Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2018	MATH1551-WE01

Title:
Mathematics for Engineers and Scientists

Time Allowed:	3 hours	
Additional Material provided:	Formula sheet	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks.

Revision:	

ED01/2018

1. (i) Compute the modulus and argument of $\left(\frac{3}{5}+\frac{4}{5}i\right)^8$.
(You do not need to find the principal value of the argument.)

(ii) Find the modulus and argument for every possible value of $(i-1)^i$.

(iii) Write down the definitions of $\cos z$ and $\sin z$ for complex numbers z in terms of exponential functions. Use them to find all complex solutions to the equation

$$\sin z = 2i\cos z.$$

2. (i) Calculate the limit of the sequence $s_n = \sqrt{4n^2+3n} - 2n$ as $n \to \infty$.

(ii) Let a be a non-zero constant. Without using l'Hôpital's Rule, evaluate the limit

$$\lim_{x\to a} \frac{x^2-a^2}{x^3-a^3}.$$

(iii) Compute the limit

$$\lim_{x\to 0} \frac{\sin x - x}{\tan x - x},$$

stating any standard results you use.

(iv) By using the definitions of continuity and differentiability in terms of limits, show that if a function $f(x)$ is differentiable at $x=a$, then it is also continuous at $x=a$.

3. (i) Find the Taylor polynomial of degree 2 about the point $x=\pi/2$ of the function $f(x) = x\cos x$. Use your result to obtain the limit

$$\lim_{x\to\pi/2} \frac{x\cos x}{x-\pi/2}.$$

(ii) State Leibniz' Rule for finding the nth derivative of a product $u(x)v(x)$.
A function $y(x)$ is known to satisfy the differential equation $y' = xy$. Using Leibniz' Rule or otherwise, show that for integer $n \geq 1$,

$$y^{(n+1)} = xy^{(n)} + ny^{(n-1)}.$$

(iii) The equation $f(x) = x^3 - 2x + 2 = 0$ has a unique real root. Write down the iteration the Newton-Raphson method gives for approximating solutions to this equation. Starting with initial estimate $x_0 = 1$, find the next two approximations x_1 and x_2 that the method produces. Do you expect the method to converge to the root ?

4. (i) Find a parametric equation for the straight line passing through the points $A = (2,-4,-3)$ and $B = (-1,2,3)$. Show that the line intersects the z-axis and find the angle between the line and the z-axis.

(ii) Determine an equation for the tangent plane to the surface $x^2+2y^2+3z^2 = 20$ at the point $(3,2,1)$ in the form $ax+by+cz=d$.

(iii) Compute the divergence $\nabla\cdot\mathbf{A}$ and curl $\nabla\times\mathbf{A}$ of the vector-valued function

$$\mathbf{A}(x,y,z) = \left(xy+z^2, \sin(xy+z^2), e^{xy+z^2}\right).$$

5. (i) Determine all the critical points of the surface $z = f(x, y)$ where

$$f(x, y) = (x^2 + y^2)^2 + 2x^2 - 2y^2$$

and classify each as a local minimum, local maximum or saddle point.

(ii) Find the solution to the first order differential equation

$$xy\frac{dy}{dx} = x^2 + y^2$$

satisfying the initial condition $y(1) = 2$.

6. Find the solution to the second order order differential equation

$$y'' - 2y' + 5y = 5x^2 + x + 4e^x$$

satisfying the initial conditions $y(0) = y'(0) = 0$.

7. (i) Consider the following set of vectors in $\mathbb{R}^3$:

$$S = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ 9 \end{pmatrix} \right\}$$

Is S a linearly independent set ? Does S span $\mathbb{R}^3$?

(ii) Find the LU decomposition of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 13 \\ -2 & -7 & 4 \end{pmatrix}.$$

Use your result with forward and backward substitution to solve

$$A\mathbf{x} = \begin{pmatrix} 2 \\ 12 \\ 12 \end{pmatrix}.$$

8. Let A be the matrix

$$A = \begin{pmatrix} -5 & 6 \\ -9 & 10 \end{pmatrix}.$$

Find a matrix Y so that $Y^{-1}AY$ is diagonal. Use your result to find a matrix B such that $B^2 = A$.