Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2018	MATH1061-WE01

Title:

Calculus and Probability I paper 1: Calculus

Time Allowed:	1 hour 30 minutes	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks.

Revision:	

ED01/2018

1. (a) Without using Taylor series or L'Hôpital's rule, calculate the limit

$$\lim_{x\to\infty}\left(\sqrt{4x^2+3x+1}-2x\right).$$

(b) Use Taylor series to calculate the limit

$$\lim_{x\to 0}\frac{\log(1+x)}{e^{2x}+\tan x-\cos x}.$$

(c) At $x=\pi/2$ a function $g(x)$ is continuous but is not differentiable.
Show that the function $f(x)=g(x)\cos x$ is differentiable at $x=\pi/2$.

2. (a) Prove that there is at least one solution in the interval $(0,\pi/2)$ of the equation $f''(x)=0$, where

$$f(x)=\int_2^{\sin^2 x}\frac{e^t\log(1+t)}{3+t^2}\,dt.$$

(b) For integer $n\geq 0$ define

$$I_n=\int_1^e(\log x)^n\,dx.$$

Derive a recurrence relation between I_n and I_{n-1}.

3. (a) Solve the initial value problem

$$xyy'+4x^2+y^2=0,\qquad y(1)=-2.$$

(b) Find the general solution of the ordinary differential equation

$$y''+2y'+5y=16e^{-3x}.$$

4. (a) Apply the change of variable $x=e^u\sin(2v)$, $y=e^u\cos(2v)$ to calculate the double integral

$$\iint_D(x^2+y^2)\,dxdy,$$

where D is the disc given by $x^2+y^2\leq 1$.

(b) Consider the function $f(x,y)=(x-y-1)^2+(x+y)^2$.
Calculate ∇f (the gradient of f) and hence identify any stationary points.

5. The function $f(x)$ has period 4π, that is $f(x+4\pi n) = f(x)$ for all integer n, and is given by

$$f(x) = \begin{cases} -1 & \text{if} \quad -2\pi < x < -\pi \\ 0 & \text{if} \quad |x| \leq \pi \\ 1 & \text{if} \quad \pi < x < 2\pi \end{cases}$$

(a) Sketch the graph of $f(x)$ for $x \in [-4\pi, 4\pi]$, including open and closed circles to indicate whether specific points are contained in the graph.

(b) Show that the Fourier series of $f(x)$ is equal to

$$\sum_{m=1}^{\infty} \frac{2}{(2m-1)\pi} \Big(\sin\big((2m-1)x/2\big) - \sin\big((2m-1)x\big) \Big).$$

(c) By evaluating the Fourier series at $x = \pi$, determine the value of

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{2m-1}.$$

(d) Apply Parseval's theorem to determine the value of

$$\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2}.$$