EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2018	MATH1561-WE01

Title:

Single Mathematics A

Time Allowed:	3 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks.

Revision:	

ED01/2018

1. (a) Differentiate $x^{(x^2)}$ with respect to x.
   (b) Find all real solutions x to $\ln(x^2+2) = \ln(x) + 3$
   (c) Compute the following limits:
   $$\lim_{x\to\infty} \frac{2x+1}{x+\cos(x)} \qquad \lim_{x\to 1} \frac{(x-1)\ln(x)}{\sin^2(\pi x)}$$
   State clearly which theorems you have used here, if any.
   (d) Use $\cosh(\operatorname{arcosh}(x)) = x$ to demonstrate that, for $x > 1$,
   $$\frac{d}{dx}\operatorname{arcosh}(x) = \frac{1}{\sqrt{x^2-1}}$$
   (e) State what it means for a function $f(x)$ to be *continuous* at a point $x = a$.

2. (a) Compute the following indefinite integral
   $$\int x(\sin(x^2) + e^{-x})\,dx$$
   (b) Compute the following definite integral
   $$\int_0^{\pi/12} \cos(3x)\sin(6x)\,dx$$
   (c) Compute the following definite integral
   $$\int_5^6 \frac{5x-12}{x^2-5x+6}\,dx$$
   Write your answer in the form $\log(a)$ for some number a.
   (d) Let
   $$w = \frac{z+1}{z-2i}$$
   and let $z = x+iy$ with x and y real. Compute $\mathrm{Re}(w)$ as a function of x and y.
   (e) Find the modulus and argument of
   $$\frac{1}{3+i} + \frac{1-i}{2-i}$$

3. (a) Find all solutions z to the equation
   $$z^4 = \sqrt{3} - i$$
   Give your answers in polar form.
   (b) Express
   $$\frac{\sin(4\theta)}{\sin(\theta)}$$
   as a polynomial in $\cos(\theta)$.
   (c) Find the numbers a, b and c such that
   $$\sin(\theta)^3 = a\sin(\theta) + b\sin(c\theta)$$
   (d) Find all solutions z to the equation
   $$\exp(z^2) = i$$
   Give your answers in polar form.

4. (a) Explain the difference between absolute convergence and conditional convergence.

(b) Determine whether the series $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^2+1}}$ converges or diverges.

(c) Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n^2+1}$ converges absolutely, converges conditionally or diverges.

(d) Find the interval of convergence for the power series $\sum_{k=2}^{\infty} (-1)^k \frac{(2x)^k}{\sqrt[4]{k^2-k}}$.

5. (a) Find the Taylor polynomial $p_2(x)$ for the function $f(x) = \frac{1}{\sqrt{\tan x}}$ about $x = \pi/4$.

(b) Compute the limit $\lim_{x \to \frac{\pi}{4}} \frac{\cot x - 1}{\left(x - \frac{\pi}{4}\right)^2} \left(\frac{1}{\sqrt{\tan x}} - 1 \right)$.

6. (a) Compute the determinant of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 3 & 0 & 1 & 1 \\ 1 & 2 & -1 & 1 \end{pmatrix}.$$

(b) Find for which values of $\lambda \in \mathbb{R}$ the following system of linear equations has one solution, infinitely many solutions or no solutions:

$$\begin{cases} x + 2y + \lambda z = 0, \\ y + 3z = 0, \\ 2x + \lambda^2 y - 7z = 1 - \lambda^2 . \end{cases}$$

7. (a) Let $A = \begin{pmatrix} 0 & 6 \\ -1 & 5 \end{pmatrix}$. Compute A^{16}.

(b) Find a matrix X such that

$$X \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 \\ 4 & 3 & 2 \\ 1 & -2 & 5 \end{pmatrix}$$