Groups and semigroups: from a duet to a chorus

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London Mathematical Society – EPSRC Durham Symposium
Permutation groups and transformation semigroups
2015-07-20 to 2015-07-30

I came to an age...

1st Question: how to deal with a bull?

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Peter Cameron's Blog

always busy counting, doubting every figured guess . . .

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← Futoshiki squares Busy times, 6: Leuven →

Busy times, 5

Posted on 21/05/2014

Then I came to a serious check. The path went through a gate into a field with a big herd of cows, many with calves, who came hurrying over to see me. With them came the bull, a very solidly built chap whose conversation and gestures made it very clear that he didn't want me to come into his field. Normally it is quite easy to hoosh cows away, but this herd, perhaps emboldened by the presence of Big Daddy, were not to be moved. So instead I had to climb over the fence and walk through the potato field next door.

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Groups & Semigroups

TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 333, Number 2, October 1992

THE MINIMAL DEGREE OF A FINITE INVERSE SEMIGROUP

BORIS M. SCHEIN

ABSTRACT. The minimal degree of an inverse semigroup S is the minimal

finding $\delta(S)$ for infinite S is not devoid of interest, we consider only finite inverse semigroups in this paper. Our main result is an exact formula for $\delta(S)$ "modulo groups." Solving semigroup problems "modulo groups" (a semigroup problem reduced to a group problem is considered solved) may raise objections,

1 In other words, people who care about semigroups *qua* semigroups consider groups to be "known." If you can solve a semigroup problem in terms of groups, then you consider the problem 'solved', even if the group-theoretic problem is not really "solved". – Arturo Magidin May 1 '12 at 19:00

www.encyclopediaofmath.org/index.php/Semi-group

When the structure of semi-groups is considered, much importance is attached to various constructions that reduce the description of the semi-groups in question to that of "better" types. Quite frequently, the latter are groups, and the principle of description "modulo groups" is common in semi-group-theoretical contexts; in fact, it already

8+1 0

0 Críticas

Escrever crítica

Algorithmic Problems in Groups and Semigroups

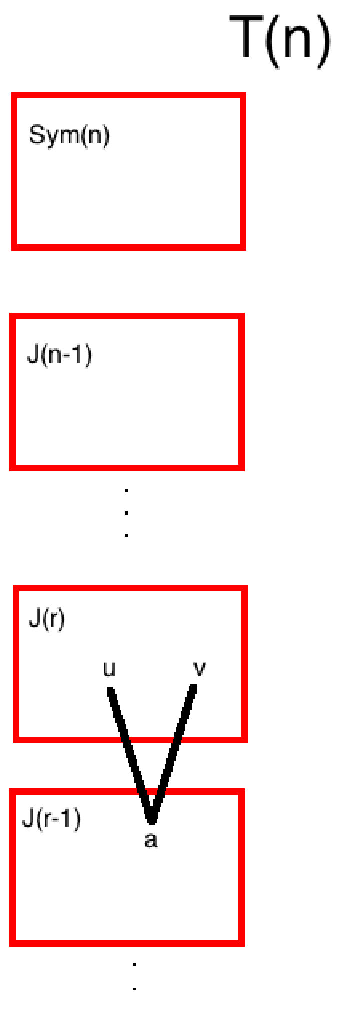
editado por Jean-Camille Birget,Stuart Margolis,John Meakin,Mark V. Sapir

in the theory of groups or with the role finite-dimensional simple algebras over a field play in the structure theory of rings.

The structure of finite 0-simple semigroups was described (modulo groups) by Sushkevich in 1928. This is why the class of finite 0-simple semigroups is considered to be one of the most transparent classes of semigroups. The following result was therefore totally unexpected.

Theorem 1 (Kublanovskii, 1994). *The* $\mathbb{S}$-*problem and the* $\mathbb{SP}$-*problem for the class* $\mathbf{C}_0$ *are undecidable.*

rank of $T(n)$



rank T(n) = rank S(n) + 1

$$t = \begin{pmatrix} \{1,2\} & 3 & 4 & \dots & n \\ 2 & 3 & 4 & \dots & n \end{pmatrix}$$

$$g^{-1}\, t\, h = \begin{pmatrix} \{1,2\}^g & 3^g & 4^g & \dots & n^g \\ 2^h & 3^h & 4^h & \dots & n^h \end{pmatrix}$$

2nd Question:

Are answers of the type

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3rd Q: How to try the best cherries?

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← Fibonacci numbers, 4 Happy birthday, Isaac New

A train journey

Posted on 09/07/2012

I like travelling by train. I have made memorable trips from Cairns to Gympie; Vancouver to Calgary; Roma to Potenza; Fort William to Mallaig; Mumbai to Pune; Paris to Milano. Now I can add another to this list.

The train starts its journey in Covilhã; the line to the north has been closed. It passes through the fertile valley of the Zêzere river, with vineyards and orchards everywhere. (The first stop, Fundão, produces "the best cherries in the world"; we ate them in the breaks at the workshop in Covilhã, and I would not quarrel with the description.) Then it climbs a rugged range, where a tunnel takes it to the other side, another wide flat valley.

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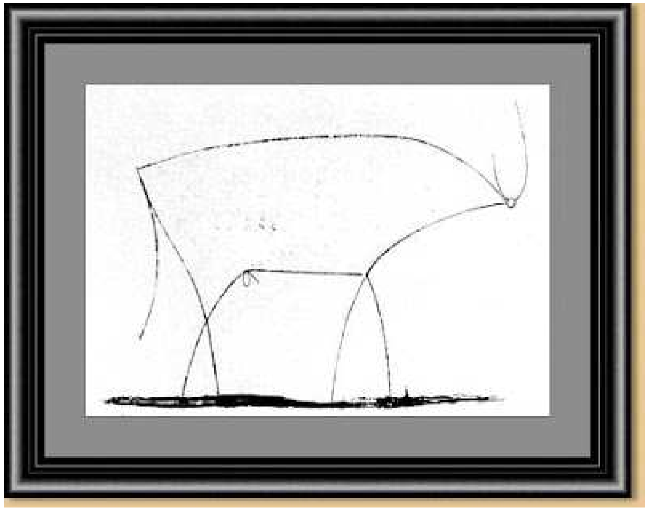
How to deal with a bull



What is good

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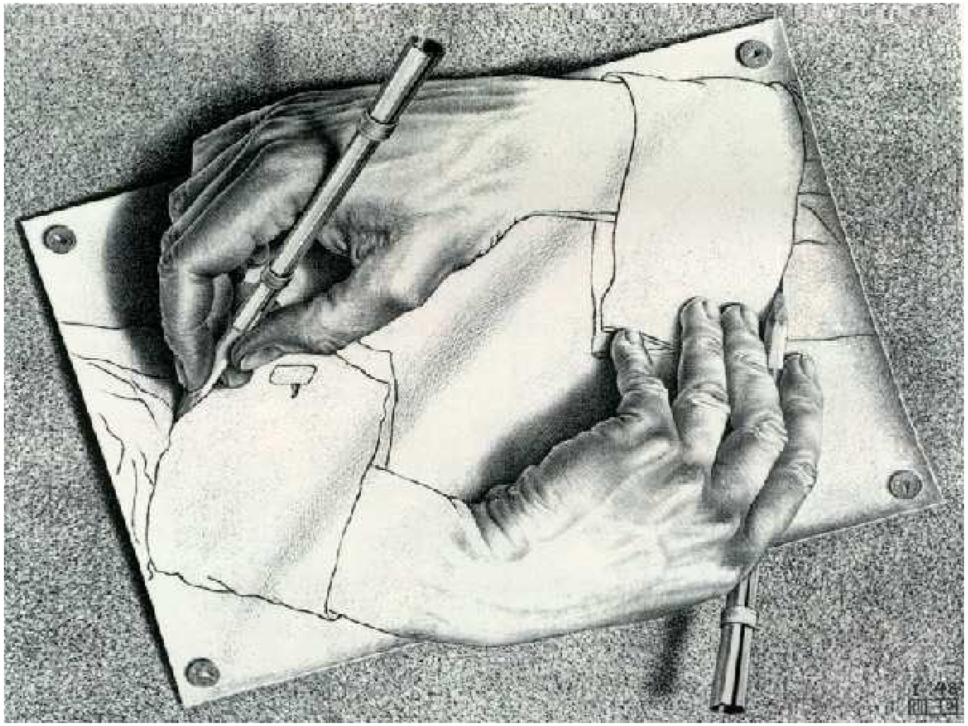
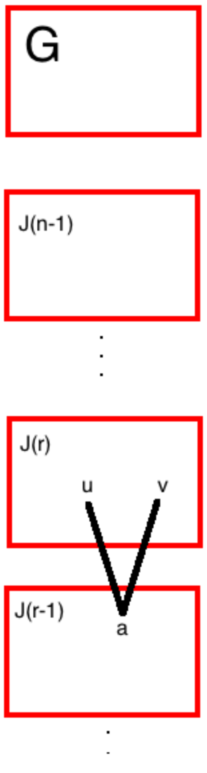


Example



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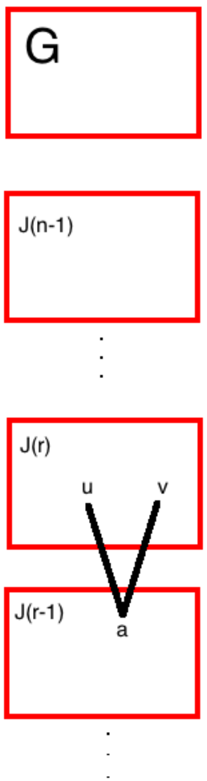

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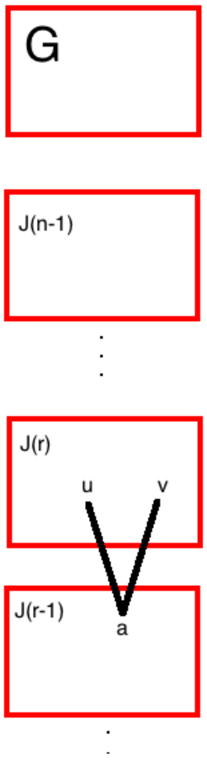

