Ramsey Classes by Partite Construction I

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Ramsey classes

We consider relational structures in language L without function symbols.

Definition

A class $\mathcal{C}$ (of finite relational structures) is Ramsey iff

$$\forall_{\mathbf{A},\mathbf{B}\in\mathcal{C}}\exists_{\mathbf{C}\in\mathcal{C}} : \mathbf{C} \longrightarrow (\mathbf{B})^{\mathbf{A}}_2.$$

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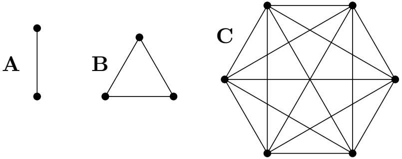

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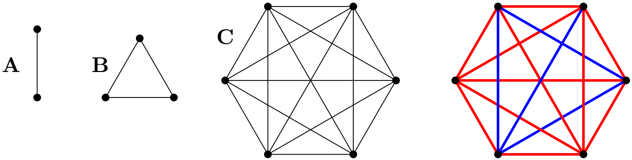

Examples of Ramsey classes

Example

The class of all finite linear orders is Ramsey.

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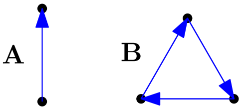
Example (Non-example)

The class of all directed graphs is not Ramsey.



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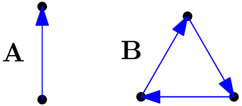
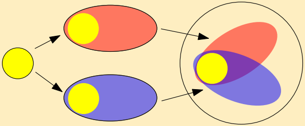


Given **C** consider arbitrary linear order. Color edges red if they go forward in the linear order and blue otherwise.

Ramsey classes are amalgamation classes

Definition (Amalgamation property of class $\mathcal{K}$)



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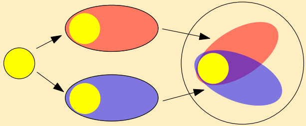
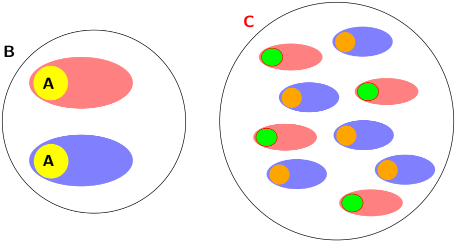


Nešetřil, 1989: Under mild assumptions Ramsey classes have amalgamation property.



Classification programme

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Ramsey classes	$\Longrightarrow$	amalgamation classes
$\Uparrow$		$\Downarrow$
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Can we always find a Ramsey lift?

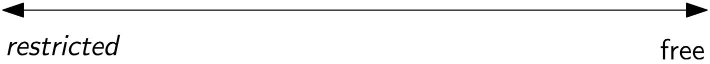
Theorem (Nešetřil, 1989)

All homogeneous graphs have Ramsey lift.

Theorem (Jasiński,Laflamme,Nguyen Van Thé,Woodrow, 2014)

All homogeneous digraphs have Ramsey lift.

Map of Ramsey Classes



Map of Ramsey Classes

interval graphs

permutations

cyclic orders

unions of complete graphs

linear orders

restricted ⟷ free

Nešetřil-Rödl Theorem

A structure **A** is called complete (or irreducible) if every pair of distinct vertices belong to a relation of **A**.

$\mathrm{Forb}_E(\mathcal{E})$ is a class of all finite structures **A** such that there is no embedding from $\mathbf{E} \in \mathcal{E}$ to **A**.

Theorem (Nešetřil-Rödl Theorem, 1977)

- *Let L be a finite relational language.*
- *Let $\mathcal{E}$ be a set of complete ordered L-structures.*
- *The then class $\mathrm{Forb}_E(\mathcal{E})$ is a Ramsey class.*

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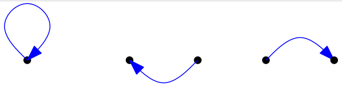
Explicitly: For every $\mathbf{A}, \mathbf{B} \in \mathrm{Forb}_E(\mathcal{E})$ there is $\mathbf{C} \in \mathrm{Forb}_E(\mathcal{E})$ such that $\mathbf{C} \longrightarrow (\mathbf{B})^{\mathbf{A}}_2$.

Examples of Ramsey lifts

Example

Graphs with order are Ramsey.



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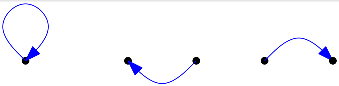

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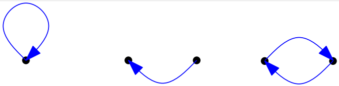
Acyclic graphs with linear extension are Ramsey.



