Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2017	MATH1551-WE01

Title: Mathematics for Engineers and Scientists

Time Allowed:	3 hours	
Additional Material provided:	Formula sheet	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks.

Revision:	

ED01/2017

1. (i) Find the modulus and principal argument of $\dfrac{1}{(1+i)^{10}}$.

 (ii) Use de Moivre's theorem to express $\sin(5\theta)$ as a polynomial in $\sin(\theta)$.

 (iii) Find all complex solutions to the equation

$$z^6 - 7z^3 - 8 = 0.$$

 (You may leave your solutions in polar form.)

2. (i) Calculate the limit of the sequence $s_n = n^2\left(\dfrac{1}{2n} - \dfrac{1}{2n+1}\right)$ as $n \to \infty$.

 (ii) Evaluate the limit $\displaystyle\lim_{x\to 0} \frac{\ln(\cos x)}{x^2}$, stating any standard results you use.

 (iii) Use the definition

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

 to find the derivative of $f(x) = x^{3/2}$ for $x > 0$ from first principles.

3. (i) Use Leibniz' Rule to calculate the fourth derivative of $f(x) = \dfrac{\cos x}{x}$.

 (ii) Find the Taylor polynomial of degree 3 about the point $x = \pi/4$ of the function $f(x) = \sin^2 x$.

 (iii) The Taylor series for $\sqrt{1+x}$ about $x = 0$ is $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$, valid for $|x| < 1$. Use this to calculate the limit

$$\lim_{y\to\infty} \left[\sqrt{y^2+y} - y\right].$$

4. (i) A plane passes through points $A = (1, 1, -2)$, $B = (3, 2, 1)$ and $C = (3, -1, 1)$. Find a vector which is normal to the plane and use it to find an equation for the plane in the form $ax + by + cz = d$.

 (ii) Find the direction in which the function

$$f(x, y) = e^x \cos y + 2e^y \cos x$$

 is increasing fastest starting from the origin $(0, 0)$. What is the rate of increase in this direction ?

 (iii) Compute the divergence $\nabla \cdot \mathbf{A}$ and the curl $\nabla \times \mathbf{A}$ of the vector-valued function

$$\mathbf{A}(x, y, z) = \left(x^2 e^{yz}, y^2 e^{zx}, z^2 e^{xy}\right).$$

5. (i) Determine all the critical points of the following function

$$f(x, y) = 3y - y^3 - x^2$$

and identify each as a local maximum, local minimum or saddle point.

(ii) Show that the following first order differential equation

$$(2y + x^2 + 1)\frac{dy}{dx} = 6x^2 - 2xy$$

is exact and find its general solution. (You may leave your answer in implicit form.)

6. Find the general solution of the equation

$$y'' + 6y' + 9y = 9x^2 + 12x - 7$$

and hence find the solution satisfying $y(1) = 0$, $y'(1) = 3$.

7. Consider the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -2 & 3 & 1 \\ 4 & 2 & 7 \end{pmatrix}.$$

(a) Determine whether or not A is strictly diagonally dominant.

(b) Determine whether or not A is positive definite.

(c) Express the matrix A in the form $A = LU$ where L and U are triangular matrices of the form

$$L = \begin{pmatrix} 1 & 0 & 0 \\ * & 1 & 0 \\ * & * & 1 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{pmatrix}.$$

8. Find the general solution to the following system of differential equations.

$$\dot{x} = -2x + 6y,$$
$$\dot{y} = -2x + 5y.$$