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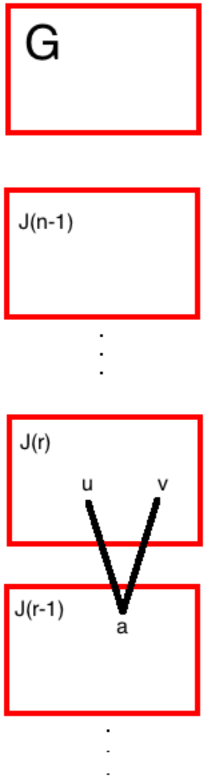

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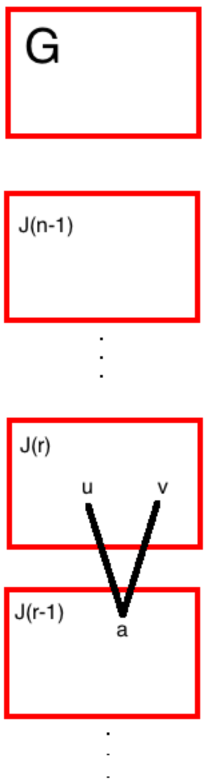

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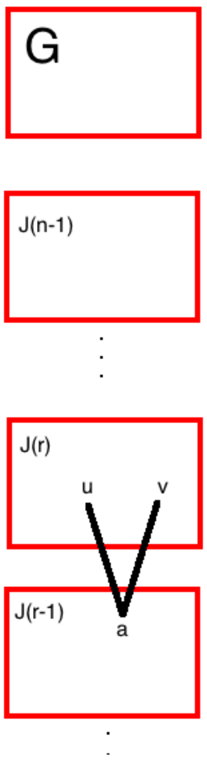

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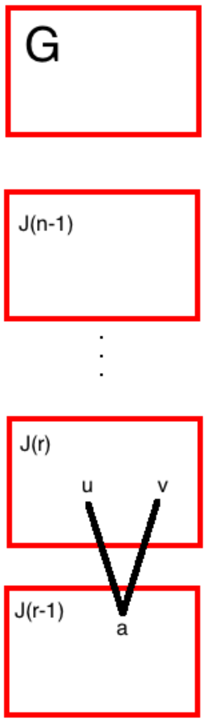

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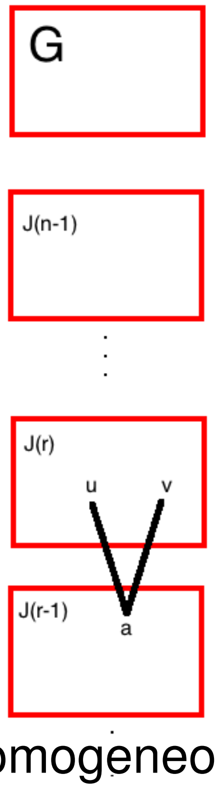

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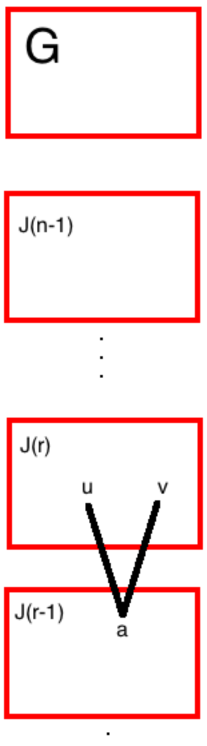

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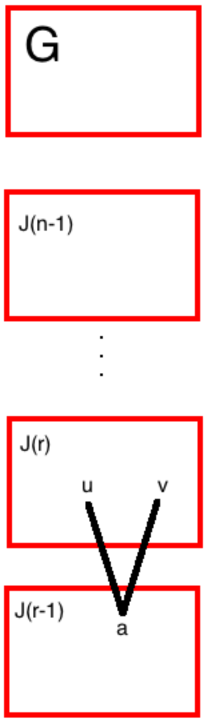

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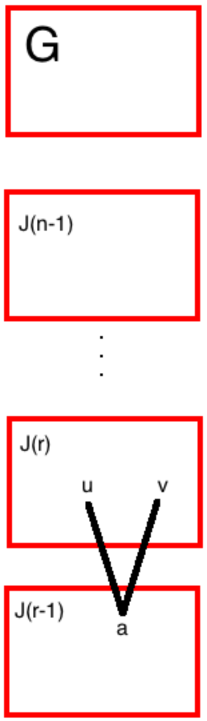

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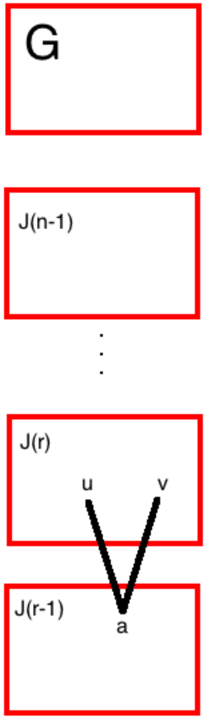

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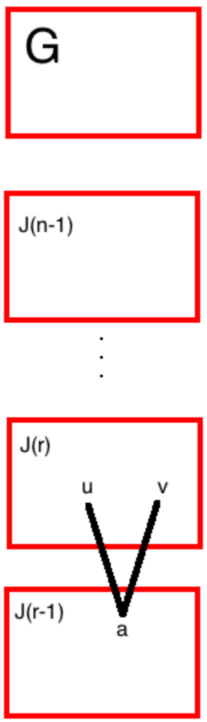

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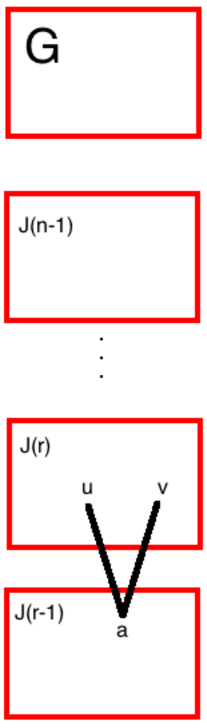

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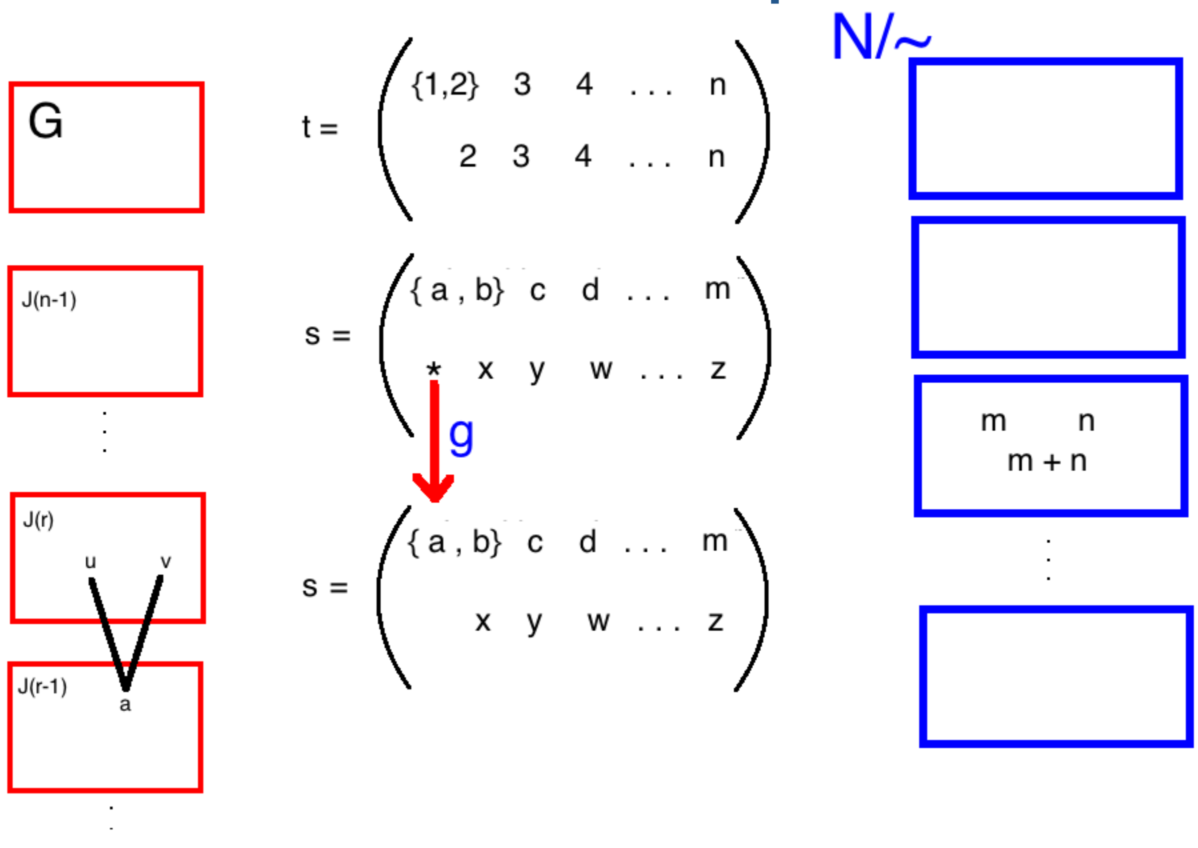