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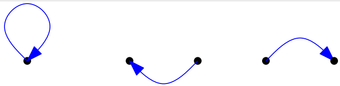

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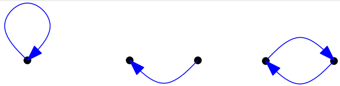
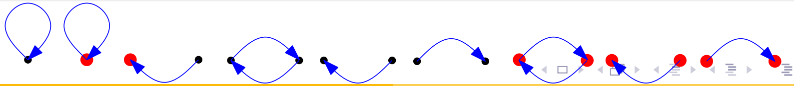


Example

Bipartite graphs with unary relation identifying bipartition and with convex linear order are Ramsey.



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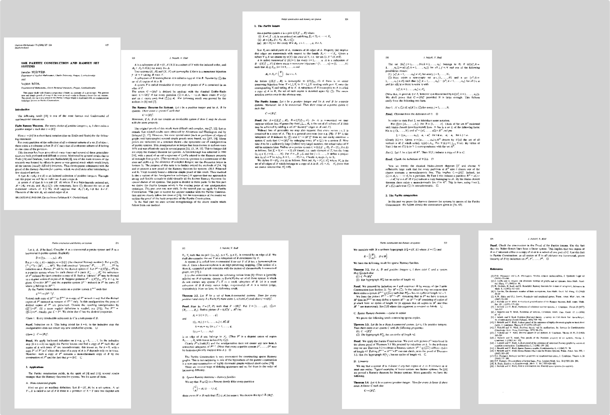

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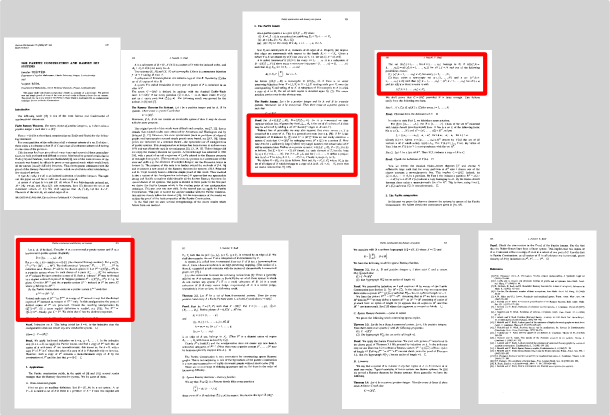


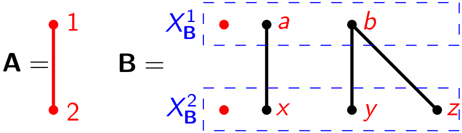
The Partite Construction

Definition (**A**-partite system)

Let **A** be an ordered relational structure on vertices $\{1, 2, \ldots a\}$.

An **A**-partite system is a tuple $(\mathbf{A}, \mathcal{X}_\mathbf{B}, \mathbf{B})$ where **B** is structure and $\mathcal{X}_\mathbf{B} = \{X^1_\mathbf{B}, X^2_\mathbf{B}, \ldots, X^a_\mathbf{B}\}$ partitions vertex set of **B** into a classes ($X^i_\mathbf{B}$ are called *parts* of **B**) such that:

1. ordering satisfies $X^1_\mathbf{B} < X^2_\mathbf{B} < \ldots < X^a_\mathbf{B}$;
2. mapping (projection) π which maps every $x \in X^i_\mathbf{B}$ to i ($i = 1, 2, \ldots, a$) is a homomorphism;
3. every tuple in every relation of **B** meets every class $X^i_\mathbf{B}$ in at most one element.



The Partite Construction

$\mathbf{A} =$  $\mathbf{B} =$  $\mathbf{C} = ?$

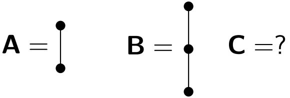
Construction outline:

- Put n such

$$n \longrightarrow (|\mathbf{B}|)^{|\mathbf{A}|}_2.$$

  (For every coloring of $|A|$ tuples in $\{1, 2, \ldots n\}$ there exists monochromatic subset of size $|\mathbf{B}|$). Here $n = 6$.

The Partite Construction



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- Picture 0: $|\mathbf{K}|_n$-partite system $\mathbf{P}_0$ s.t. for every coloring of copies of A in $\mathbf{P}_0$ where the color of a copy $\widetilde{\mathbf{A}}$ depends only on a projection $\pi(\widetilde{\mathbf{A}})$ there exists a monochromatic copy of $\mathbf{B}$.

