Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2017	MATH2627-WE01

Title:
Geometric Topology II

Time Allowed:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for the best **TWO** answers from Section A and the best **TWO** answers from Section B. Questions in Section B carry **ONE and a HALF times** as many marks as those in Section A.

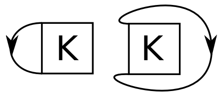
Revision:	

ED01/2017

SECTION A

1. (i) What does it mean to say that a knot is achiral? What does it mean to say that a knot is invertible?

   (ii) Using only Reidemeister moves, show that the trefoil knot is invertible. Indicate clearly on your diagrams where the moves are being applied and only do one move per diagram. (Hint: It may help for you to break the process into stages, noting that the following two diagrams



   are always equivalent.)

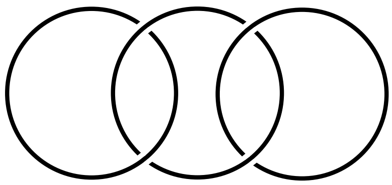
   (iii) Explain how one may prove that the trefoil knot is not achiral. You may refer to any of the techniques seen in the course but need not provide the full accompanying calculations.

2. (a) Given a link diagram D, a *crossing change* is an alteration of D which switches the roles of the overpass and the underpass at a single crossing. Prove that for every link diagram D of n components there is a sequence of crossing changes which may be applied to D resulting in a diagram of the n-component unlink.

   (b) State the defining relations of the absolute polynomial.

   (c) Compute the absolute polynomial of the n-component unlink.

   (d) Calculate the absolute polynomial of the following link:



3. (a) For a map $\gamma\colon S^1 \to S^1$, define the winding number $\omega(\gamma)$ of γ.

   (b) For two maps $\gamma_1, \gamma_2\colon S^1 \to S^1$, prove that $\omega(\gamma_1 \cdot \gamma_2) = \omega(\gamma_1) + \omega(\gamma_2)$. The product function $\gamma_1 \cdot \gamma_2$ is defined by considering S^1 as a subset of $\mathbb{C}$ and setting $(\gamma_1 \cdot \gamma_2)(z) \coloneqq \gamma_1(z) \cdot \gamma_2(z)$.

   (c) Given $f_i\colon \mathbb{C} \to \mathbb{C}$, define $\gamma_i\colon S^1 \to S^1$ by

$$\gamma_i(z) \coloneqq \frac{f_i(z)}{|f_i(z)|}.$$

   Determine $\omega(\gamma_i)$ for:

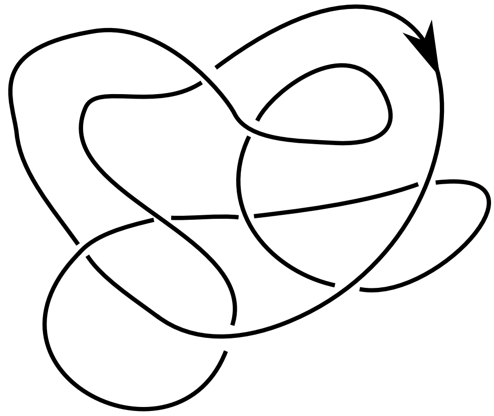
   (i) $f_1(z) = z^4 - \frac{z^2}{4}$;

   (ii) $f_2(z) = \frac{z+4}{z-\frac{1}{3}}$;

   (iii) $f_3(z) = \frac{\bar{z}^2+2\bar{z}}{z^2+3z-\frac{i}{2}z-\frac{3i}{2}}$.

SECTION B

4. (i) (a) Apply Seifert's algorithm to the oriented knot diagram below and compute the genus of the resulting surface.

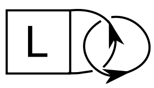


(b) Find an isotopic diagram of the above knot and a corresponding Seifert surface whose genus realises the genus of the knot. Justify your answer.

(ii) For any given oriented knot diagram D, show that the genus of the Seifert surface produced from Seifert's algorithm is unchanged after applying a Reidemeister move of type 1 to D. On the other hand, demonstrate through examples that applying a Reidemeister move of type 2 may or may not change the genus of the resulting surface.

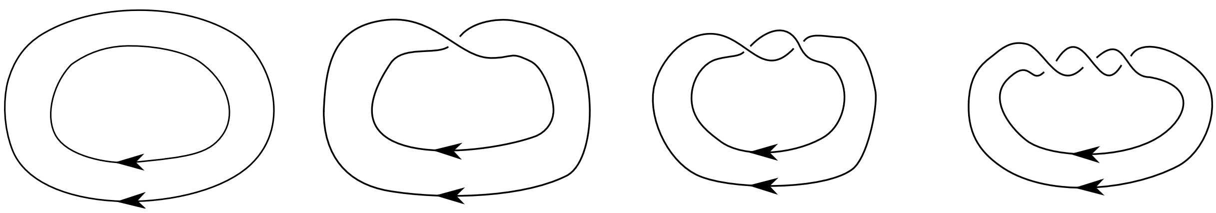
5. (i) Define the Alexander–Conway polynomial ∇_L for an oriented link L. Define the Jones polynomial V_L in terms of the bracket polynomial.

(ii) Given an oriented link L, let L' be a link of the following form:



Find a formula for $\nabla_{L'}$ in terms of ∇_L.

(iii) An infinite family of oriented knots and links $(L^n)_{n\in\mathbb{N}_0}$ is defined according to the picture below (which depicts L^0, L^1, L^2 and L^3), where the diagram L^n has n crossings. Compute $\nabla_{L^n}(1)$ for each $n \in \mathbb{N}$.



6. (i) Define the index of an isolated singularity of a vector field in $\mathbb{R}^2$.

(ii) Draw a diagram of the tangent curves to a vector field of the plane which has an isolated singularity of index -2. Draw another such diagram for a vector field with an isolated singularity of index 0.

(iii) State the Poincaré–Hopf theorem.

(iv) The following surface is given a vector field with a single isolated singularity. The direction of the vector field is illustrated only near to the surface's four boundary components. Determine the index of the singularity.