Exceptional field theory for affine algebras

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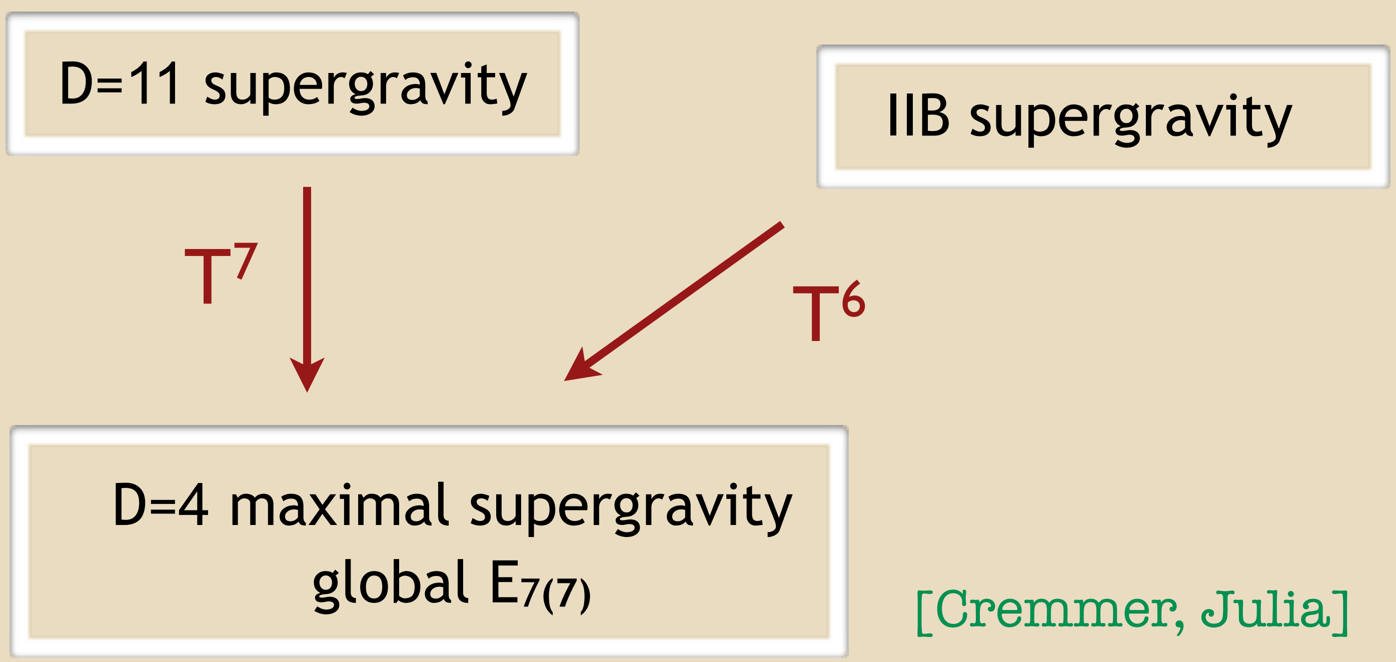
motivation

exceptional field theory (ExFT)

- upon toroidal reduction on T^d, eleven-dimensional supergravity exhibits the global exceptional symmetry group $E_{d(d)}$

  after proper dualisation/reorganisation of the fields



[Cremmer, Julia]

- ExFT: reformulate D=11 supergravity such that $E_{d(d)}$ (or its remnants) become manifest before dimensional reduction

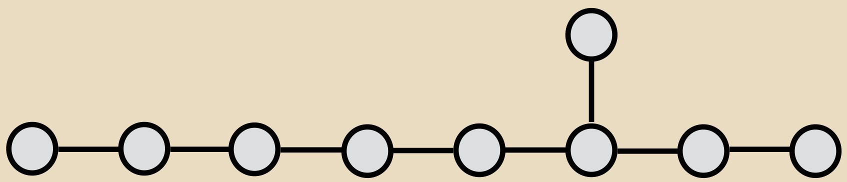
motivation

structure of exceptional field theory (ExFT)

- $E_{d(d)}$ generalized diffeomorphisms
- unique invariant two-derivative actions
- reproduce the bosonic sector of maximal supergravity
- powerful tool for construction of vacua and consistent truncations

