Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2017	MATH2607-WE01

Title: Actuarial Mathematics II

Time Allowed:	2 hours	
Additional Material provided:	Life Tables and Commutation Columns	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for the best **TWO** answers from Section A and the best **TWO** answers from Section B. Questions in Section B carry **ONE and a HALF times** as many marks as those in Section A.

Revision:	

ED01/2017

SECTION A

1. (i) Consider an account where an initial capital F_0 is invested, and at the end of year k, for $k = 1, \ldots, n$, an additional amount r_k is invested (no further transactions). Let F_k be the balance at the end of year k, including r_k, and assume a constant AER $i > 0$.

   (a) Define the discount factor v. Prove that $v^n F_n = F_0 + \sum_{k=1}^{n} v^k r_k$, and give an interpretation of this equality.

   (b) Suppose that $F_0 = 10,000$, $i = 0.06$, and $r_k = r$ for $k = 1, \ldots, 5$. Calculate the required r for a balance of $F_5 = 20,000$ at the end of year 5.

   (ii) Let $i^{(m)}$ be the annual nominal interest rate, convertible m times per year.

   (a) Derive an expression for $i^{(m)}$ in terms of m and AER i.

   (b) Suppose that you invest 5,000 in an account with interest compounded monthly, so $m = 12$, with $i^{(12)}$ equal to 5%. Calculate the corresponding AER. Calculate the balance of this account after 4 months, and also after 6 years.

2. (a) Calculate the present value of an immediate annuity with 15 annual payments of 8,000, and AER 4%.

   (b) Consider an annuity-due with 15 annual payments, and with present value 50,000. Calculate the annual payments in case the AER is 6%.

   (c) Calculate the annual payments of a perpetuity-due with present value 50,000 and AER 6%.

   (d) Suppose that you have a debt of 20,000 at the start of year 1. You decide to repay this in 20 equal payments, one at the end of each of years 1 to 20, calculated on the basis of an AER of 8%. However, at the start of year 4 of this contract, you are offered the opportunity to switch the outstanding debt to an AER of 7%, but this option involves the payment of a penalty cost of $C > 0$ which will be added to the outstanding debt at that moment in time. Calculate all values of C for which this switch is to your benefit.

3. (a) Suppose that μ_{x+t}, the force of mortality at age $x+t$ of a life aged x, is constant for $0 \leq t < 1$, and that $q_x = 0.12$. Calculate the value of t for which ${}_tp_x = 0.9$.

   (b) Derive, as functions of all ages x, the probabilities ${}_{50}p_x$, q_x and ${}_{10|}q_x$ according to the De Moivre model with maximum possible age 120.

   (c) Calculate the following probabilities, using the illustrative life tables provided: ${}_{50}p_{25}$, ${}_{50}q_{25}$, ${}_{40}p_{25+10}$, and ${}_{40|10}q_{25}$, and explain carefully for which events these are the probabilities.

SECTION B

4. Consider a 25-year endowment with benefit payment of 100,000 issued to a life aged 40, based on the provided life and commutation tables with constant AER $i = 0.05$. In case of death of the insured during the period of the contract, the benefit is paid at the end of the year of death.

   (a) Calculate the net single premium of this contract.

   (b) Calculate the level net annual premiums for this contract, assuming that these premiums are paid at the start of each year of the contract if the insured is alive.

   (c) Calculate the level net annual premiums for this contract, assuming that these premiums are paid at the start of each of the first 15 years of the contract if the insured is alive.

   (d) Consider the scenario of part (b), with the following change: 10 years after the start of the contract, the insured decides that he would like to reduce the annual premium payment by 50%. In order to achieve this, he agrees with the company to change the contract such that the death benefit payment remains unchanged, but the pure endowment part of the contract would have payment C. The company further charges, at the moment of this conversion, 1,500 for the costs of the administration, these costs are also to be covered by the future premium payments. Compute the value of C.

5. Consider a 20-year family income insurance policy on a life aged 50, paying 30,000 annually from the end of the year of death. Level net annual premiums are paid at the start of each of the first 10 years of the contract if the policy holder is alive. The AER is 5% and the provided life and commutation tables are used.

   (a) Calculate the level net annual premiums for this contract.

   (b) The insurance company will charge expenses for this contract: collection expenses of 2% of the premiums and administration expenses of 300 per year. Calculate the expense-loaded annual premium.

   (c) Suppose that, at the start of year 8 of the contract, before paying the eighth premium, the insured would like to stop paying premiums. The company agrees not to receive the final 3 premium payments, but of course changes the benefit payments accordingly to match, at that moment in time, the reserve built up from the first 7 premium payments. Calculate the new benefit payments (assuming that no expenses are charged).

6. Consider a 15-year deferred life annuity contract for a life aged 50, with annual payments of 10,000 at the end of each year of the contract if the insured is alive at the time of the benefit payment. However, assume that the first 5 payments are guaranteed. This means that these payments will be made to the estate of the insured in case he is not alive at the moment of payment. Level net annual premiums are to be paid at the start of the first 15 years of this contract, AER is $i = 0.05$.

   (a) Calculate the net annual premiums using the provided life and commutation tables.

   (b) Suppose that the future lifetime of the insured is distributed according to the De Moivre model with maximum age 100. Briefly explain, without calculations, whether these premiums will be larger, smaller or equal to those calculated in part (a).

   (c) For the scenario of part (a), calculate the net premium reserves for this contract at the end of years 1, 14 and 40 of this contract, and briefly discuss these reserves.

END