duals of 4-form fluxes

$$M^{IJ} = (\mathtt{Im}\, G_{IJ})^{-1}, \quad G_{IJ}(\varphi) = \partial_I \partial_J \mathcal{G}(\varphi)$$ - prepotential of special Kaehler space of scalar moduli

$$S^I \equiv \frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\Big(M^{IJ}(S)(\Sigma_J - \bar\Sigma_J)\Big), \qquad (\bar{\mathcal{D}}^2 - 8\mathcal{R})\Sigma_J = 0, \quad A_3 \in \Sigma$$

complex linear superfields

$$L = -3\int d^4\theta |S^0|^{\frac{2}{3}} e^{-\frac{1}{3}K(S^i, \bar S^j)}\, \mathtt{Ber}\, E^A_M(x, \theta, \bar\theta)$$

No superpotential

Conformal compensator

supervielbein of N=1, D=4 sugra

Dual (conventional) description in terms of usual chiral superfields

$$\Phi^I = \varphi^I + \theta^\alpha \psi^I_\alpha + \theta^2 F^I, \qquad I = 0, 1, \ldots, n = (0, i)$$

independent auxiliary fields

$$L = -3 \int d^4\theta |\Phi^0|^{\frac{2}{3}} e^{-\frac{1}{3}K(\Phi^i, \bar{\Phi}^j)} \, \mathtt{Ber}\, E^A_M(x,\theta,\bar{\theta}) + \int d^2\theta \, \underbrace{\left(e_I \Phi^i - m^I \partial_I \mathcal{G}(\Phi)\right)}_{W(\Phi)}$$

It is obtained by solving the equations of motion of the 3-form potentials in the higher-form formulation

$$*F_4^I = g^I(e, m, \varphi, \bar{\varphi}), \quad *\tilde{F}_{4I} = \tilde{g}_I(e, m, \varphi, \bar{\varphi})$$

Coupling to supermembranes

(I. Bandos, *F. Farakos, S. Lanza, L. Martucci, D.S*. '18)

Generalization of previous results of *Ovrut & Waldram '97, Huebscher, Meessen & T. Ortin '09, Bandos & Meliveo '10-'12, Buchbinder, Hutomo, Kuzenko & Tartaglino-Mazzucchelli '17*

Kappa-symmetry invariant action

$$S = S_{SG+matter} - 2\int d^3\xi \sqrt{-\det h_{pq}}\,|q_I S^I - p^I \partial_I \mathcal{G}(S)| + \int (q_I \mathcal{A}_3^I - p^I \tilde{\mathcal{A}}_{3I})$$

$\xi^p \quad (p = 0,1,2)$ - membrane worldvolume coordinates $q_I,\ p^J - \texttt{membrane charges}$

Kappa-symmetry variations:

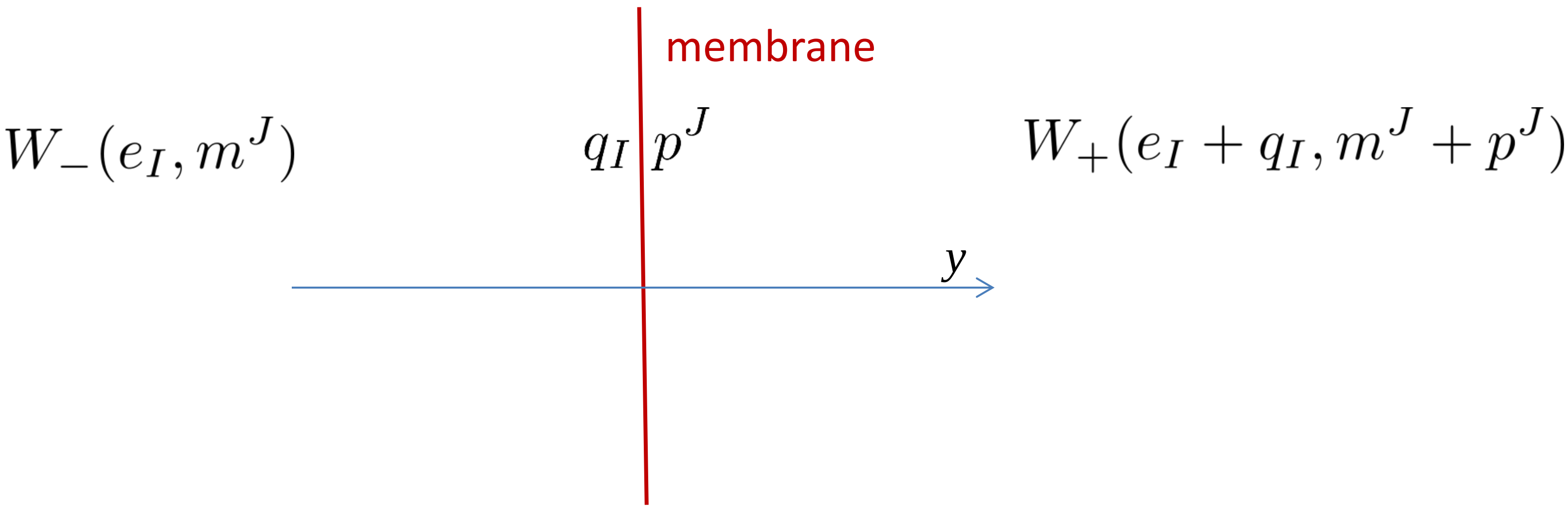
$$\delta z^M(\xi) = \kappa^\alpha(\xi) E_\alpha^M(z(\xi)) + \bar{\kappa}^{\dot\alpha}(\xi) E_{\dot\alpha}^M(z(\xi)), \qquad z^M = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot\alpha})$$

$$\kappa_\alpha = \frac{q_I \Phi^I - p^I \partial_I \mathcal{G}}{|q_I \Phi^I - p^I \partial_I \mathcal{G}|}\Gamma_{\alpha\dot\alpha}\bar{\kappa}^{\dot\alpha}, \quad \Gamma_{\alpha\dot\alpha} \equiv \frac{\mathrm{i}\epsilon^{pqr}}{3!\sqrt{-\det h}}\epsilon_{abcd}E_p^b E_q^c E_r^d\, \sigma^a_{\alpha\dot\alpha}$$

The action can be used for studying e.g. BPS domain walls associated with membranes separating different susy vacua, as well as the Brown-Teitelboim mechanism

½-BPS domain walls

(Cvetic et. al. '92-'96, Ovrut & Waldram '97, Ceresole et. al. '06, Huebscher, Meessen & Ortin '09 ….)



Flat static domain wall ansatz:

$$
\begin{aligned}
ds^2 &= e^{2D(y)} dx^\mu dx_\mu + dy^2 \\
\phi^i &= \phi^i(y) \\
A_3^I &= \alpha^I(y) dx^0 \wedge dx^1 \wedge dx^2 \\
\tilde{A}_{3J} &= \tilde{\alpha}_J(y) dx^0 \wedge dx^1 \wedge dx^2
\end{aligned}
$$

Flow equations
(consequences of susy preservation)

$$\partial_y \phi^i = 2K^{i\bar{j}}\, \partial_{\bar{j}} |\mathcal{Z}(\phi)|,$$

$$\partial_y D = -|\mathcal{Z}|$$

$$\mathcal{Z} = e^{\frac{1}{2}K(\phi,\bar{\phi})}\, [\Theta(-y) W_-(\phi) + \Theta(y) W_+(\phi)]$$

functions are continuous but not smooth

Simple example of an analytic solution generated by four 4-form fluxes $(F_4^I, \tilde{F}_{4J})$

(I. Bandos, F. Farakos, S. Lanza, L. Martuci & D.S. '18)

$$(I,J=0,1) \qquad W(\Phi)=(e_0+\mathrm{i}m^1)-\mathrm{i}(e_1+\mathrm{i}m^0)\Phi, \qquad K(\Phi,\bar\Phi)=-\log(4\,\mathrm{Im}\,\Phi)$$