- unlike DFT: based on a split external / internal coordinates $\{x^\mu,\, Y^M\}$ $\mu = 1, \ldots, D$
- constructed separately for every exceptional group
- so far: until $E_{8(8)}$ (symmetry of 3D sugra) — D=3 external dimensions

⟶ now : $E_{9(9)}$ (symmetry of 2D sugra) — D=2 external dimensions



infinite-dimensional algebra and representations
(integrable structure of 2D sugra)

plan

exceptional field theory

- generalized diffeomorphisms
- tensor hierarchy

affine symmetries in 2D supergravity

- 2D supergravity
- affine symmetries

exceptional field theory for the affine algebra E_9

- generalized diffeomorphisms & section constraints
- invariant actions

based on work with Olaf Hohm, Guillaume Bossard, Martin Cederwall, Axel Kleinschmidt, Jakob Palmkvist, Franz Ciceri, Gianluca Inverso

exceptional field theory (ExFT)

- generalized diffeomorphisms
- tensor hierarchy
- invariant actions

exceptional field theory (ExFT)

- generalized diffeomorphisms parameter ξ^M vector $V^M \in \mathcal{R}_1$ irrep of $\mathfrak{g}$

$$\mathcal{L}_\xi V^M = \xi^N \partial_N V^M + Z^{MN}{}_{PQ}\, \partial_N \xi^P V^Q$$

[Coimbra, Strickland-Constable, Waldram]
[Berman, Cederwall, Kleinschmidt, Thompson]

$$= \xi^N \partial_N V^M + \kappa\, (\mathbb{P}_{\rm adj})^N{}_P{}^M{}_Q\, (\partial_N \xi^P)\, V^Q + \beta\, \partial_N \xi^N\, V^M \qquad \beta = \frac{1}{D-2}$$

compatible with $E_{d(d)}$ structure (respects invariant tensors)

$$= \xi^N \partial_N V^M - V^N \partial_N \xi^M + Y^{MN}{}_{PQ}\, \partial_N \xi^P V^Q$$

closure of the algebra

— up to section condition $Y^{MN}{}_{PQ}\, \partial_M \otimes \partial_N = 0$

— $\exists$ trivial gauge parameters

— E-bracket (not associative ! Jacobiator ...)

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— up to section condition $Y^{MN}{}_{PQ}\, \partial_M \otimes \partial_N = 0$

— $\exists$ trivial gauge parameters

— E-bracket (not associative ! Jacobiator ...)

- infinite-dimensional (Y^M) local (x^μ) gauge structure of ExFT

covariant derivatives $\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$

- modified YM field strengths

$$\mathcal{F}_{\mu\nu}{}^M = 2\,\partial_{[\mu} A_{\nu]}{}^M - [A_\mu, A_\nu]_{\rm E}^M - Y^{MN}{}_{PQ}\, \partial_N B_{\mu\nu}{}^{PQ}$$

Stückelberg type coupling to 2-forms

exceptional field theory — tensor hierarchy

- generalized diffeomorphisms   parameter ξ^M vector $V^M \in \mathcal{R}_1$ irrep of $\mathfrak{g}$

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Stückelberg type coupling to 2-forms

- tensor hierarchy (schematically)

$$\mathcal{F}^{[p+1]} \;=\; D\mathcal{C}^{[p]} + \ldots + \mathcal{D}\mathcal{C}^{[p+1]} \qquad\qquad \mathcal{D} \;:\; \mathcal{R}_{p+1} \longrightarrow \mathcal{R}_p$$

similar to gauged supergravity, where $\mathcal{D}$ is the *embedding tensor*

(embedding of gauged supergravity by suitable Scherk-Schwarz ansatz)

precisely compatible with the supergravity field content

- tensor hierarchy (schematically)

$$\mathcal{F}^{[p+1]} = D\mathcal{C}^{[p]} + \ldots + \mathcal{D}\mathcal{C}^{[p+1]} \qquad \mathcal{D} : \mathcal{R}_{p+1} \longrightarrow \mathcal{R}_p$$

precisely compatible with the supergravity field content

- until (D−3) forms $\mathcal{C}_M \in \overline{\mathcal{R}_1}$ (dual to the vector fields)

whose field strength requires further correction

$$\mathcal{F}_M = D\mathcal{C}_M + \ldots + (t^\alpha)_M{}^N \partial_N \mathcal{C}_\alpha + \mathcal{B}_M$$

by a *covariantly constrained* (D−2) form $\mathcal{B}_M$ $\qquad Y^{MK}{}_{NL}\,\mathcal{B}_M \otimes \partial_K = 0$

related to symmetries from higher-dim dual graviton $\qquad Y^{MK}{}_{NL}\,\mathcal{B}_M\,\mathcal{B}_K = 0$

