Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2017	MATH1061-WE01

Title: Calculus & Probability I paper 1: Calculus

Time Allowed:	1 hour 30 minutes	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks.

Revision:	

ED01/2017

1. (i) State the pinching theorem and use it to calculate the following limit

$$\lim_{x \to 0} \left(5 + 3x^2 + 2x^2 \sin(\log |x|) \right).$$

(ii) Explain why L'Hôpital's rule can be applied to only one of the two limits

$$\lim_{x \to 0} \frac{\log(1-x)}{e^{3x} - \tan x + \cos x}, \qquad \lim_{x \to 0} \frac{\log(1+x)}{e^{3x} - \tan x - \cos x}$$

and apply it to calculate this limit. Calculate the other limit using any other approach.

(iii) Use Taylor series to calculate the limit

$$\lim_{x \to 0} \frac{e^{2x} - \cos(x) - \sin(2x)}{3xe^x - \sin(3x)}.$$

2. (i) Show that $(x+y)^{-1}$ is an integrating factor for the ordinary differential equation

$$dx + \left(1 + e^y(x+y)\right) dy = 0$$

and hence find an implicit form for the solution $y(x)$ that satisfies the initial condition $y(1) = 0$.

(ii) Find the general solution of the ordinary differential equation

$$y'' + 4y' + 4y = 6e^{-2x}.$$

3. (i) Calculate the integral

$$\int_{-\pi/4}^{\pi/4} \frac{[(1+2x)^4 + x\log(1+x^2)]\sec^2 x}{1 + 24x^2 + 16x^4}\, dx.$$

(ii) For integer $n \geq 0$ define

$$I_n = \int_0^{\pi} x^n \sin x \, dx.$$

Derive a recurrence relation between I_{n+2} and I_n and hence calculate I_4.

4. (a) Let D be the region of the (x, y) plane where the following inequalities are all satisfied: $\quad x^2 + y^2 \leq 2, \quad x \geq 0, \quad y \geq 0, \quad y \leq x^2.$
Sketch the region D and list the (x, y) coordinates of the three points on the boundary of D where at least two of the above inequalities become equalities.

(b) Calculate the double integral

$$\iint_D xy \, dx dy$$

where D is the region given in part (a).

5. The function $f(x)$ has period 4 and is given by

$$f(x) = \begin{cases} 3 & \text{if } \quad |x| \leq 1 \\ -1 & \text{if } 1 < |x| \leq 2. \end{cases}$$

The Fourier series of $f(x)$ can be written in the form

$$\alpha + \beta \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)} \cos\left((2m+1)\frac{\pi x}{2}\right) + \gamma \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} \sin\left((2m+1)\frac{\pi x}{2}\right)$$

where α, β, γ are constants.

(a) Determine the values of the constants α, β, γ.

(b) By evaluating the Fourier series at $x = 0$, determine the value of

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{2m+1}.$$

(c) Show that evaluating the Fourier series at $x = 1$ gives a result that is consistent with Dirichlet's theorem.

(d) Apply Parseval's theorem to determine the value of

$$\sum_{m=0}^{\infty} \frac{1}{(2m+1)^2}.$$