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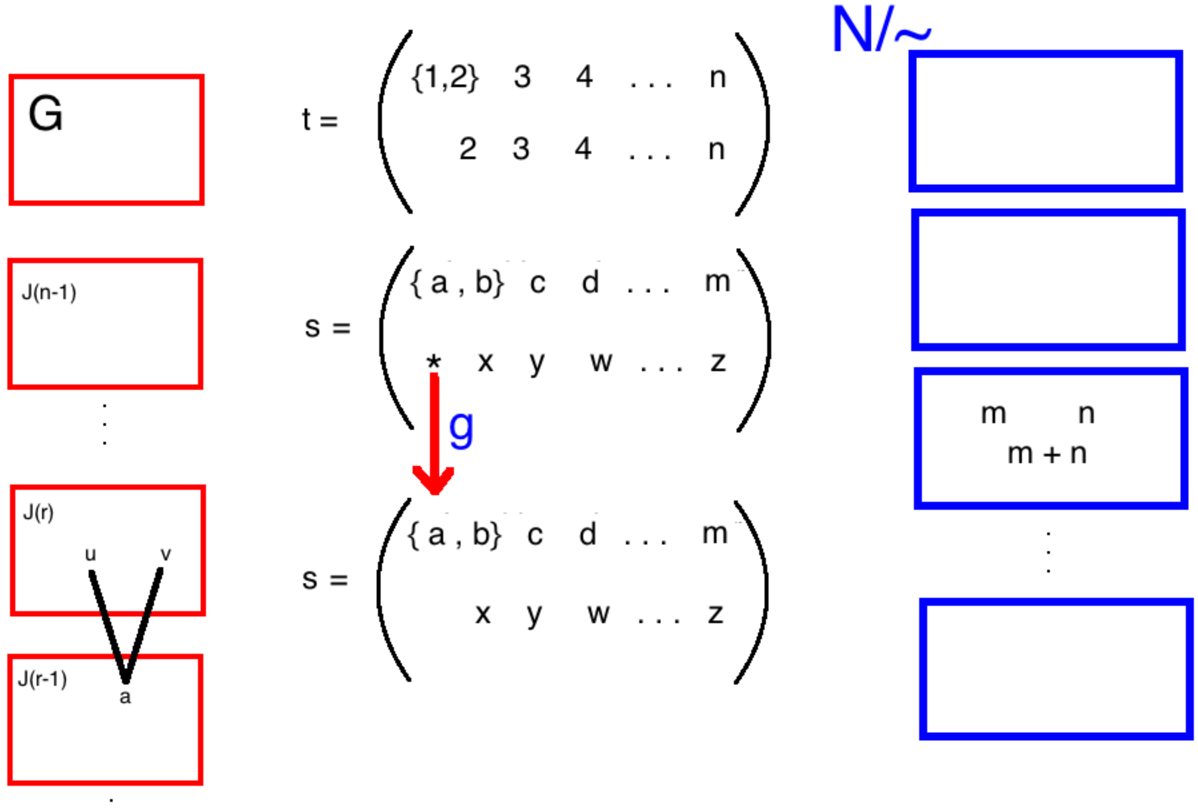
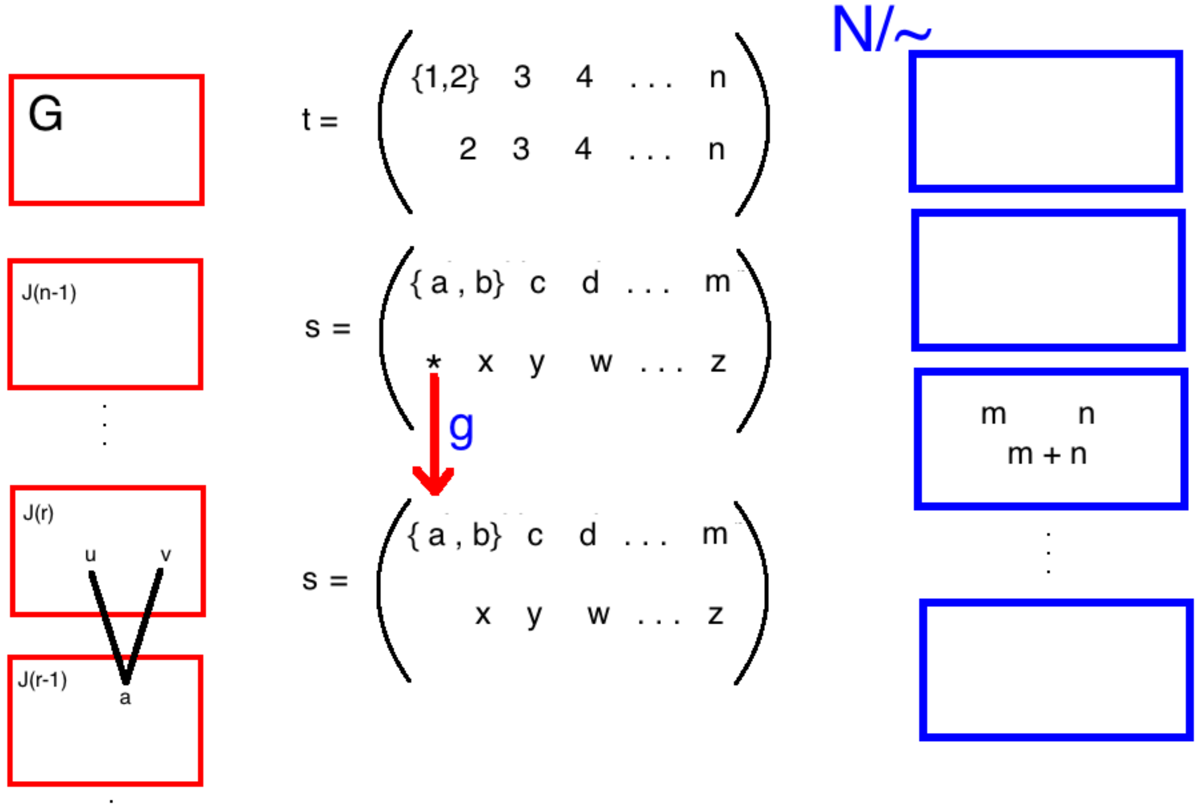


$(sg)^n$ has the same rank as s

Example



$(sg)^n$ has the same rank as s and $b \in Xsg$.

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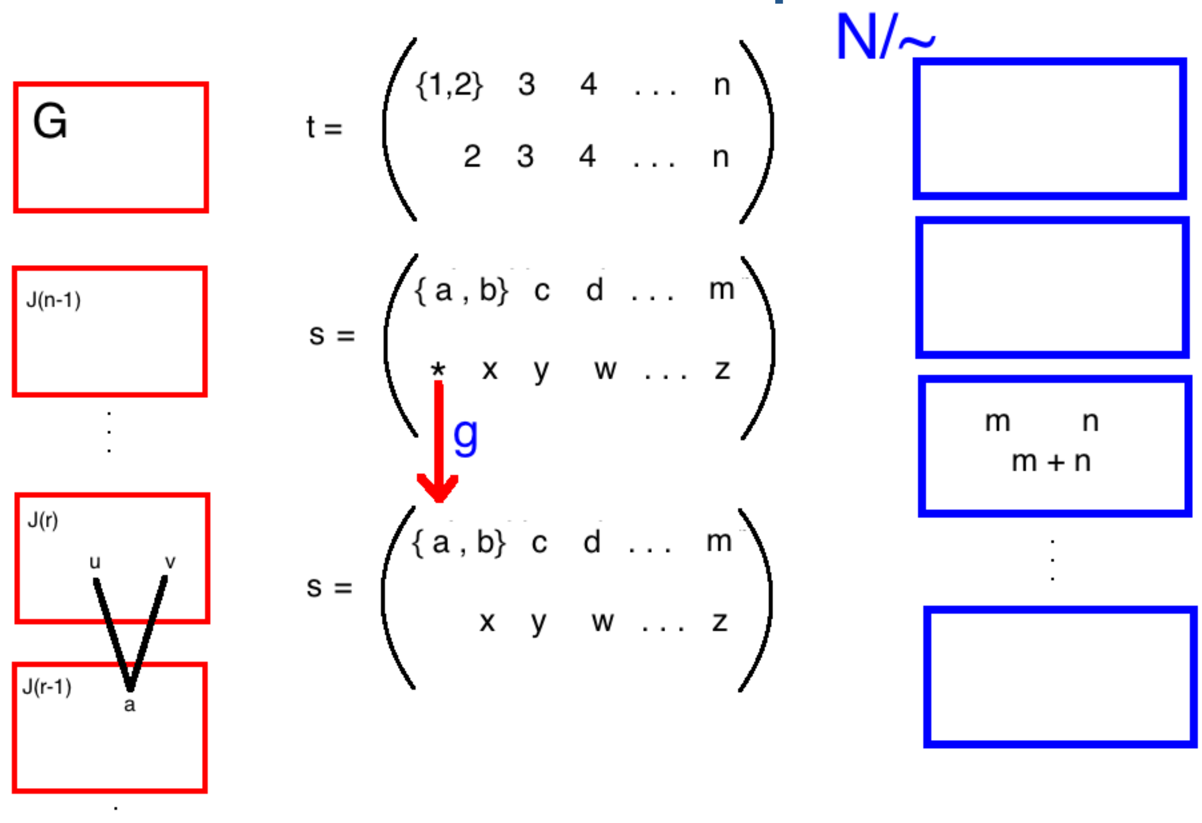