- D=4, $E_{7(7)}$FT : modified YM field strength (vectors dual to vectors)

$$\mathcal{F}_{\mu\nu}{}^M \equiv 2\partial_{[\mu}A_{\nu]}{}^M - 2\,[A_\mu, A_\nu]_{\mathrm{E}}^M - 12\,(t^\alpha)^{MN}\,\partial_N B_{\mu\nu\,\alpha} - \frac{1}{2}\,\Omega^{MN}\,\mathcal{B}_{\mu\nu\,N}$$

- D=3, $E_{8(8)}$FT : modified scalar currents (vectors dual to scalars)

$$D_\mu\mathcal{V}\,\mathcal{V}^{-1} \equiv \partial_\mu\mathcal{V}\,\mathcal{V}^{-1} - \mathcal{L}_{A_\mu}\mathcal{V}\,\mathcal{V}^{-1} + \mathcal{B}_{\mu\,M}\,T^M$$

in particular: standard algebra of generalized diffeomorphisms no longer closes

exceptional field theory — tensor hierarchy

- tensor hierarchy (schematically)

$$\mathcal{F}^{[p+1]} = D\mathcal{C}^{[p]} + \ldots + \mathcal{D}\mathcal{C}^{[p+1]} \qquad \mathcal{D} : \mathcal{R}_{p+1} \longrightarrow \mathcal{R}_p$$

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- D=3, $E_{8(8)}$FT : modified scalar currents

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in particular: standard algebra of generalized diffeomorphisms no longer closes

- D=2, $E_{9(9)}$FT : modified scalar field content! $\left\{ \widehat{\mathcal{V}},\ \chi_M \right\}$

together with additional vector fields $\left\{ A_\mu{}^{M},\ \mathcal{B}_{\mu\,M}{}^{N} \right\}$

again: standard algebra of generalized diffeomorphisms no longer closes

exceptional field theory — tensor hierarchy

- invariant actions

e.g. $E_{7(7)}$ ExFT with D=4 $\qquad \mathcal{M}_{MN} \in \mathrm{E}_{7(7)}/\mathrm{SU}(8)$

$$\mathcal{L} = \widehat{R} + \frac{1}{24}\,\mathcal{D}_\mu \mathcal{M}_{MN}\,\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,\mathcal{F}_{\mu\nu}{}^M\,\mathcal{F}^{\mu\nu\,N} + \mathcal{L}_{\rm top} - V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})$$

with "potential" (invariant under generalised diffeomorphisms)

$$V = -\frac{1}{48}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\,\partial_N\mathcal{M}_{KL} + \frac{1}{2}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\partial_L\mathcal{M}_{NK} - \frac{1}{2}g^{-1}\partial_M g\,\partial_N\mathcal{M}^{MN} - \frac{1}{4}\mathcal{M}^{MN}g^{-1}\partial_M g\,g^{-1}\partial_N g - \frac{1}{4}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\partial_N g_{\mu\nu}\,.$$

- unique action with generalized diffeomorphism invariance (modulo section condition)

- upon explicit solution of the section condition (breaking $E_{7(7)}$) the theory coincides with the full D=11 / IIB supergravity

exceptional field theory — tensor hierarchy

- invariant actions

e.g. $E_{7(7)}$ ExFT with D=4

$$\mathcal{L} \;=\; \widehat{R} + \frac{1}{24}\,\mathcal{D}_\mu \mathcal{M}_{MN}\,\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,\mathcal{F}_{\mu\nu}{}^{M}\,\mathcal{F}^{\mu\nu\,N} + \mathcal{L}_{\rm top} \;-\; V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})$$

with "potential" (invariant under generalised diffeomorphisms)

$$V \;=\; -\frac{1}{48}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\,\partial_N\mathcal{M}_{KL} + \frac{1}{2}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\partial_L\mathcal{M}_{NK} - \frac{1}{2}g^{-1}\partial_M g\,\partial_N\mathcal{M}^{MN} - \frac{1}{4}\mathcal{M}^{MN}g^{-1}\partial_M g\,g^{-1}\partial_N g - \frac{1}{4}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\partial_N g_{\mu\nu}\,.$$