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- Enumerate by $\mathbf{A}_1, \ldots \mathbf{A}_N$ all possible projections of copies of **A** in $\mathbf{P}_0$.

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- Enumerate by $\mathbf{A}_1, \ldots \mathbf{A}_N$ all possible projections of copies of $\mathbf{A}$ in $\mathbf{P}_0$.
- Pictures 1…n: $\mathbf{K}_n$-partite systems $\mathbf{P}_1, \ldots \mathbf{P}_N$ s.t. for every coloring of copies of $\mathbf{A}$ in $\mathbf{P}_i$ there exists a copy of $\mathbf{P}_{i-1}$ where all copies of $\mathbf{A}$ with projection $\mathbf{A}_i$ are monochromatic.

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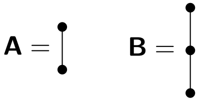

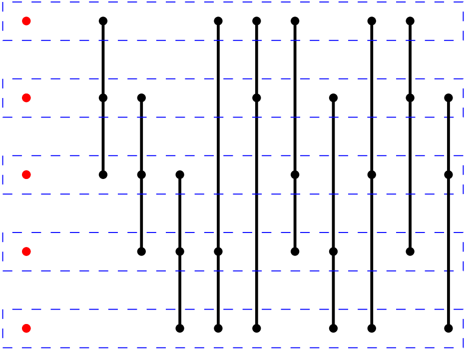

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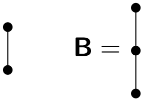
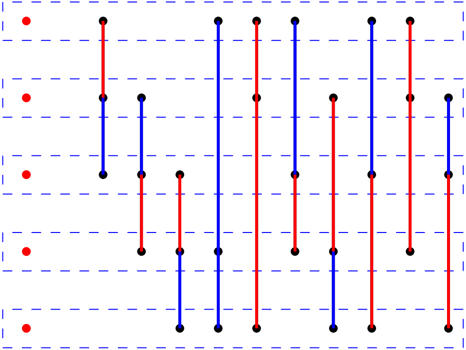
$\mathbf{A} =$  $\mathbf{B} =$



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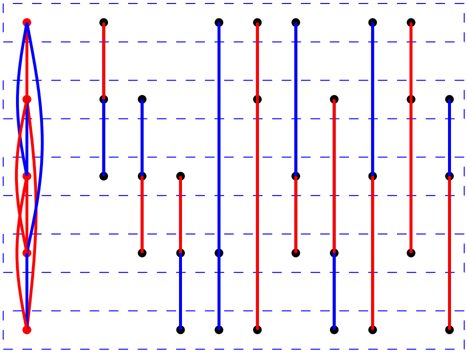

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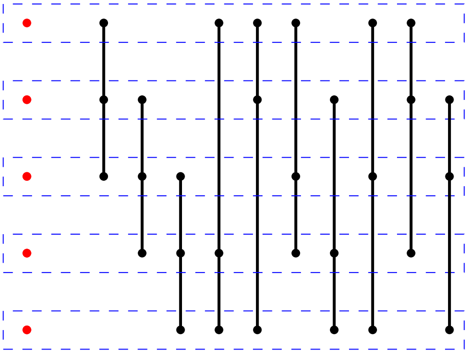

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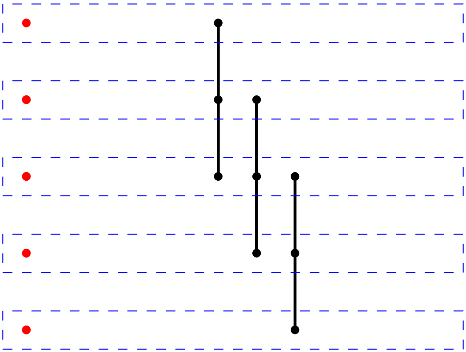

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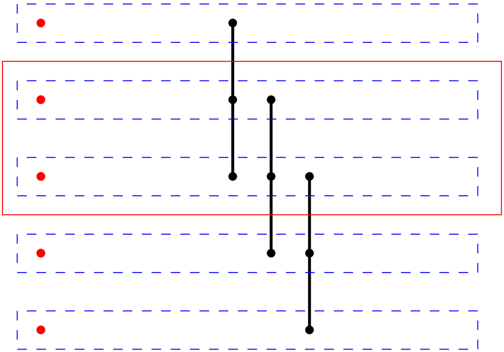