Brane charges $\quad q_1=p^0=0, \quad q_0=ke_1, \quad p^1=km^0$

Susy AdS vacua

$$<\phi>_-=\mathrm{i}\frac{e_0-\mathrm{i}m^1}{e_1-\mathrm{i}m^0} \qquad \Big| \qquad <\phi>_+=\mathrm{i}\frac{e_0-\mathrm{i}m^1}{e_1-\mathrm{i}m^0}+ik$$

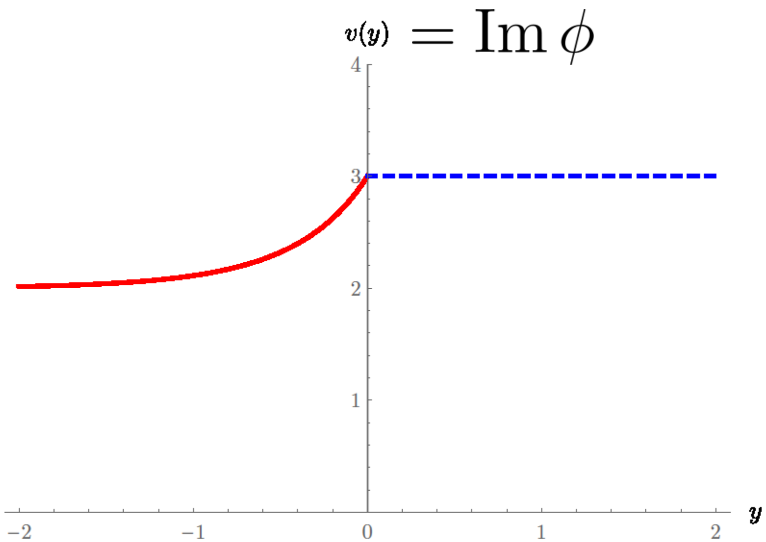
Interpolating domain wall solution of the flow equations:

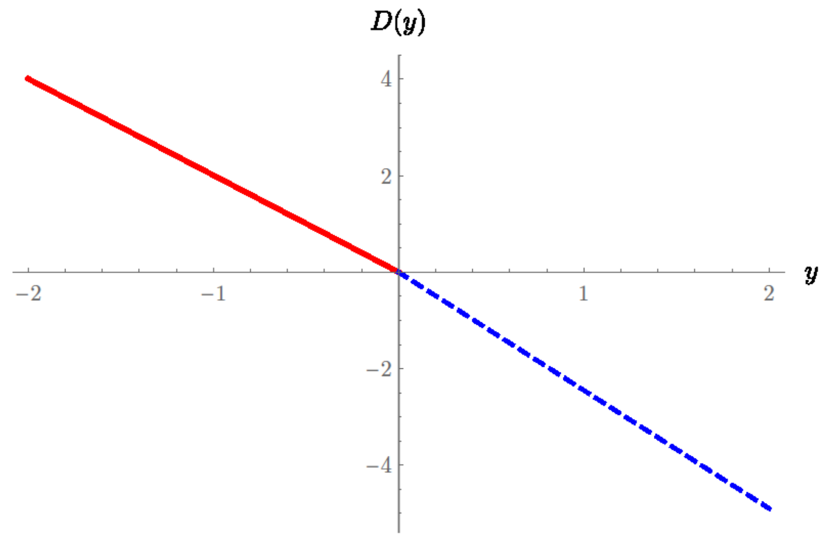
$$\mathrm{Im}\,\phi(y)=\begin{cases} <\mathrm{Im}\,\phi>_-\ \coth^2 u(y) & \text{for } y\leq 0\,,\\ <\mathrm{Im}\,\phi>_+ & \text{for } y\geq 0\,.\end{cases}$$