- consistent truncations

  - $\partial_M \;\longrightarrow\; 0$: Cremmer-Julia theory, D=4 $E_{7(7)}$
  - $\mathcal{M}_{MN} \;=\; U_M{}^A(Y)U_N{}^B(Y)\,M_{AB}(x)$:
    Scherk-Schwarz reduction to gauged supergravity

affine symmetries in D=2 supergravity

- D=2 supergravity
- affine symmetries
- vector fields

affine symmetries in D=2 supergravity

- 2D Lagrangian (dimensional reduction of D=11 supergravity)

$$\mathcal{L} \;=\; -\tfrac{1}{4}\sqrt{-g}\,\rho\,\Big(-R+\mathrm{tr}[P^\mu P_\mu]\Big) \;+\; \mathcal{L}_{\rm ferm}(\psi^I,\psi^I_2,\chi^{\dot A})$$

coset space sigma model coupled to dilaton gravity $\quad \mathcal{V}^{-1}\partial_\mu\mathcal{V} = Q_\mu + P_\mu \;\in\; \mathfrak{e}_{8(8)}$, with $Q_\mu \in \mathfrak{so}(16)$

off-shell symmetry (target space isometries): $E_{8(8)}$

- symmetries

  has a remarkable structure : $\quad \partial_\mu\tilde\rho \;=\; \varepsilon_{\mu\nu}\,\partial^\nu\rho \quad$ dual dilaton

  (infinite tower of) dual scalar potentials $\quad \partial_\mu Y_1 \;=\; \varepsilon_{\mu\nu}\,J^\nu_{\rm Noeth} \quad$ dual scalars

  ⟶ classical integrability, affine Lie-Poisson symmetry E_9

▷ realised as a coset action $\quad \mathrm{SL}(2)\ltimes \mathrm{E}_{9(9)}\big/\mathrm{SO}(1,1)\ltimes K(\mathrm{E}_9)$

$$\hat{\mathcal{V}} \;=\; \ldots e^{Y_3 w^{-3}} e^{Y_2 w^{-2}} e^{Y_1 w^{-1}}\mathcal{V}\;\; e^{\tilde\rho\, L_{-1}}\,\rho^{L_0}$$

Virasoro $\quad L_{-1}\,\tilde\rho = 1$

central extension $\quad k\,\sigma = 1 \quad$ [Julia]

affine symmetries in 2D supergravity

- vector fields $A_\mu{}^M \in \mathcal{R}_1$

(non-propagating in D=2)
restore by embedding known examples: level 1 (basic) representation of **E₉**

L₀ grading

$$1$$
$$248$$
$$1 + 248 + 3875$$
$$1 + 2\cdot 248 + 3875 + 30380$$
$$2\cdot 1 + 3\cdot 248 + 2\cdot 3875 + 30380 + 27000 + 147250$$
$$\vdots$$

$L_0^{\mathrm{SL}(9)}$ grading

$$9$$
$$36'$$
$$126$$
$$\vdots$$

$$\chi_{\omega 0} = 1 + 248q + 4124q^2 + 34752q^3 + 213126q^4 + 1057504q^5 + 4530744q^6 + \ldots$$

- $\longrightarrow$ ExFT coordinates $Y^M \in \mathcal{R}_1$ $\qquad Y^M \longrightarrow \{x^i, y_{ij}, y_{ijklm}, \ldots\}$

construct the associated generalised diffeomorphisms

exceptional field theory for the affine algebra E_9

- section constraints
- generalized diffeomorphisms

exceptional field theory for the affine algebra E_9

- section constraints $\quad Y^{MN}{}_{PQ}\,\partial_M \otimes \partial_N = 0 \quad ??$

lives in the tensor product $\mathcal{R}_1 \otimes \mathcal{R}_1$: linear combination of level 2 representations $\mathcal{R}(2\Lambda_0),\ \mathcal{R}(\Lambda_7),\ \mathcal{R}(\Lambda_1)$

$$\mathcal{R}_1 \otimes \mathcal{R}_1 \;=\; (1+q^2+q^3+\ldots)\,\mathcal{R}(2\Lambda_0) \;\oplus\; (q^2+q^3+q^4+\ldots)\,\mathcal{R}(\Lambda_7) \;\oplus\; (q+q^2+q^3+\ldots)\,\mathcal{R}(\Lambda_1)$$

where the multiplicities carry representations of the $c=\frac{1}{2}$ coset CFT $\quad \dfrac{(E_9)_1 \oplus (E_9)_1}{(E_9)_2}$