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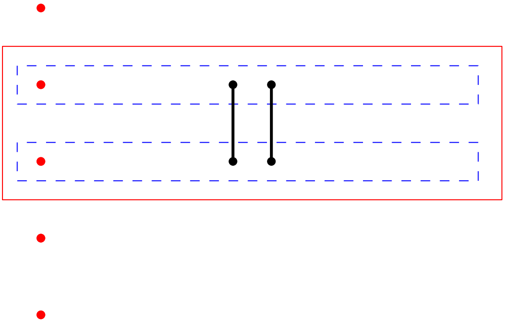

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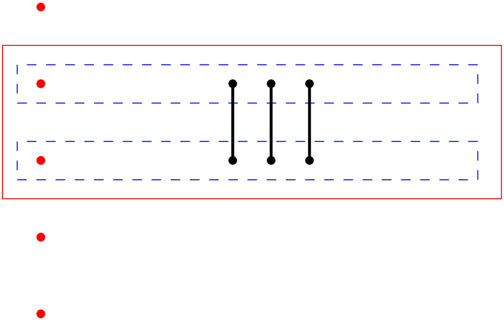

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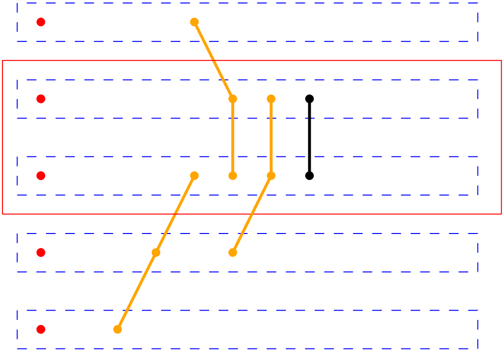

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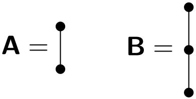

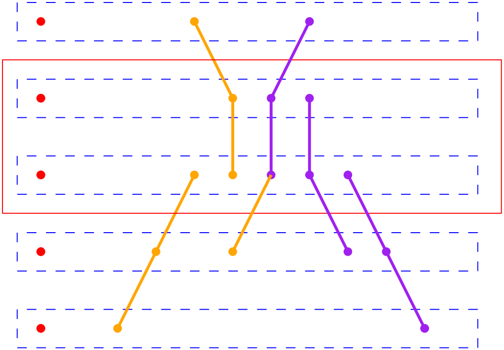

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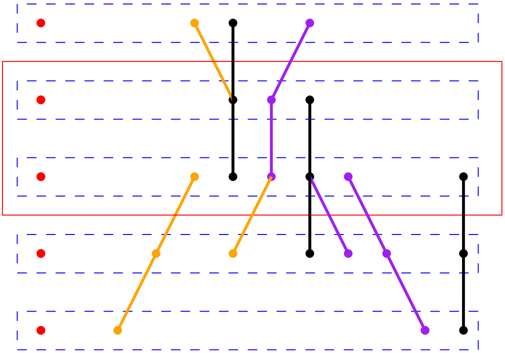

The Partite Construction: Picture 2

Picture 2: $\mathbf{K}_n$-partite system $\mathbf{P}_2$ s.t. for every coloring of copies of $\mathbf{A}$ in $\mathbf{P}_2$ there exists a copy of $\mathbf{P}_1$ where all copies of $\mathbf{A}$ with projection $\mathbf{A}_2$ are monochromatic.

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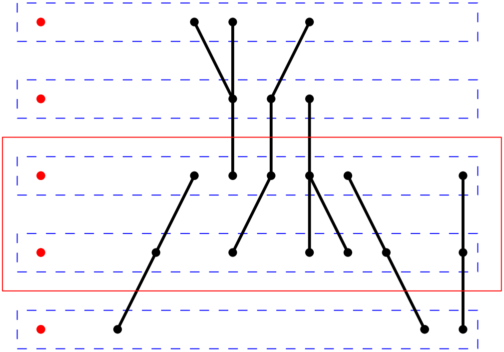

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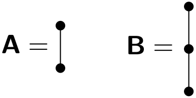

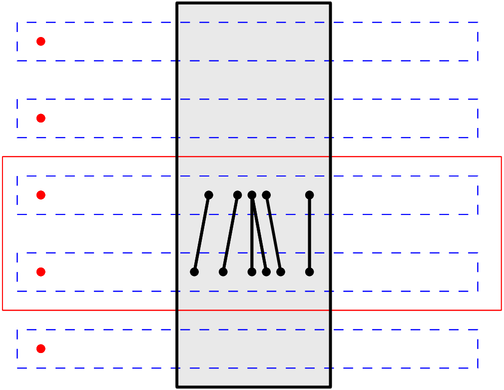

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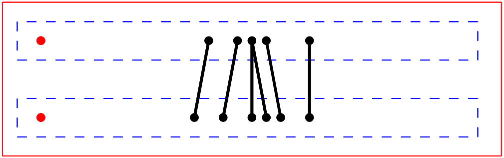