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Now define the following relation on the natural numbers:

$$n \sim m \Leftrightarrow (sg)^n = (sg)^m$$

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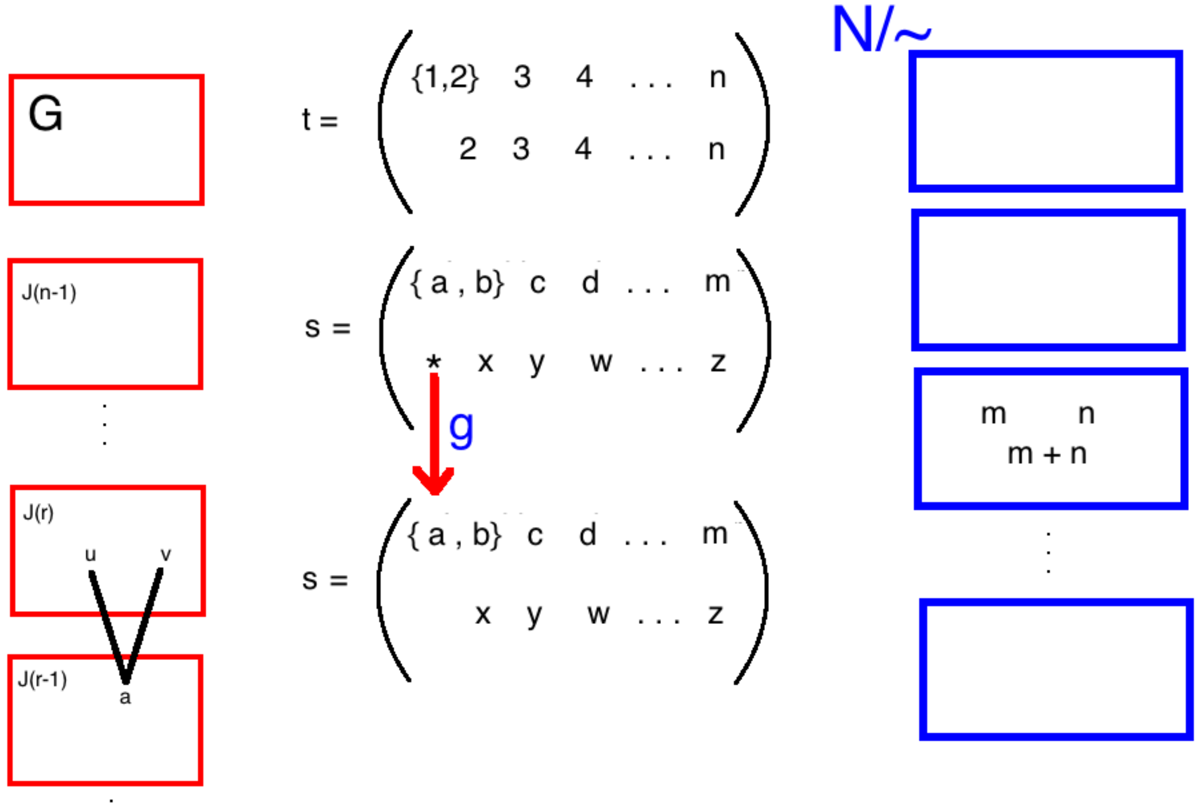

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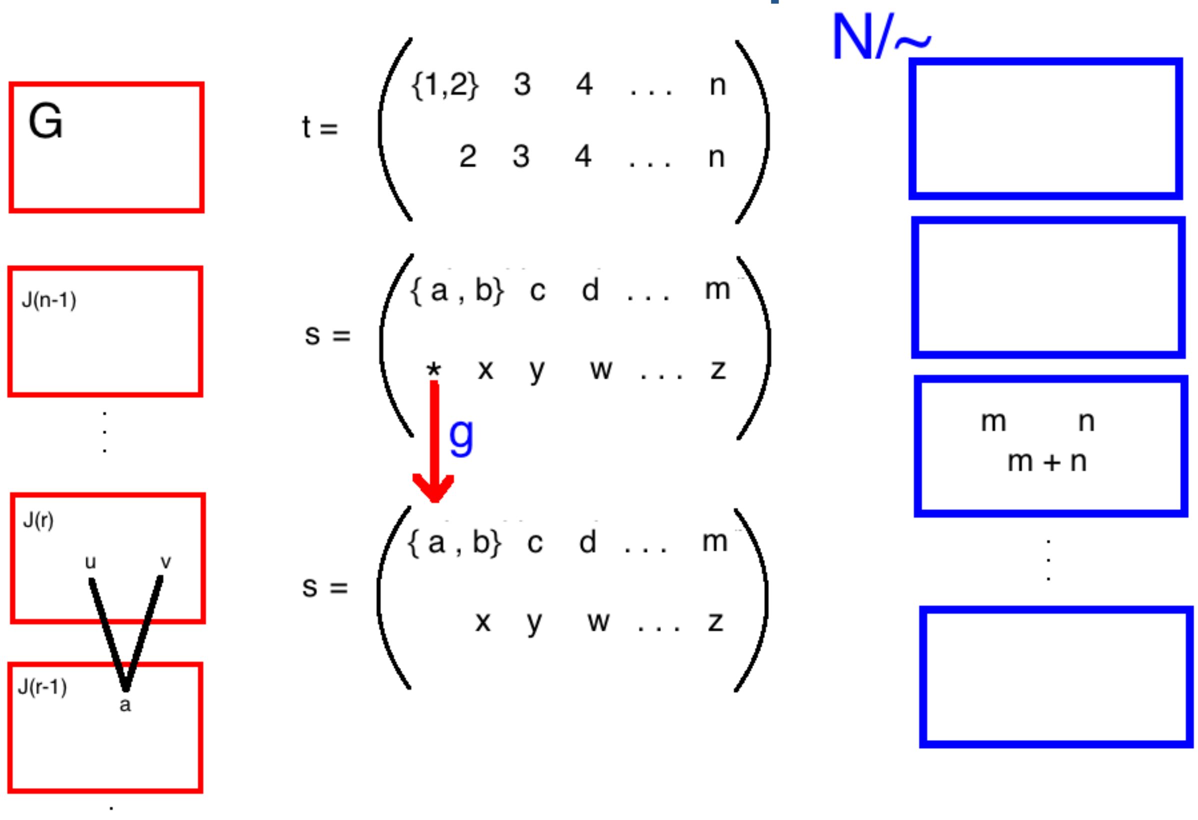

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$$(sg)^n = \begin{pmatrix} \{a,b\} & c & \dots & m \\ a & c & \dots & m \end{pmatrix} \quad \text{or} \quad (sg)^n = \begin{pmatrix} \{a,b\} & c & \dots & m \\ b & c & \dots & m \end{pmatrix}$$

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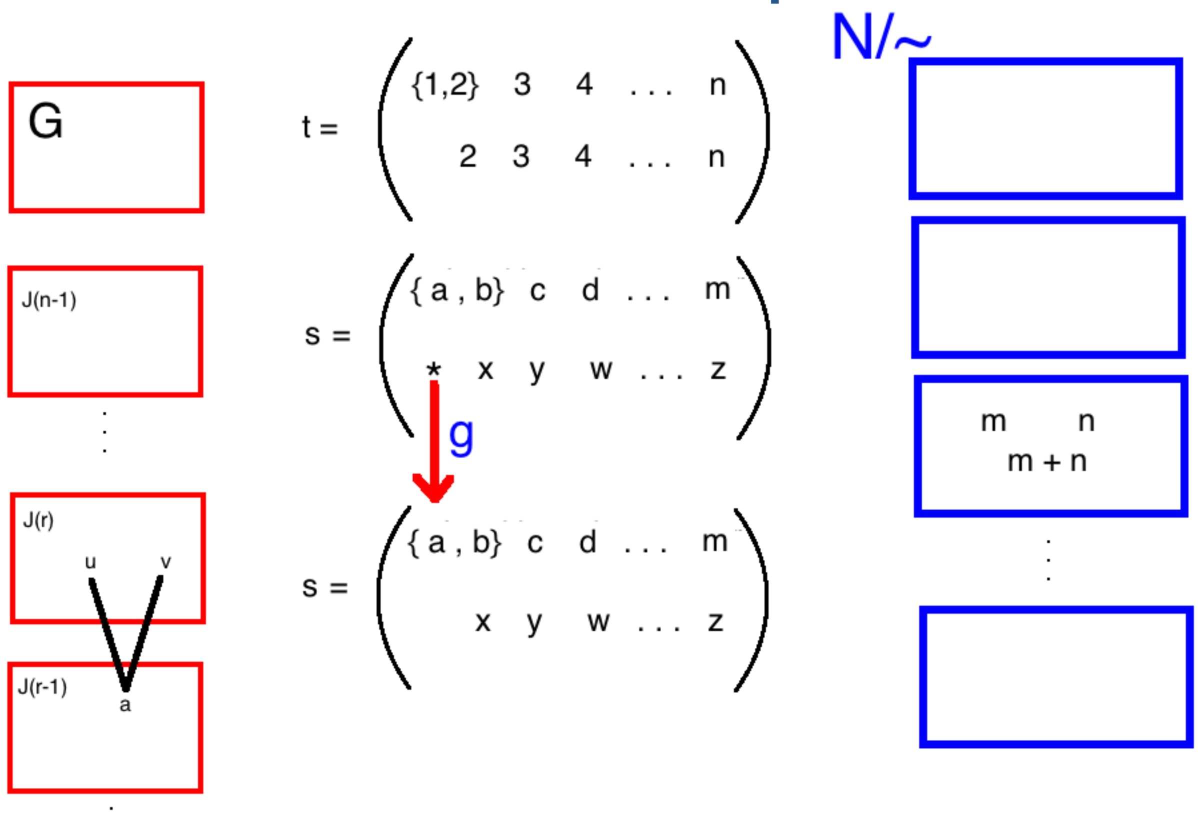

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But that only gives the rank $n-1$ idempotents; not all the rank $n-1$ maps...

Example

Ahah! You are right —said the semigroupist—

Example

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Corollary Let G be 2-homogeneous and t be a rank $n-1$ map. Then the rank of $\langle G, t\rangle$ is at most 4, and we know exactly what it is for each group G.