[Goddard,Olive]

specifically

$$\mathcal{R}_1 \otimes \mathcal{R}_1 \;=\; \chi_{(1,1)} \otimes \mathcal{R}(2\Lambda_0) \;\oplus\; \chi_{(2,1)} \otimes \mathcal{R}(\Lambda_7) \;\oplus\; \chi_{(2,2)} \otimes \mathcal{R}(\Lambda_1)$$

rescaled coset Virasoro generators $\quad C_n \;\equiv\; 32\,L_n^{\rm coset} \;=\; 32\left(\mathbb{I}\otimes L_n^{(1)} + L_n^{(1)}\otimes\mathbb{I} - L_n^{(2)}\right)$

exceptional field theory for the affine algebra E_9

$$\mathcal{R}_1 \otimes \mathcal{R}_1 = \chi_{(1,1)} \otimes \mathcal{R}(2\Lambda_0) \oplus \chi_{(2,1)} \otimes \mathcal{R}(\Lambda_7) \oplus \chi_{(2,2)} \otimes \mathcal{R}(\Lambda_1)$$

rescaled coset Virasoro generators $C_n \equiv 32\, L_n^{\text{coset}} = 32 \left(\mathbb{I} \otimes L_n^{(1)} + L_n^{(1)} \otimes \mathbb{I} - L_n^{(2)} \right)$

- section constraints

$$\begin{aligned} \left(\langle \partial_1 | \otimes \langle \partial_2 | + \langle \partial_2 | \otimes \langle \partial_1 | \right) C_1 &= 0 \\ \left(\langle \partial_1 | \otimes \langle \partial_2 | \right) \left(C_0 - \mathbb{I} + \sigma \right) &= 0 \\ \forall n > 0 \ : \quad \left(\langle \partial_1 | \otimes \langle \partial_2 | \right) C_{-n} &= 0 \end{aligned}$$

$\Longleftrightarrow$ $\partial_{(M} \otimes \partial_{N)}\, C_1{}^{MN}{}_{PQ} = 0$

etc.

- reproduce correct higher-dimensional constraints & solutions
- sufficient in order to define consistent generalised diffeomorphisms

exceptional field theory for the affine algebra E9

- generalised diffeomorphisms full affine algebra $\left\{ T^{\mathcal{A}} \right\} = \left\{ T_{\alpha,n}, L_0, K \right\}$

$$\mathcal{L}_\xi V^M = \xi^N \partial_N V^M + \kappa\, (\mathbb{P}_{\rm adj})^N{}_P{}^M{}_Q\, (\partial_N \xi^P)\, V^Q + \beta\, \partial_N \xi^N\, V^M$$

$$= \xi^N \partial_N V^M - \underbrace{\eta_{\mathcal{A}\mathcal{B}}\, (T^{\mathcal{A}})_P{}^N\, (T^{\mathcal{B}})_Q{}^M}_{(C_0)^{NM}{}_{PQ}}\, (\partial_N \xi^P)\, V^Q - \partial_N \xi^N\, V^M$$

$$(C_0)^{NM}{}_{PQ} = 32 \left(\mathbb{I} \otimes L_0^{(1)} + L_0^{(1)} \otimes \mathbb{I} - L_0^{(2)} \right)^{NM}{}_{PQ}$$

index-free notation

$$\mathcal{L}_\xi |\overset{1}{V}\rangle = \langle \overset{2}{\partial}_V | \overset{2}{\xi} \rangle \otimes |\overset{1}{V}\rangle + \langle \overset{2}{\partial}_\xi | (\overset{12}{C_0} - \mathbb{I}) | \overset{2}{\xi} \rangle \otimes |\overset{1}{V}\rangle$$