$\mathbf{A} =$  $\mathbf{B} =$ 

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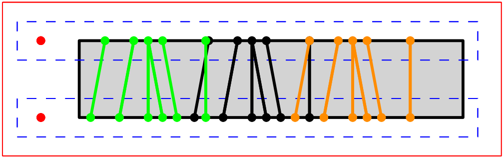
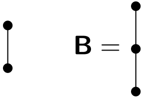


$\mathbf{A} =$  $\mathbf{B} =$

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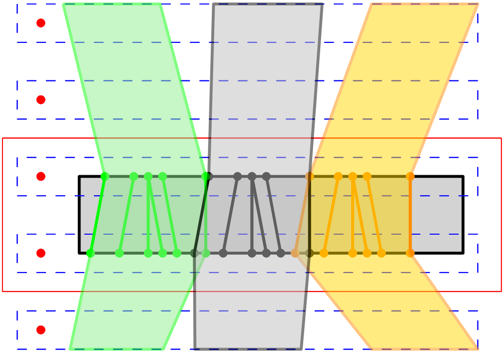

The Partite Construction: Summary

- Ramsey Theorem:
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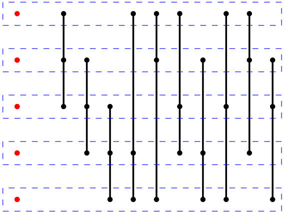

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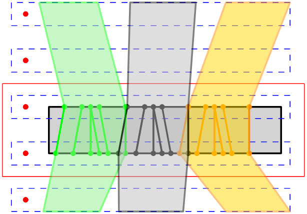

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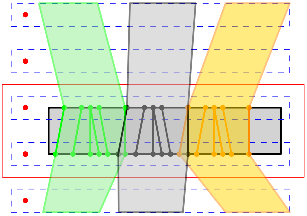

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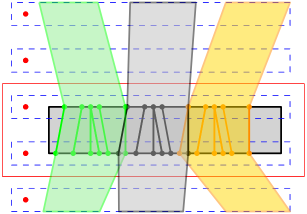

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Lemma

Let $\mathbf{A}$ *be a structure s.t.* $A = \{1, 2, \ldots, a\}$ *and* $\mathbf{B}$ *be an* $\mathbf{A}$*-partite system.*

Then there exists a $\mathbf{A}$*-partite system* $\mathbf{C}$ *s.t.*

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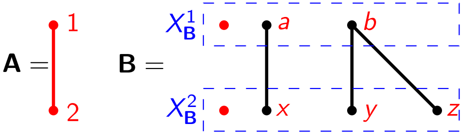

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Proof by application of Hales-Jewett theorem

Theorem (Hales-Jewett theorem)

For every finite alphabet Σ there exists $N = HJ(\Sigma)$ so that for every 2-coloring of functions $h : \{1, 2, \ldots, N\} \to \Sigma$ there exists a monochromatic combinatorial line.

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Definition

For non-empty $\omega \subseteq \{1, 2, \ldots, N\}$ and $f : \{1, 2, \ldots, N\} \setminus \omega \to \Sigma$ combinatorial line (ω, f) is the set of all functions $f' : \{1, 2, \ldots, N\} \to \Sigma$ such that

$$f'(i) = \begin{cases} \text{constant for } i \in \omega, \\ f(i) \text{ otherwise.} \end{cases}$$

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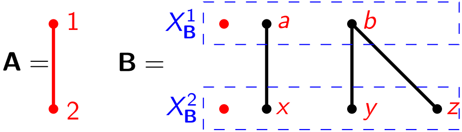

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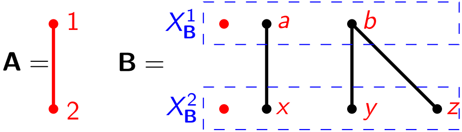

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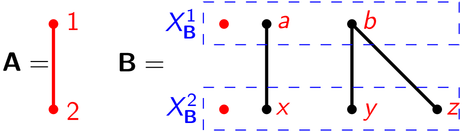

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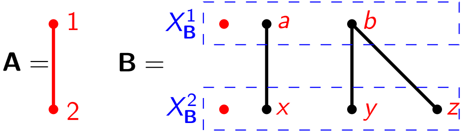

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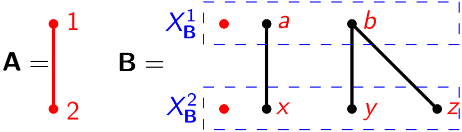