What is good

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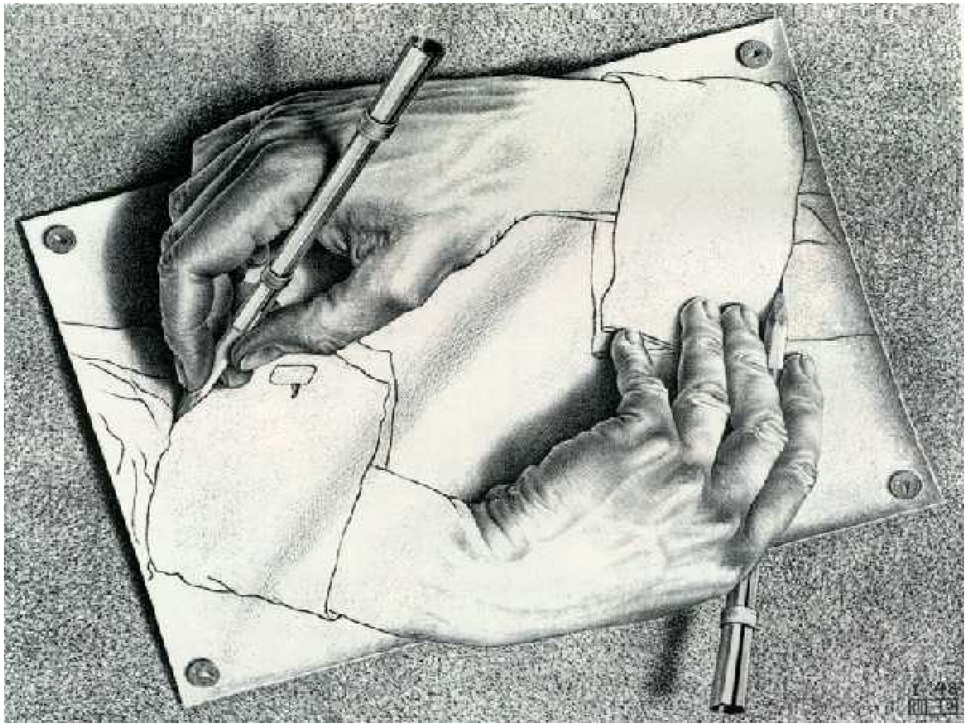

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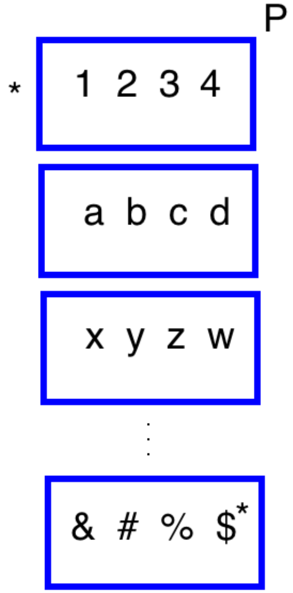

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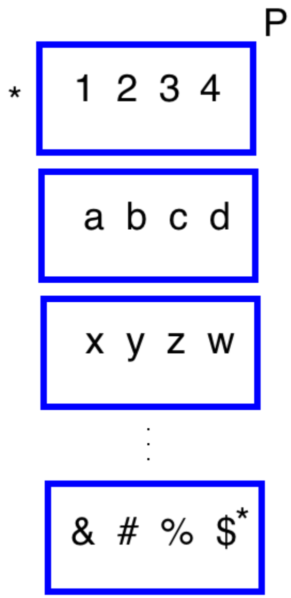

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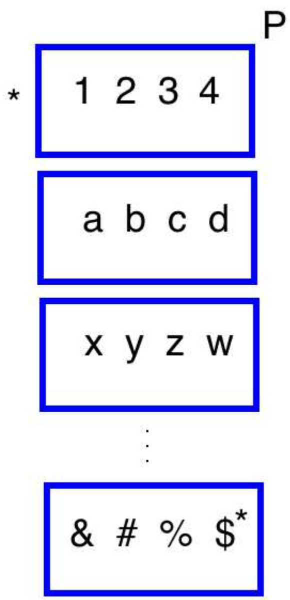

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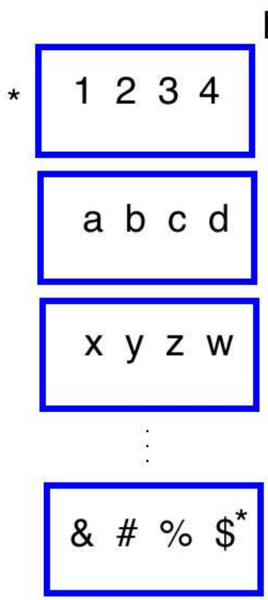

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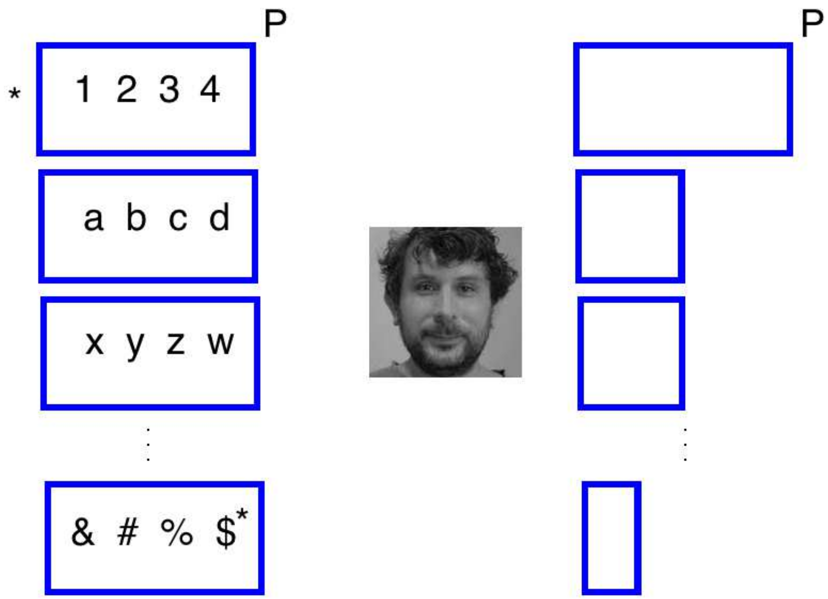

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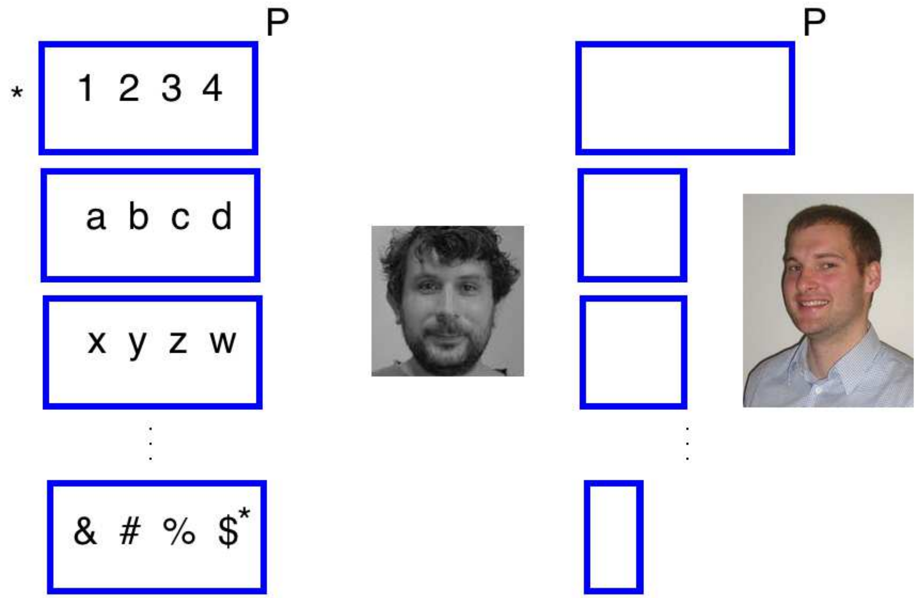

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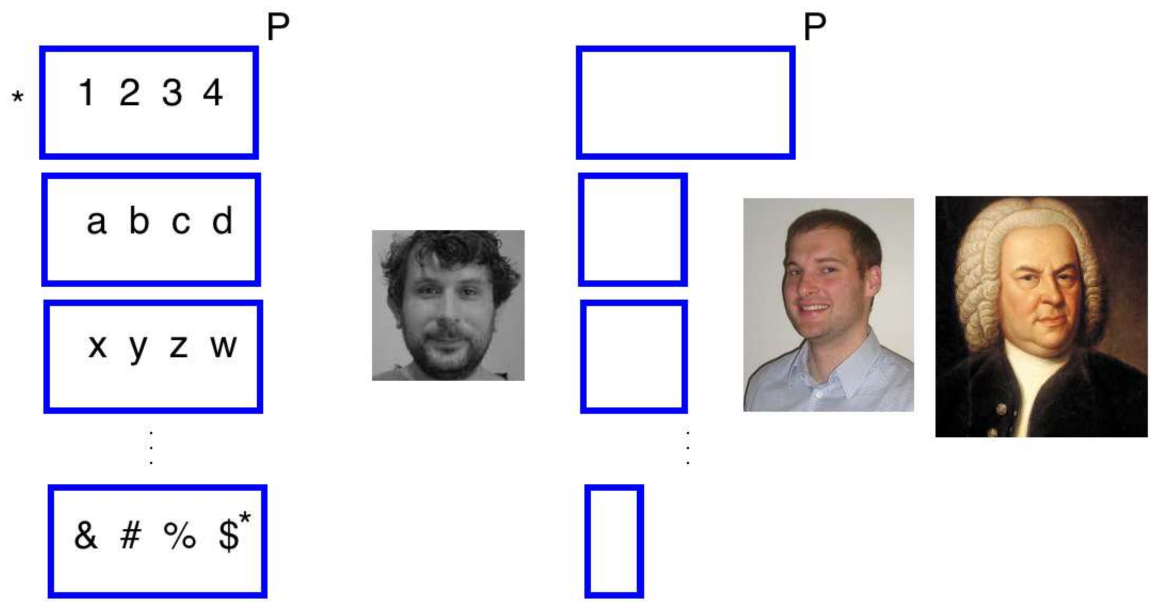