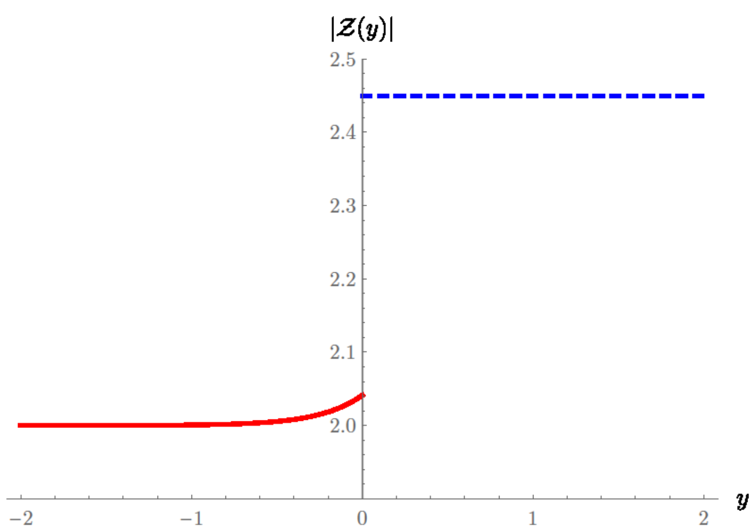
$$u(y)=\frac{1}{2}|W_-|\,(y+c)$$

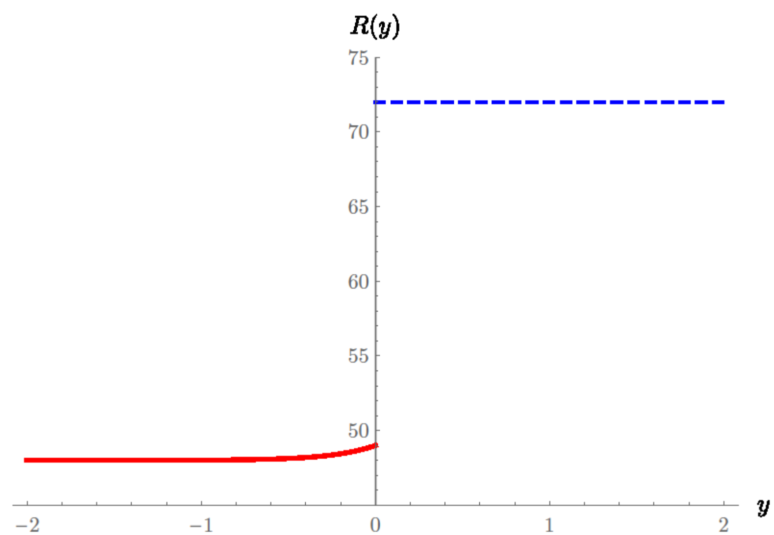
$$D(y)=\begin{cases} c'+e^{-\frac{1}{2}\hat{K}_0}\left(\log(-\sinh u)+\log\cosh u\right) & \text{for } y\leq 0\,,\\ -|W_+|\,y & \text{for } y\geq 0\,,\end{cases}$$

Profiles of the domain wall









Conclusiion

- Three-form gauge fields in D=4 have interesting physical implications:
  - Generation of a cosmological constant and its diminishing via nucleation of membrane bubbles
  - non-zero vevs of four-form fluxes trigger spontaneous symmetry breaking
- String Theory compactifications provide plenty of such fields in effective matter-coupled D=4 supergravities where they play the role of auxiliary fields of special chiral supermultiplets
- We have constructed manifestly supersymmetric action describing the effective N=1 4d theory of type IIA string compactifications with RR fluxes, coupled it to a kappa-symmetric supermembrane and find new analytic ½-BPS domain wall solutions in this system.

Outlook

- To extend this 3-form supergravity construction to phenomenologically more relevant compactification setups in type IIA and IIB theories which include
  - the NS-NS fluxes
  - D-branes (which are required for introducing a gauge field sector)
- To further study phenomenological and cosmological implications of these models within the lines of *Bousso & Polchinski ’00, Kaloper & Sorbo ’08,…, Bielleman, Ibanez & Valenzuela ’15, Carta, Marchesano, Staessens & Zoccarato ’16,…*