

EXAMINATION PAPER

Examination Session: May

2018

Year:

Exam Code:

MATH1061-WE01

Title:

Calculus and Probability I paper 1: Calculus

Time Allowed:	1 hour 30 minutes			
Additional Material provided:	None			
Materials Permitted:	None			
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.		
Visiting Students may use dictionaries: No				

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks.				

Revision:

1. (a) Without using Taylor series or L'Hôpital's rule, calculate the limit

$$\lim_{x \to \infty} \left(\sqrt{4x^2 + 3x + 1} - 2x \right).$$

(b) Use Taylor series to calculate the limit

$$\lim_{x \to 0} \frac{\log(1+x)}{e^{2x} + \tan x - \cos x}.$$

- (c) At $x = \pi/2$ a function g(x) is continuous but is not differentiable. Show that the function $f(x) = g(x) \cos x$ is differentiable at $x = \pi/2$.
- 2. (a) Prove that there is at least one solution in the interval $(0, \pi/2)$ of the equation f''(x) = 0, where

$$f(x) = \int_{2}^{\sin^2 x} \frac{e^t \log(1+t)}{3+t^2} dt.$$

(b) For integer $n \ge 0$ define

$$I_n = \int_1^e (\log x)^n \, dx.$$

Derive a recurrence relation between I_n and I_{n-1} .

3. (a) Solve the initial value problem

$$xyy' + 4x^2 + y^2 = 0, \qquad y(1) = -2.$$

(b) Find the general solution of the ordinary differential equation

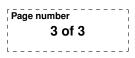
$$y'' + 2y' + 5y = 16e^{-3x}.$$

4. (a) Apply the change of variable $x = e^u \sin(2v)$, $y = e^u \cos(2v)$ to calculate the double integral

$$\iint_D (x^2 + y^2) \, dx dy,$$

where D is the disc given by $x^2 + y^2 \leq 1$.

(b) Consider the function $f(x, y) = (x - y - 1)^2 + (x + y)^2$. Calculate ∇f (the gradient of f) and hence identify any stationary points.



5. The function f(x) has period 4π , that is $f(x + 4\pi n) = f(x)$ for all integer n, and is given by

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$$f(x) = \begin{cases} -1 & \text{if } -2\pi < x < -\pi \\ 0 & \text{if } |x| \le \pi \\ 1 & \text{if } \pi < x < 2\pi \end{cases}$$

- (a) Sketch the graph of f(x) for $x \in [-4\pi, 4\pi]$, including open and closed circles to indicate whether specific points are contained in the graph.
- (b) Show that the Fourier series of f(x) is equal to

$$\sum_{m=1}^{\infty} \frac{2}{(2m-1)\pi} \bigg(\sin\big((2m-1)x/2\big) - \sin\big((2m-1)x\big) \bigg).$$

(c) By evaluating the Fourier series at $x = \pi$, determine the value of

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{2m-1}.$$

(d) Apply Parseval's theorem to determine the value of

$$\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2}.$$