

## **EXAMINATION PAPER**

Examination Session: May

2018

Year:

Exam Code:

MATH1551-WE01

Title:

## Mathematics for Engineers and Scientists

Time Allowed:	3 hours		
Additional Material provided:	Formula sheet		
Materials Permitted:	None		
Calculators Permitted:	No	Models Permitted:	
		Use of electronic calculators is forbidden.	
Visiting Students may use dictionaries: No			

Instructions to Candidates:	Credit will be given for your answers All questions carry the same marks.	to each ques	stion.

**Revision:** 

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- 1. (i) Compute the modulus and argument of  $\left(\frac{3}{5} + \frac{4}{5}i\right)^8$ . (You do not need to find the principal value of the argument.)
  - (ii) Find the modulus and argument for every possible value of  $(i-1)^i$ .
  - (iii) Write down the definitions of  $\cos z$  and  $\sin z$  for complex numbers z in terms of exponential functions. Use them to find all complex solutions to the equation

$$\sin z = 2i \cos z.$$

- 2. (i) Calculate the limit of the sequence  $s_n = \sqrt{4n^2 + 3n} 2n$  as  $n \to \infty$ .
  - (ii) Let a be a non-zero constant. Without using l'Hôpital's Rule, evaluate the limit

$$\lim_{x \to a} \frac{x^2 - a^2}{x^3 - a^3}.$$

(iii) Compute the limit

$$\lim_{x \to 0} \frac{\sin x - x}{\tan x - x},$$

stating any standard results you use.

- (iv) By using the definitions of continuity and differentiability in terms of limits, show that if a function f(x) is differentiable at x = a, then it is also continuous at x = a.
- 3. (i) Find the Taylor polynomial of degree 2 about the point  $x = \pi/2$  of the function  $f(x) = x \cos x$ . Use your result to obtain the limit

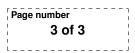
$$\lim_{x \to \pi/2} \frac{x \cos x}{x - \pi/2}.$$

(ii) State Leibniz' Rule for finding the *n*th derivative of a product u(x)v(x). A function y(x) is known to satisfy the differential equation y' = xy. Using Leibniz' Rule or otherwise, show that for integer  $n \ge 1$ ,

$$y^{(n+1)} = xy^{(n)} + ny^{(n-1)}.$$

- (iii) The equation  $f(x) = x^3 2x + 2 = 0$  has a unique real root. Write down the iteration the Newton-Raphson method gives for approximating solutions to this equation. Starting with initial estimate  $x_0 = 1$ , find the next two approximations  $x_1$  and  $x_2$  that the method produces. Do you expect the method to converge to the root ?
- 4. (i) Find a parametric equation for the straight line passing through the points A = (2, -4, -3) and B = (-1, 2, 3). Show that the line intersects the z-axis and find the angle between the line and the z-axis.
  - (ii) Determine an equation for the tangent plane to the surface  $x^2 + 2y^2 + 3z^2 = 20$ at the point (3, 2, 1) in the form ax + by + cz = d.
  - (iii) Compute the divergence  $\nabla \cdot \mathbf{A}$  and curl  $\nabla \times \mathbf{A}$  of the vector-valued function

$$\mathbf{A}(x, y, z) = \left(xy + z^2, \sin(xy + z^2), e^{xy + z^2}\right).$$





5. (i) Determine all the critical points of the surface z = f(x, y) where

$$f(x,y) = (x^2 + y^2)^2 + 2x^2 - 2y^2$$

and classify each as a local minimum, local maximum or saddle point.

(ii) Find the solution to the first order differential equation

$$xy\frac{dy}{dx} = x^2 + y^2$$

satisfying the initial condition y(1) = 2.

6. Find the solution to the second order order differential equation

$$y'' - 2y' + 5y = 5x^2 + x + 4e^x$$

satisfying the initial conditions y(0) = y'(0) = 0.

7. (i) Consider the following set of vectors in  $\mathbb{R}^3$ :

$$S = \left\{ \begin{pmatrix} 1\\0\\0 \end{pmatrix}, \begin{pmatrix} 1\\2\\3 \end{pmatrix}, \begin{pmatrix} -1\\4\\6 \end{pmatrix}, \begin{pmatrix} 2\\6\\9 \end{pmatrix} \right\}$$

Is S a linearly independent set ? Does S span  $\mathbb{R}^3$  ?

(ii) Find the LU decomposition of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 13 \\ -2 & -7 & 4 \end{pmatrix}.$$

Use your result with forward and backward substitution to solve

$$A\mathbf{x} = \begin{pmatrix} 2\\12\\12 \end{pmatrix}.$$

8. Let A be the matrix

$$A = \begin{pmatrix} -5 & 6\\ -9 & 10 \end{pmatrix}.$$

Find a matrix Y so that  $Y^{-1}AY$  is diagonal. Use your result to find a matrix B such that  $B^2 = A$ .