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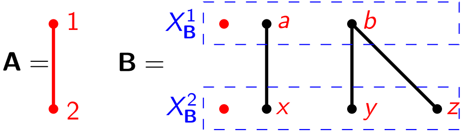

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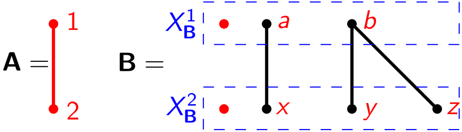

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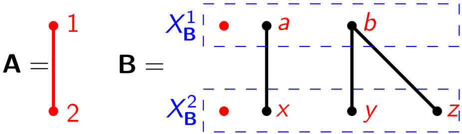
  - Fact: It is possible to add tuples to relations as needed to make this work

Easy description of **C**:



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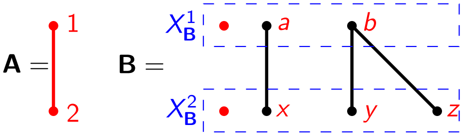

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- Add as many tuples to relations as possible such that all the evaluation maps $g_i(f) = f(i)$ are homomorphisms from **C** to **B**.

Uses of the partite construction

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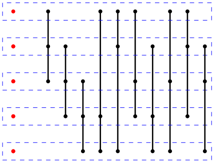
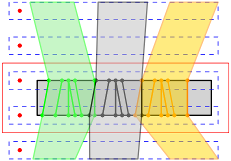
The Partite Construction





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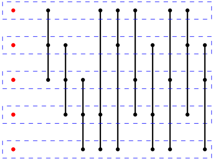

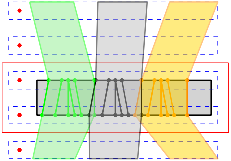

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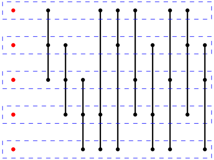

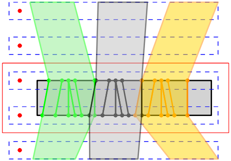

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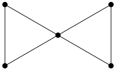
If **K** is irreducible and **A**, **B** are **K**-free, then so is **C**.

An exotic example

Bow-tie graph:



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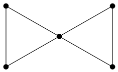
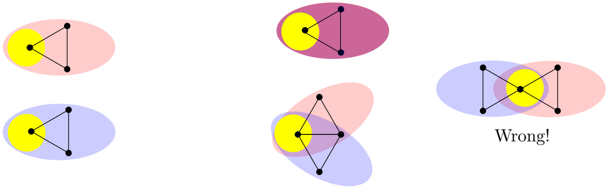
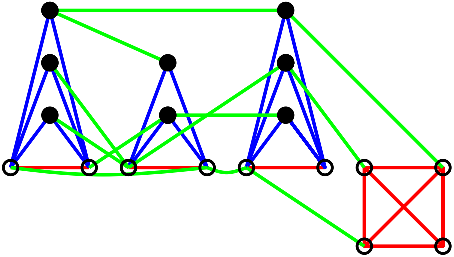


Amalgamation of two triangles must unify vertices.



Structure of bow-tie-free graphs



Edges in no triangles Edges in 1 triangle Edges in 2+ triangles

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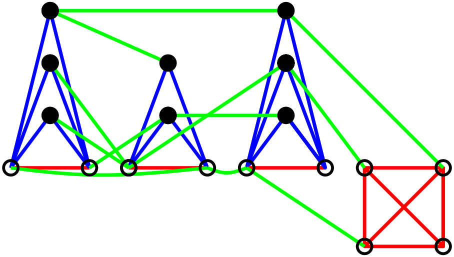

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Definition

Chimney is a graph created by gluing multiple triangles over one edge.

Definition

Graph is good if every vertex is either in a chimney or K_4.

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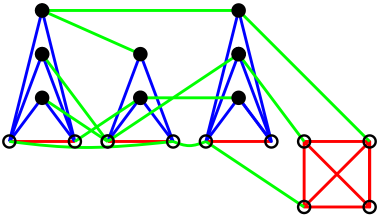

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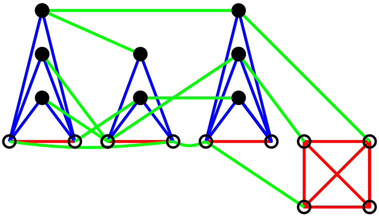

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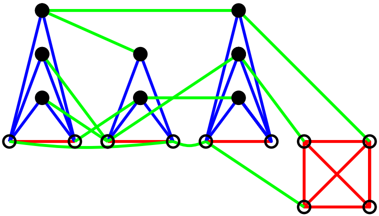