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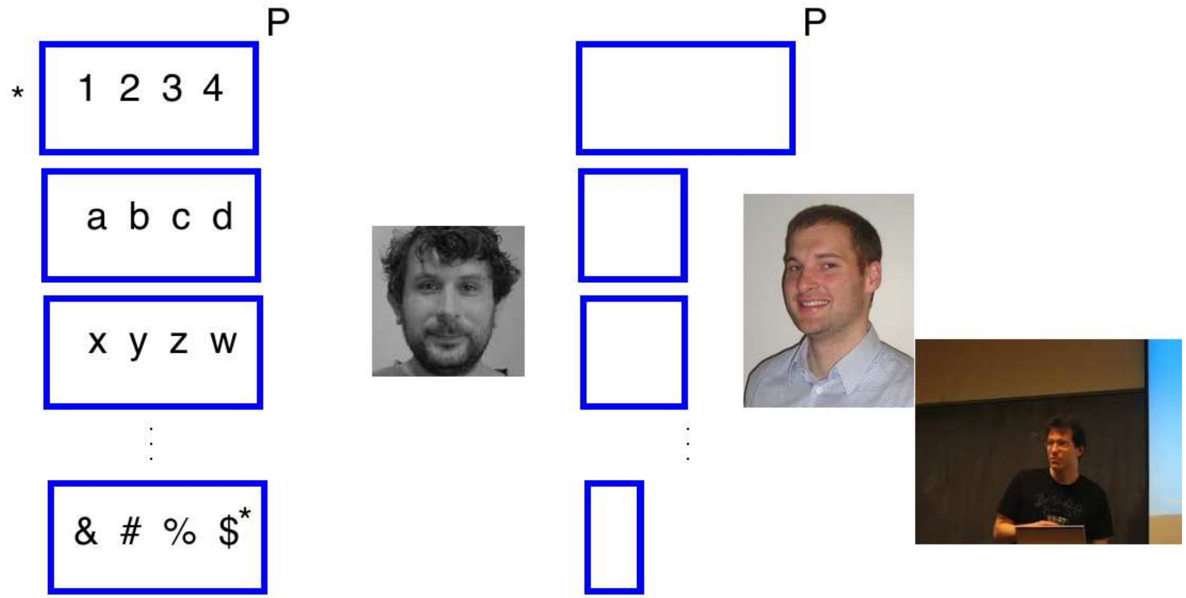

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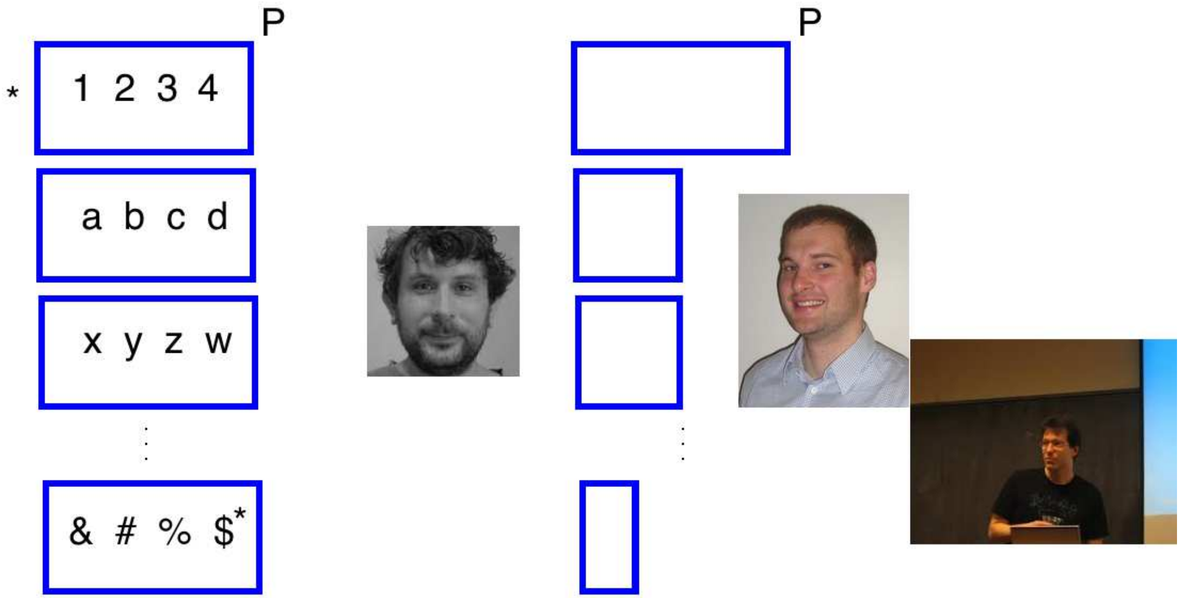

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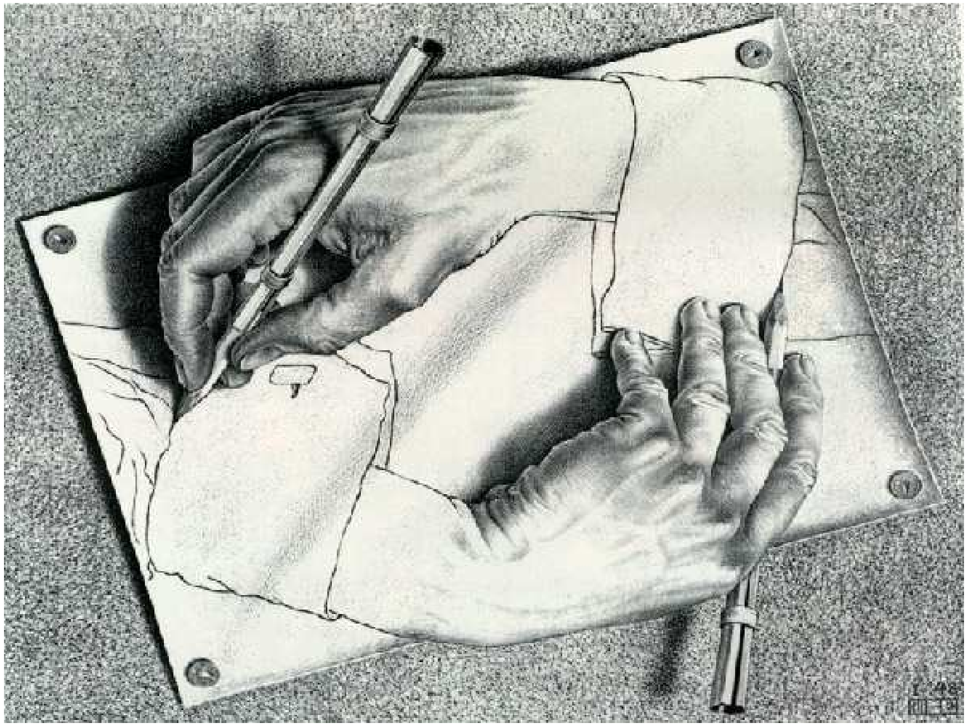
The first paper in transformation semigroups accepted in this journal in the last 20 years.

This is what is good!



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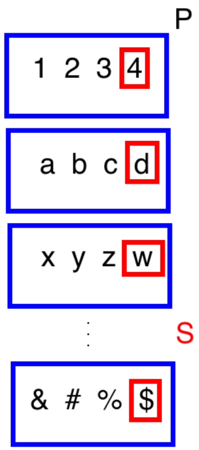

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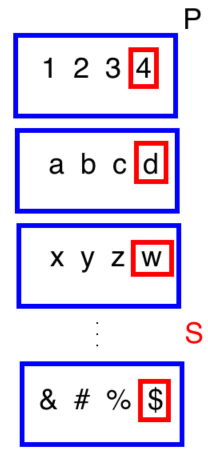

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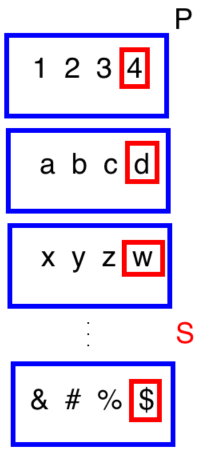

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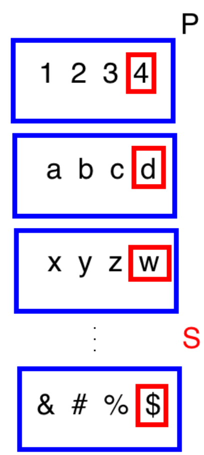

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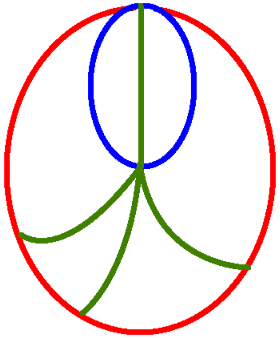

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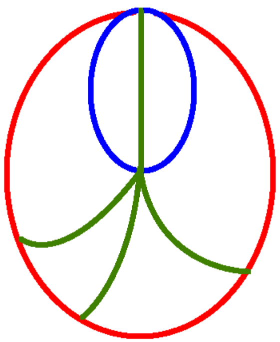

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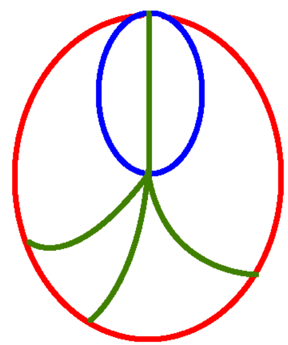

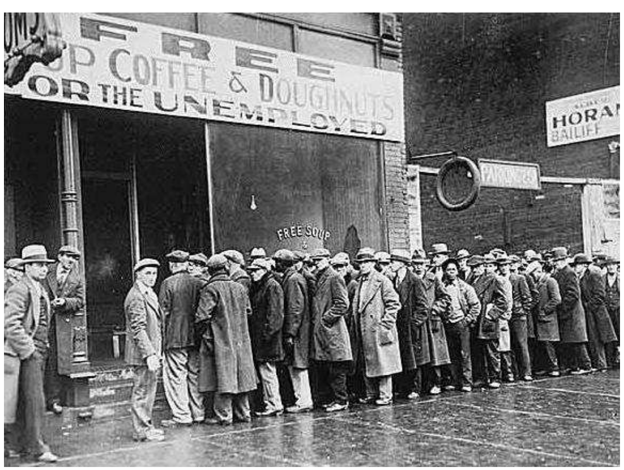

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Are there natural (easy) digraphs in which every arrow is contained in a cycle?