- as expected: do not close into an algebra (but come close...)
- additional symmetry — weight compensating internal derivative

$$\mathcal{L}_\Sigma V^M = C_{-1}{}^{NM}{}_{PQ}\, \Sigma_N{}^P\, V^Q$$

with covariantly constrained parameter $\Sigma_N{}^P$

exceptional field theory for the affine algebra E₉

- generalised diffeomorphisms full affine algebra $\left\{ T^{\mathcal{A}} \right\} = \left\{ T_{\alpha,n}, L_0, K \right\}$

$$\mathcal{L}_\xi V^M = \xi^N \partial_N V^M + \kappa\, (\mathbb{P}_{\rm adj})^N{}_P{}^M{}_Q\, (\partial_N \xi^P)\, V^Q + \beta\, \partial_N \xi^N\, V^M$$

$$= \xi^N \partial_N V^M - \underbrace{\eta_{\mathcal{AB}}\, (T^{\mathcal{A}})_P{}^N\, (T^{\mathcal{B}})_Q{}^M}_{(C_0)^{NM}{}_{PQ}}\, (\partial_N \xi^P)\, V^Q - \partial_N \xi^N\, V^M$$

$$(C_0)^{NM}{}_{PQ} = 32 \left(\mathbb{I} \otimes L_0^{(1)} + L_0^{(1)} \otimes \mathbb{I} - L_0^{(2)} \right)^{NM}{}_{PQ}$$

index-free notation

$$\mathcal{L}_\xi |\overset{1}{V}\rangle = \langle \overset{2}{\partial}_V | \overset{2}{\xi} \rangle \otimes |\overset{1}{V}\rangle + \langle \overset{2}{\partial}_\xi | (\overset{12}{C}_0 - \mathbb{I}) | \overset{2}{\xi} \rangle \otimes |\overset{1}{V}\rangle + \langle \overset{2}{\pi}_\Sigma | \overset{12}{C}_{-1} | \overset{2}{\Sigma} \rangle \otimes |\overset{1}{V}\rangle$$

- additional symmetry

$$\mathcal{L}_\Sigma V^M = C_{-1}{}^{NM}{}_{PQ}\, \Sigma_N{}^P\, V^Q$$

with covariantly constrained parameter $\Sigma_N{}^P \sim |\Sigma\rangle\langle \pi_\Sigma|$

- close into an algebra (modulo section constraints)

- generalised diffeomorphisms

$$\mathcal{L}_\xi \overset{1}{|V\rangle} = \langle \overset{2}{\partial}_V | \overset{2}{\xi}\rangle \otimes \overset{1}{|V\rangle} + \langle \overset{2}{\partial}_\xi | (\overset{12}{C}_0 - \mathbb{I}) \overset{2}{|\xi\rangle} \otimes \overset{1}{|V\rangle} + \langle \overset{2}{\pi}_\Sigma | \overset{12}{C}_{-1} \overset{2}{|\Sigma\rangle} \otimes \overset{1}{|V\rangle}$$

- algebra closes $\left[\mathscr{L}_{\xi_1,\Sigma_1}, \mathscr{L}_{\xi_2,\Sigma_2}\right] = \mathscr{L}_{\xi_{12},\Sigma_{12}}$

$$\xi_{12} \equiv [\![\xi_1, \xi_2]\!] \equiv \frac{1}{2}\left(\mathscr{L}_{\xi_1}\xi_2 - \mathscr{L}_{\xi_2}\xi_1\right)$$

$$|\Sigma_{12}\rangle\langle\pi_{\Sigma_{12}}| \equiv \mathscr{L}_{\xi_1}\left(|\Sigma_2\rangle\langle\pi_{\Sigma_2}|\right) + \frac{1}{2}\langle\pi_{\Sigma_1}|C_{-1}|\Sigma_2\rangle \otimes |\Sigma_1\rangle\langle\pi_{\Sigma_2}| + \frac{1}{4}\langle\partial_{\xi_2}|C_1|\big(|\xi_2\rangle\otimes|\xi_1\rangle - |\xi_1\rangle\otimes|\xi_2\rangle\big)\langle\partial_{\xi_2}| - (1\leftrightarrow 2)$$

modulo section constraints, non-associative, Jacobiator

- the computation is remarkably simple !