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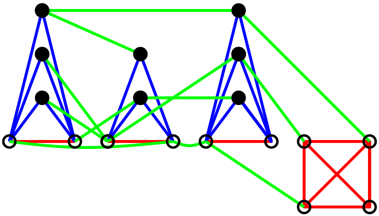

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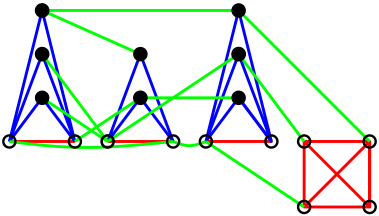
- Every bowtie-free graph G is a subgraph of some good graph G'.
- For every good graph $G = (V, E)$ the graph $G_\Delta = (V, E_\Delta)$ (E_Δ are edges in triangles) is a disjoint union of copies of chimneys and K_4.

Structure of bow-tie-free graphs



Graph is good if every vertex is either in a chimney or K_4.

- Every bowtie-free graph G is a subgraph of some good graph G'.
- For every good graph $G = (V, E)$ the graph $G_\Delta = (V, E_\Delta)$ (E_Δ are edges in triangles) is a disjoint union of copies of chimneys and K_4.

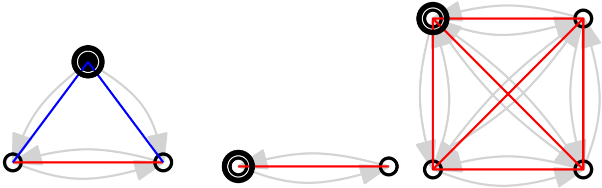
Closure of a vertex v = all endpoints of red edges contained in triangles containing v.

Lemma

Bow-tie-free graphs have free amalgamation over closed structures.

Ramsey property of bow-tie free graphs

3 types of vertices and their closures:



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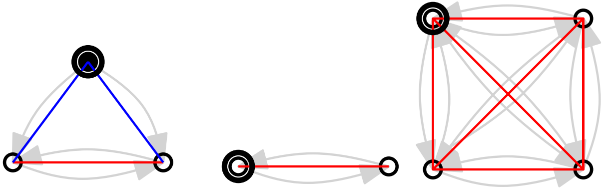

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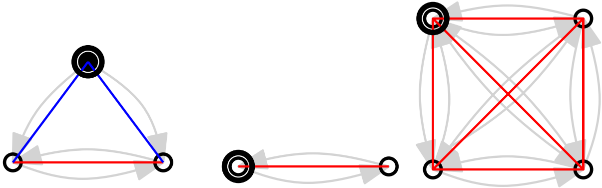

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Theorem (H., Nešetřil, 2014)

The class of graphs not containing bow-tie as non-induced subgraph have Ramsey lift.

Unary closures = relations with out-degree 1

Unary closure description $\mathcal{C}$ is a set of pairs (R^U, R^B) where R^U is unary relation and R^B is binary relation.

We say that structure $\mathbf{A}$ is $\mathcal{C}$-closed if for every pair (R^U, R^B) the B-outdegree of every vertex of $\mathbf{A}$ that is in U is 1.

Theorem (H., Nešetřil, 2015)

Let $\mathcal{E}$ be a family of complete ordered structures and $\mathcal{U}$ an unary closure description. Then the class of all $\mathcal{C}$-closed structures in $\mathrm{Forb}_E(\mathcal{E})$ has Ramsey lift.

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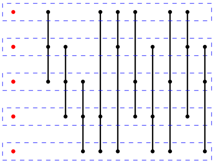
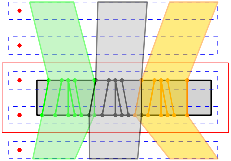
All Cherlin Shelah Shi classes with unary closure can be described this way!

The Induced Partite Construction with unary closure





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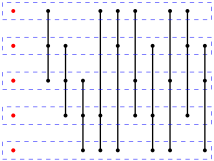

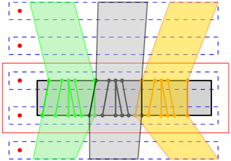

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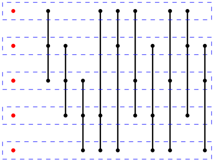

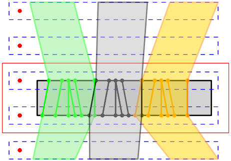

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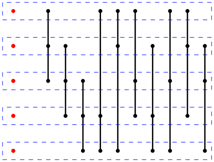

