Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2018	MATH2627-WE01

Title: Geometric Topology II

Time Allowed:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for the best **TWO** answers from Section A and the best **TWO** answers from Section B. Questions in Section B carry **ONE and a HALF times** as many marks as those in Section A.

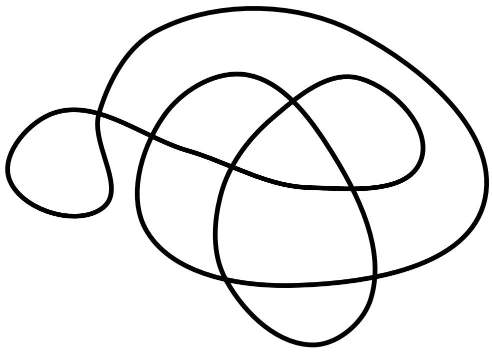
Revision:	

ED01/2018

SECTION A

1. (a) List the Reidemeister moves.

   (b) Add over/underpasses to the crossings of the following diagram to make it a diagram of the unknot. Briefly explain why your choices have produced the unknot.



   (c) Show that the standard diagram of the figure-8 knot can be turned into a diagram of the unknot by flipping the roles of the over/underpass at a single crossing.

2. (a) What is the *writhe* of an oriented link diagram?

   (b) Does the writhe of a *knot* diagram (one of a single component) depend on the orientation assigned to the knot? Is the writhe invariant under any of the Reidemeister moves? Justify your answers.

   (c) Repeat the above question but for oriented link diagrams: does the writhe of an oriented link depend on the orientations we assign to its components? Is the writhe invariant under any of the Reidemeister moves? Justify your answers.

   (d) Prove, using whichever techniques from the course that you wish, that there are infinitely many links of two components up to isotopy.

3. (a) Apply Seifert's algorithm to the standard diagram of the trefoil knot. What is the genus of this surface?

   (b) Why can we be sure that the genus of this surface does equal the genus of the trefoil knot?

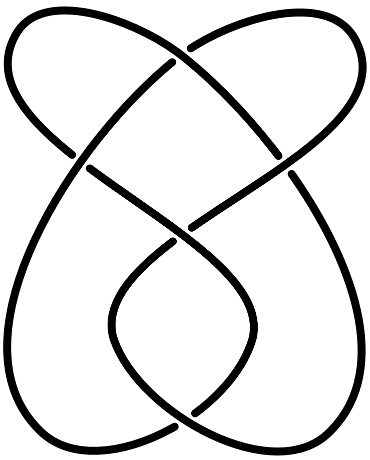
   (c) Draw another diagram of the trefoil knot so that applying Seifert's algorithm produces a surface of different genus to that of the trefoil.

SECTION B

4. (a) State the defining relations of the bracket polynomial. Define the Jones polynomial.

   (b) How are the Jones polynomials of a knot and of the knot's mirror image related? Prove this relationship.

   (c) Calculate the Jones polynomial of the following knot. Is this knot achiral?



5. (a) What is a *simplicial complex* in $\mathbb{R}^n$? What is a *triangulation* of a subspace $X \subseteq \mathbb{R}^n$?

   (b) Suppose that $\mathcal{K}$ is a simplicial complex in $\mathbb{R}^n$ which is a union of two subsets $\mathcal{L}$ and $\mathcal{M}$ which are both also simplicial complexes (these are called *subcomplexes*). Prove that their intersection $\mathcal{L} \cap \mathcal{M}$, if non-empty, is also a simplicial complex. Find a relationship between the Euler characteristics of $\mathcal{K}$, $\mathcal{L}$, $\mathcal{M}$ and $\mathcal{L} \cap \mathcal{M}$.

   (c) State and prove the formula for the Euler characteristic of Σ_g, the surface of genus g.

   (d) A football is stitched together from faces that are pentagons (5-sided polygons) and hexagons (6-sided polygons). It is done in such a way that any two faces, if they intersect at all, intersect along a common edge, and so that at each vertex precisely 3 faces meet. See the figure below. How many pentagons must be used?

6. (a) Explain how to define the *winding number* of a continuous map $\gamma\colon S^1 \to S^1$.

(b) Show that the following function has a well defined winding number and calculate it:

$$\gamma(z) = \frac{3\overline{z}^3 - 4\overline{z}^2 + 2\overline{z}}{|3\overline{z}^3 - 4\overline{z}^2 + 2\overline{z}|}.$$

(c) Briefly explain how we define the index of a vector field at an isolated singularity on an orientable surface.

(d) For each of the following surfaces, either draw a vector field with precisely two singularities of equal, non-zero index, or give a justification as to why such a vector field can not exist:

i. S^2.

ii. The torus.

iii. The surface of genus 2.