Circulant digraphs

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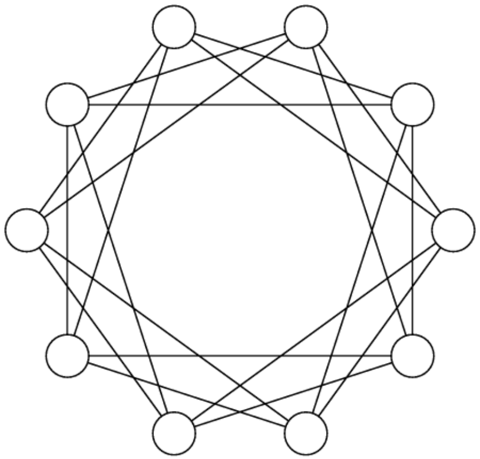
$C_n^S := ((i, i+s))_{i \in n, s \in S}$



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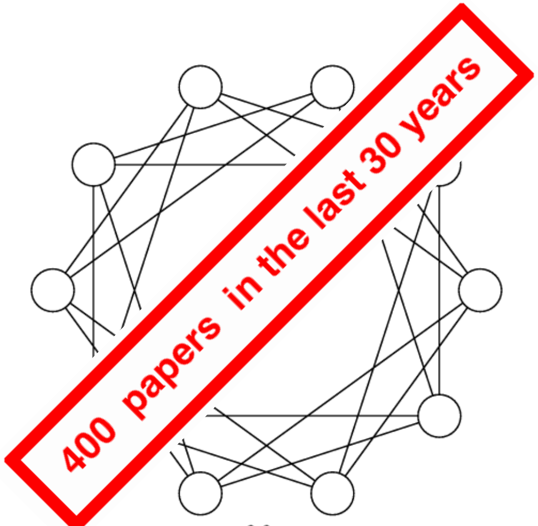

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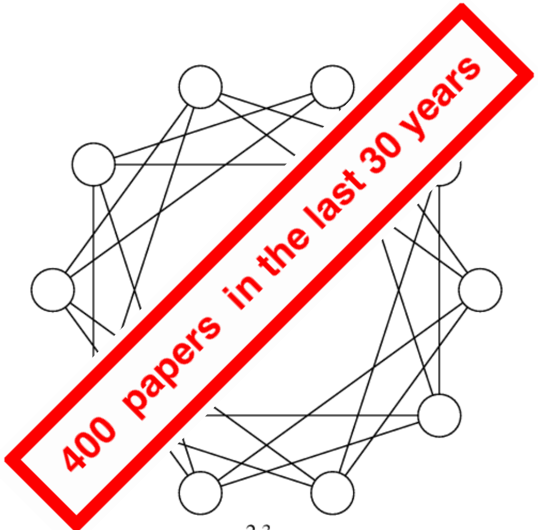

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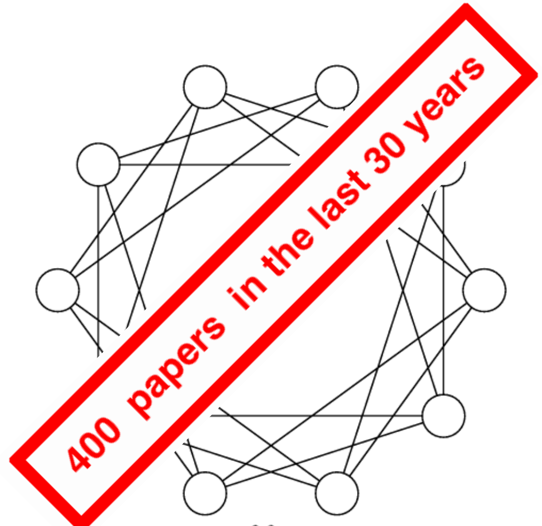

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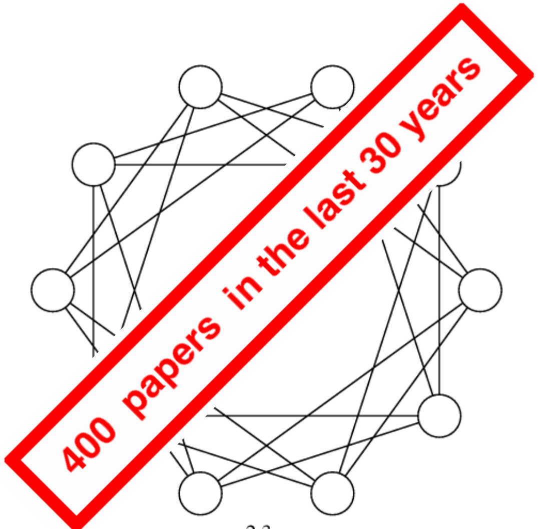

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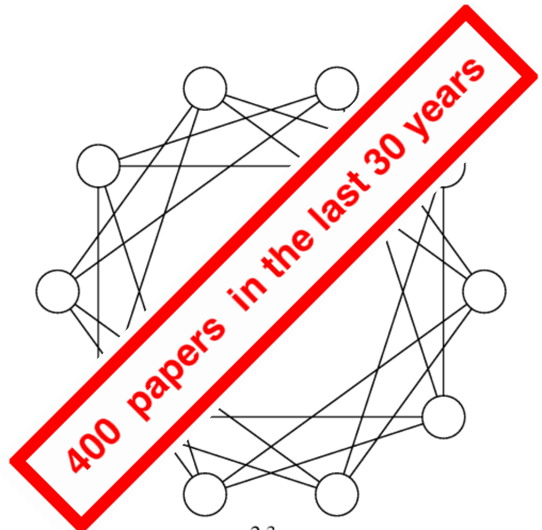

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(Worth eternal fame!)

From *Pushed by the father*

- profinite groups / profinite semigroups
- hyperbolic groups / hyperbolic semigroups
- (bi)automatic groups / (bi)automatic semigroups
- conjugation in groups / conjugation in semigroups

To *Pushing the father*

By beauty

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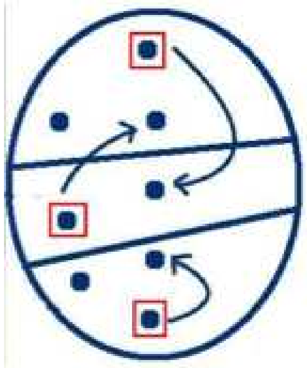
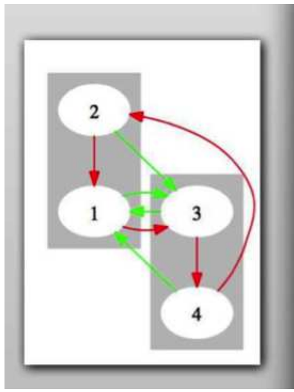


Pirates of Pangaea

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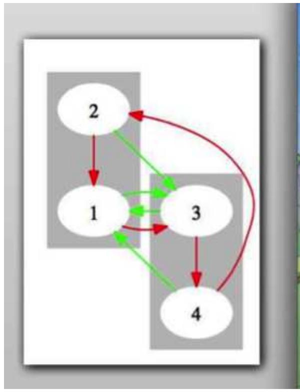
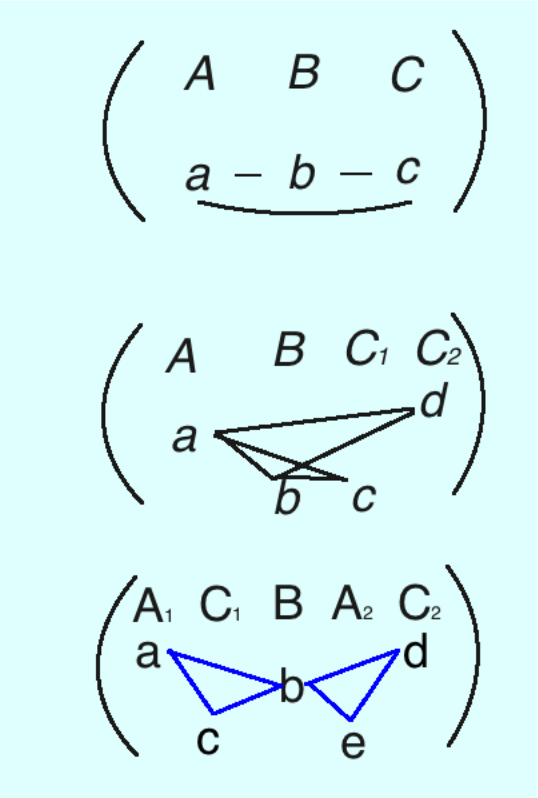


Rank 5



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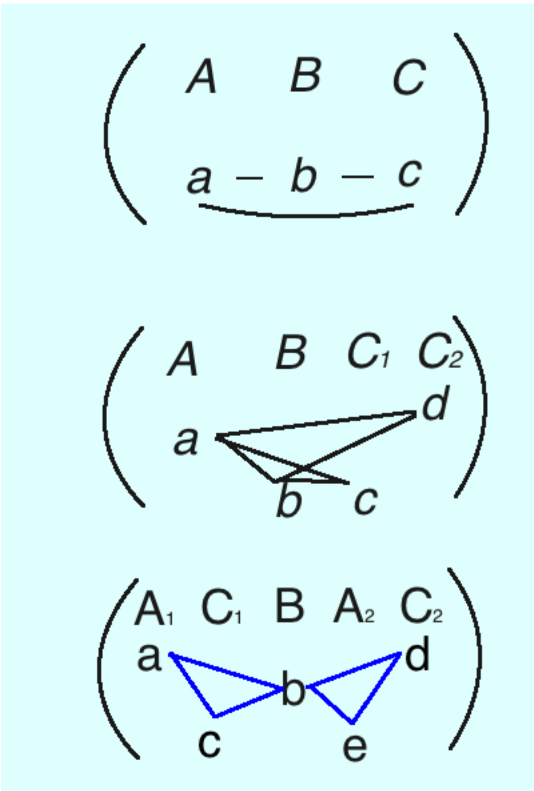