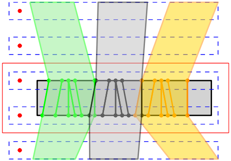

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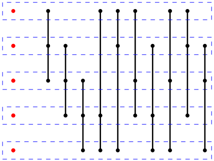

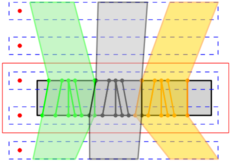

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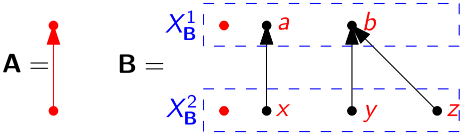
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_i$ are $\mathcal{C}$-closed. Only potential problem is the partite construction.

Easy description of **C**:



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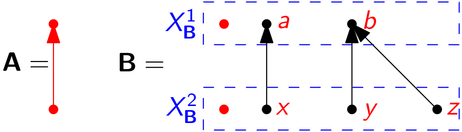

- Vertices in partition $\mathbf{X}^i_{\mathbf{C}}$: Functions $f : \{1, 2, \ldots, N\} \to X^i_{\mathbf{B}}$
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The Partite Lemma

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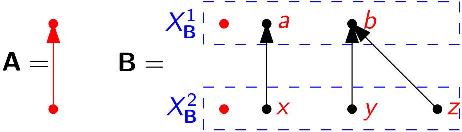

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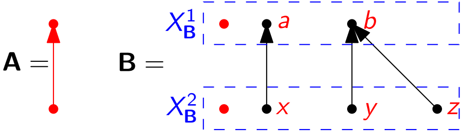

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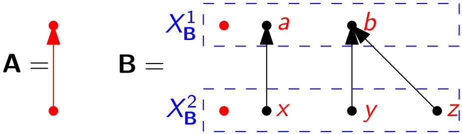

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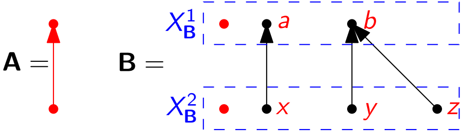

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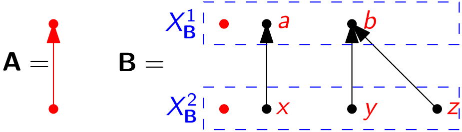

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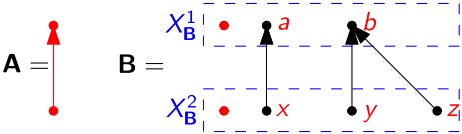

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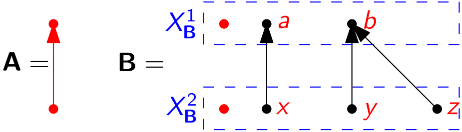

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Map of Ramsey Classes

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interval graphs

permutations

cyclic orders

unions of complete g

linear orders

metric spaces

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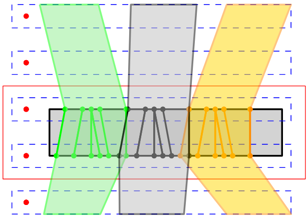

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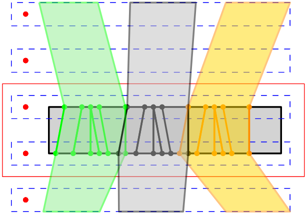

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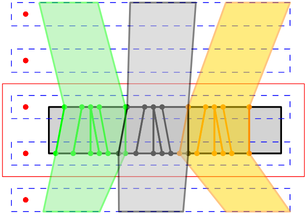

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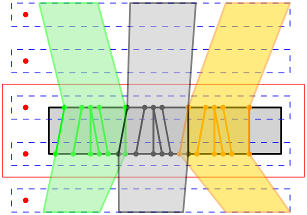

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  - unify vertices to preserve out-degree 1 of non-unary closure edges

Example: $\mathbb{QQ}$

Definition

Denote by $\mathbb{QQ}$ the structure with binary relation $\leq$ and ternary relation $\prec$ with the following properties

1. relation $\leq$ forms the generic linear order
2. for every vertex $a \in \mathbb{QQ}$ the relation $\{(b, c) : (a, b, c) \in \prec\}$ forms the generic linear oder on $\mathbb{QQ} \setminus \{a\}$ that is free to $\leq$.

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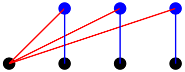

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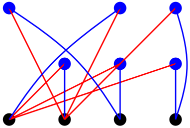

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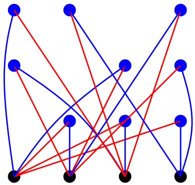

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