only relies on commutation relations of the coset Virasoro generators

$$\left[\overset{13}{C}_m, \overset{23}{C}_n\right] = \frac{m-n}{2}\left(\overset{13}{C}_{m+n} + \overset{23}{C}_{m+n} - \overset{12}{C}_{m+n}\right) + \frac{2}{3}m(m^2-1)\,\delta_{m+n,0} + \overset{123}{C}_{m+n}$$

not on any details on the $E_{8(8)}$ structure constants…

exceptional field theory for the affine algebra E9

- generalised diffeomorphisms

$$\mathcal{L}_\xi |\overset{1}{V}\rangle = \langle \overset{2}{\partial}_V | \overset{2}{\xi}\rangle \otimes |\overset{1}{V}\rangle + \langle \overset{2}{\partial}_\xi | (\overset{12}{C}_0 - \mathbb{I}) | \overset{2}{\xi}\rangle \otimes |\overset{1}{V}\rangle + \langle \overset{2}{\pi}_\Sigma | \overset{12}{C}_{-1} | \overset{2}{\Sigma}\rangle \otimes |\overset{1}{V}\rangle$$

- additional symmetry

  with a covariantly constrained gauge parameter $\Sigma_N{}^P \sim |\Sigma\rangle\langle\pi_\Sigma|$

$$\langle \overset{2}{\pi}_\Sigma | \overset{12}{C}_{-1} | \overset{2}{\Sigma}\rangle \otimes |\overset{1}{V}\rangle \sim \eta^{(-1)}_{\mathcal{AB}} (T^{\mathcal{A}})_P{}^N (T^{\mathcal{B}})_Q{}^M \Sigma_N{}^P V^Q$$

  with shifted Cartan-Killing form and sum over the extended algebra $\left\{ T^{\mathcal{A}} \right\} = \left\{ T_{\alpha,n},\, L_{-1},\, K \right\}$

  - brings in additional generator L_{-1}
  - upon Scherk-Schwarz reduction: matches gauge structure of 2D sugra!

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exceptional field theory for the affine algebra E_9

- invariant action

exceptional field theory for the affine algebra E_9

- recall: invariant actions

e.g. $E_{7(7)}$ ExFT with D=4

$$\mathcal{L} = \widehat{R} + \frac{1}{24}\mathcal{D}_\mu \mathcal{M}_{MN}\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\mathcal{M}_{MN}\mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu N} + \mathcal{L}_{\rm top} - V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})$$

with 'potential' (invariant under generalised diffeomorphisms) for $\mathcal{M} = \mathcal{V}\mathcal{V}^T$

$$V = -\frac{1}{48}\mathcal{M}^{MN}\partial_M \mathcal{M}^{KL}\partial_N \mathcal{M}_{KL} + \frac{1}{2}\mathcal{M}^{MN}\partial_M \mathcal{M}^{KL}\partial_L \mathcal{M}_{NK} - \frac{1}{2}g^{-1}\partial_M g\,\partial_N \mathcal{M}^{MN} - \frac{1}{4}\mathcal{M}^{MN}g^{-1}\partial_M g\,g^{-1}\partial_N g - \frac{1}{4}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\partial_N g_{\mu\nu}\,.$$

- main challenge: construct the invariant 'potential' for $E_{9(9)}$

building blocks: $(J_M)^K{}_L = \mathcal{M}_{LP}\,\partial_M \mathcal{M}^{PK}$

in a Borel gauge, such that at given level, $\mathcal{M}^{MN}$ has only finitely many entries

exceptional field theory for the affine algebra E9

- main challenge: construct the invariant 'potential' for E9(9)

building blocks: $(J_M)^K{}_L = \mathcal{M}_{LP}\,\partial_M \mathcal{M}^{PK}$

expand on E9(9), (for simplicity truncate to $\tilde\rho = 0$)

$$J_M\Big|_{\tilde\rho=0} = J_M{}^{\alpha\, m}\, T_{\alpha\, m} + J_M{}^0\, L_0 + J_M{}^{(K)}\, K = J_M{}^{\mathcal{A}}\, T_{\mathcal{A}}$$