Table 2
Vertex-primitive arc-transitive graphs of valency 4

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$PGL_2(7)$	D_{16}	1	21	1	Cayley
$Aut(A_6)$	$[2^5]$	1	45	1	non-Cayley
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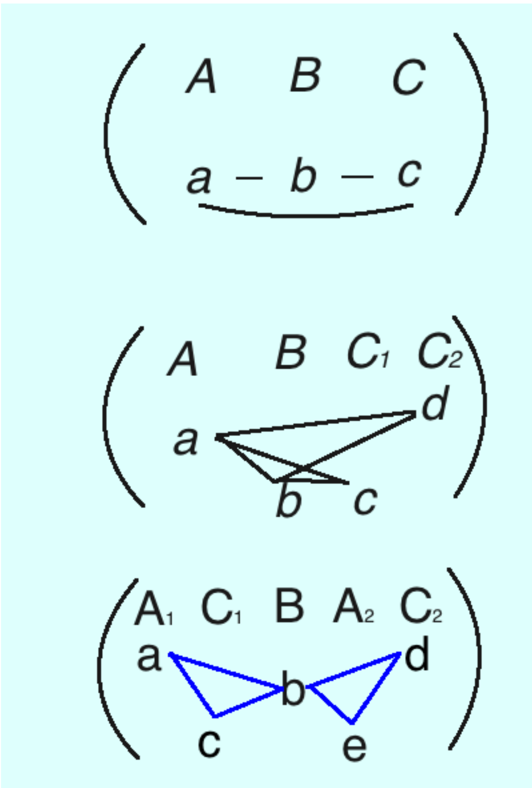

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$PSL_3(7)$	$(A_4{:}Z_3){:}Z_2$	3	26 068	1	non-Cayley

Peter J. Cameron Prize (worth 200 euros) Suppose G is primitive, A and B are 2-sets, and there exists $g \in G$ such that $Ag = B$. Let S be any generating set for G. Prove that there exists a word w on S, of length $n-1$, such that $Aw = B$?

Example

A group $G \leq S_n$ is $(k, k+1)$-homogeneous ($k \leq n/2$) if for every k-set A and every $(k+1)$-set B there is $g \in G$ such that $Ag \subseteq B$.

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Problem Classify the groups $G \leq S_n$ that are $(k, k+1)$-homogeneous, for all $k \leq n/2$.
Answer C_5, D_5, AGL$(1,5)$ (degree 5), PSL$(2,5)$ or PGL$(2,5)$ (dg 6), AGL$(1,7)$ (dg 7), PGL$(2,7)$ (dg 8), PSL$(2,8)$ or PΓL$(2,8)$ (dg 9), or A_n, S_n.

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Theorem (JA, JD Mitchell, C. Schneider; J. Algebra) Let $G \leq S_n$. Then $\langle G, t\rangle$ is regular for all $t \in T_n$ if and only if G is one of the groups in the list above. (Regular means: for every $a \in S$ exists $b \in S$ s.t. $a = aba$.)

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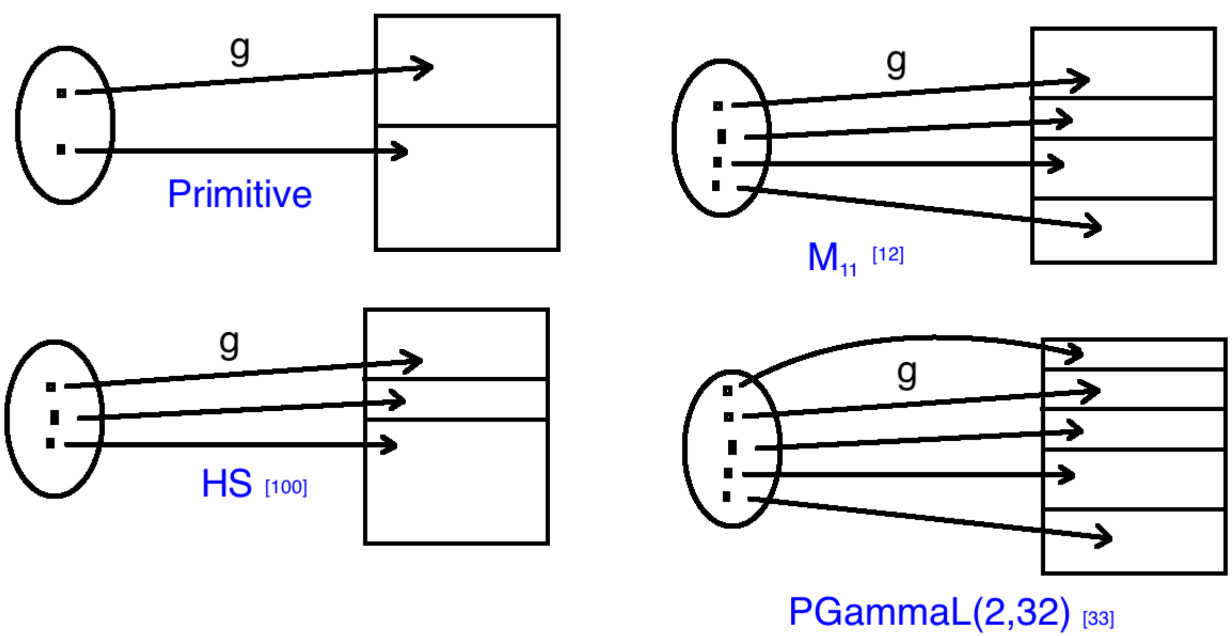

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Theorem Let $G \leq S_n$ and $k \leq n/2$; then $\langle G, t\rangle$ is regular, for all rank k maps t, iff in the orbit (under G) of any k-set there exists a section for any k-partition.

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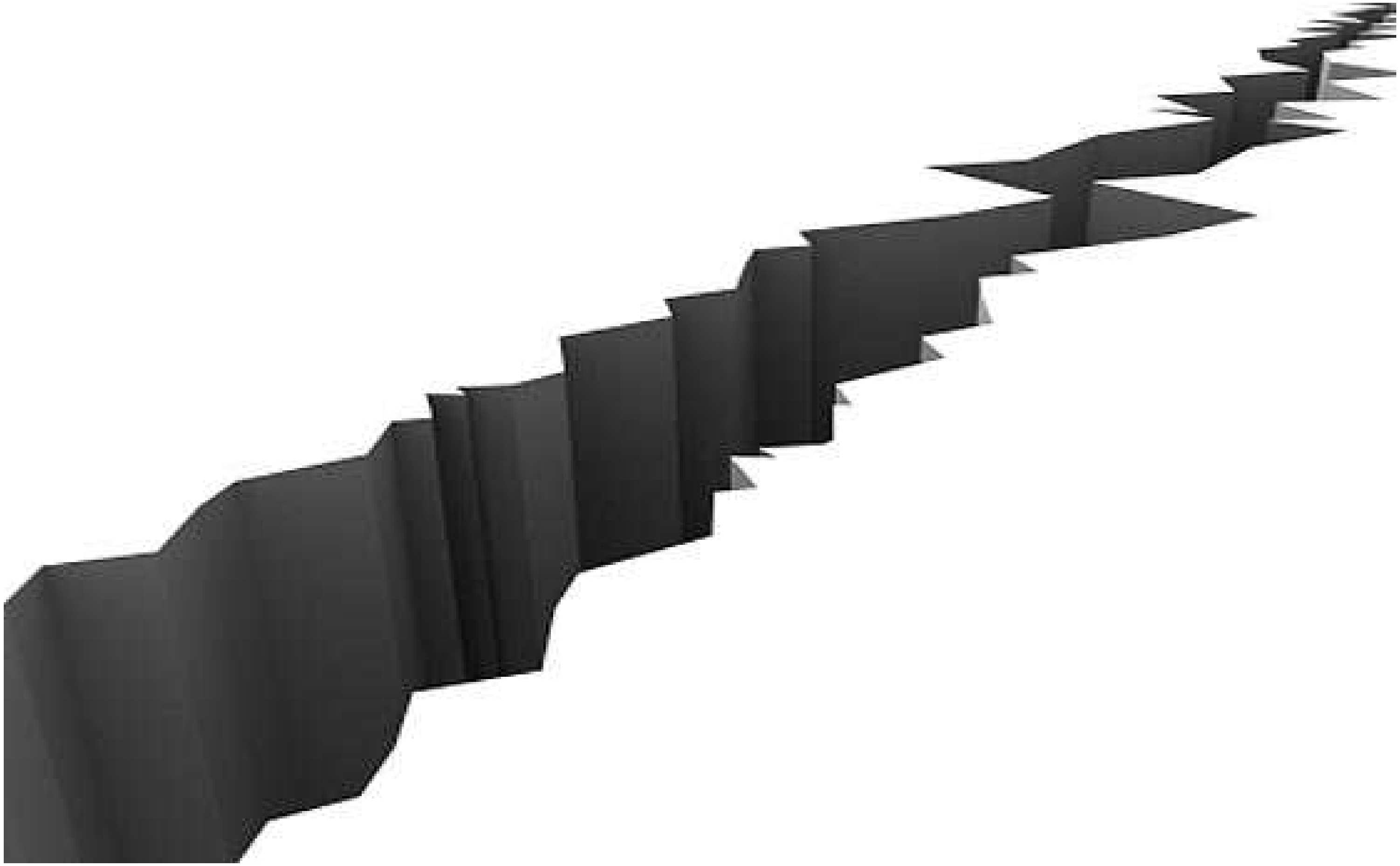
Epic split



Fundao cherries

Fundao cherries

Inbox (32,620)
Starred
Important
Sent Mail
Drafts (547)
Circles

From: **Ricardo Gonçalves** <ricardogoncalves@cm-fundao.pt>
Date: 27 July 2015 at 15:03
Subject: Message from Paulo Fernandes, Mayor fo Fundão
To: jjrsga@gmail.com

Special Message to the participants in the London Mathematical Society Conference in Durham.

As Mayor of Fundão, I am very honoured to know that Professor Cameron is an admirer of the quality of Fundão's cherries; if possible, we are even more honoured to know that he has been acting as a sales representative for the Fundão cheries to the world's mathematical community!

Next time you visit Portugal please come to visit us.

I will be very pleased to receive you in the Town Hall and offer you the best of the best cherries, that only in situ can be tasted.

This invitation, of course, applies also to any of the participants of your conference. Thank you very much for what you have been doing for Fundão. And for mathematics!

Big hug my friend,

Paulo Fernandes