- generic contribution $V_1 = \rho^{-1}\, \mathcal{M}^{MN}\, J_M{}^{\mathcal{A}}\, J_M{}^{\mathcal{B}}\, \eta_{\mathcal{A}\mathcal{B}}$
- generic contribution $V_2 = \rho^{-1}\, \mathcal{M}^{MN}\, (J_M)_K{}^L\, (J_L)_N{}^K$
- specific contribution $V_3 = \rho\, \mathcal{M}^{MN}\, (\mathbb{J}_{-1,K})_M{}^L\, (\mathbb{J}_{-1,L})_N{}^K$

in terms of the shifted current

$$(\mathbb{J}_{-1})_M\Big|_{\tilde\rho=0} = J_M{}^{\alpha\, m}\, T_{\alpha, m-1} + J_M{}^0\, L_{-1} + \chi_M\, K$$

bringing in the extra scalar fields χ_M

exceptional field theory for the affine algebra E_9

- final step: restore $\tilde{\rho}$ dependence

$$J_M\Big|_{\tilde{\rho}=0} = J_M{}^{\alpha\, m}\, T_{\alpha\, m} + J_M{}^0\, L_0 + J_M{}^{(K)}\, K \;+\; J_M{}^+\, L_{+1} + J_M{}^-\, L_{-1}$$

  - generic contribution

$$V_1 = \rho^{-1}\, \mathcal{M}^{MN}\, J_M{}^{\mathcal{A}}\, J_M{}^{\mathcal{B}}\, \eta_{\mathcal{A}\mathcal{B}}$$

there is no longer a bilinear invariant on $\mathrm{SL}(2) \ltimes \mathrm{E}_{9(9)}$

exceptional field theory for the affine algebra E9

- final step: restore $\tilde\rho$ dependence

$$J_M\Big|_{\tilde\rho=0} = J_M{}^{\alpha\,m}\,T_{\alpha\,m} + J_M{}^0\,L_0 + J_M{}^{(K)}\,K \;+\; J_M{}^+\,L_{+1} + J_M{}^-\,L_{-1}$$

- complete contribution

$$V_1 \;=\; \rho^{-1}\,\mathcal{M}^{MN}\,J_M{}^{\mathcal{A}}\,J_M{}^{\mathcal{B}}\,\eta_{\mathcal{A}\mathcal{B}} \;+\; \mathcal{M}^{MN}\left(J_M{}^+\,\chi_{+,N} + J_M{}^-\,\chi_{-,N}\right)$$

with the $\check\chi_{\pm,M}$ related to the above χ_M as

$$\chi_{+,M} \;=\; \chi_M$$
$$\chi_{-,M} \;=\; \rho^{-2}\,\chi_M - \omega_-(\mathcal{M})_{\mathcal{A}}\,J_M{}^{\mathcal{A}}$$

- final result $\quad V\left(\mathcal{M}^{MN},\,\chi_M\right) \;=\; \widehat{V}_1 + \widehat{V}_2 + \widehat{V}_3 + \ldots$

(upon completing into $\tilde\rho$ power series and adding standard terms …)

invariant under generalized diffeomorphisms (up to total derivatives)

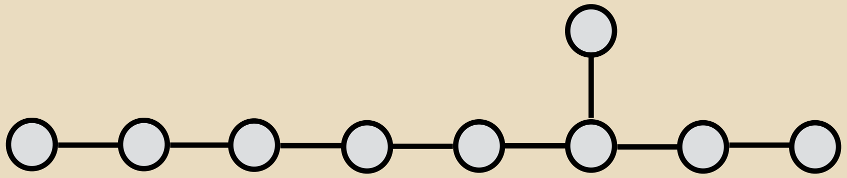
conclusions

exceptional field theory

- unique theory with generalized diffeomorphism invariance in all coordinates (modulo section condition)
- exceptional group structure manifest
- universal formulation of IIA and IIB supergravity
- powerful tool for construction of vacua and consistent truncations

exceptional field theory for affine algebras

- infinite-dimensional algebra and representations
- algebra of generalized diffeomorphisms with additional gauge symmetry
- dynamics: invariant action — potential



outlook

- **complete the E_9 construction:**
  - external part of the action
  - duality equations for scalar fields ⟶ linear system
  - remnants of the integrable structure
- **supersymmetry**
  - representation theory of $K(E_9)$
  - potential as an internal curvature scalar
  - predictions for D=2 supergravity
- **extend to the higher-rank algebras:**
  - (E_{10}, E_{11}, Borcherds, tensor hierarchy, ...)
  - relate to [Damour,Henneaux,Nicolai][West]
- **understand / weaken / relax the section constraints**
- **tool for analyzing existing theories or hints towards a more fundamental structure ..?**