**FORMULA SHEET for MATH 3101/4081 : CONTINUUM MECHANICS**

Some vector identities:

$$\nabla\cdot\big(f\boldsymbol{A}\big) = \big(\nabla f\big)\cdot\boldsymbol{A} + f\nabla\cdot\boldsymbol{A} \tag{1}$$

$$\nabla\times\big(f\boldsymbol{A}\big) = \big(\nabla f\big)\times\boldsymbol{A} + f\nabla\times\boldsymbol{A} \tag{2}$$

$$\nabla\times\big(\nabla\times\boldsymbol{A}\big) = \nabla\big(\nabla\cdot\boldsymbol{A}\big) - \Delta\boldsymbol{A} \tag{3}$$

$$\boldsymbol{A}\cdot\nabla\boldsymbol{A} = \tfrac{1}{2}\nabla|\boldsymbol{A}|^2 - \boldsymbol{A}\times\big(\nabla\times\boldsymbol{A}\big) \tag{4}$$

$$\nabla\cdot\big(\boldsymbol{A}\times\boldsymbol{B}\big) = \boldsymbol{B}\cdot\big(\nabla\times\boldsymbol{A}\big) - \boldsymbol{A}\cdot\big(\nabla\times\boldsymbol{B}\big) \tag{5}$$

$$\nabla\times\big(\boldsymbol{A}\times\boldsymbol{B}\big) = \boldsymbol{B}\cdot\nabla\boldsymbol{A} - \boldsymbol{A}\cdot\nabla\boldsymbol{B} + \boldsymbol{A}\nabla\cdot\boldsymbol{B} - \boldsymbol{B}\nabla\cdot\boldsymbol{A} \tag{6}$$

$$\nabla\big(\boldsymbol{A}\cdot\boldsymbol{B}\big) = \boldsymbol{A}\cdot\nabla\boldsymbol{B} + \boldsymbol{B}\cdot\nabla\boldsymbol{A} + \boldsymbol{A}\times\big(\nabla\times\boldsymbol{B}\big) + \boldsymbol{B}\times\big(\nabla\times\boldsymbol{A}\big) \tag{7}$$

In cylindrical coordinates $(r,\theta,z)$:

$$\nabla f = \text{grad}\, f = \boldsymbol{e}_r\partial_r f + \frac{\boldsymbol{e}_\theta}{r}\partial_\theta f + \boldsymbol{e}_z\partial_z f \tag{8}$$

$$\nabla\cdot\boldsymbol{A} = \text{div}\,\boldsymbol{A} = \frac{1}{r}\partial_r(rA_r) + \frac{1}{r}\partial_\theta A_\theta + \partial_z A_z \tag{9}$$

$$\nabla\times\boldsymbol{A} = \text{rot}\,\boldsymbol{A} = \Big(\frac{1}{r}\partial_\theta A_z - \partial_z A_\theta\Big)\boldsymbol{e}_r + \big(\partial_z A_r - \partial_r A_z\big)\boldsymbol{e}_\theta + \frac{1}{r}\big(\partial_r(rA_\theta) - \partial_\theta A_r\big)\boldsymbol{e}_z \tag{10}$$

$$\Delta f = \frac{1}{r}\partial_r\big(r\,\partial_r f\big) + \frac{1}{r^2}\partial_{\theta\theta}f + \partial_{zz}f \tag{11}$$

$$\Delta\boldsymbol{A} = \Big(\Delta A_r - \frac{1}{r^2}A_r - \frac{2}{r^2}\partial_\theta A_\theta\Big)\boldsymbol{e}_r + \Big(\Delta A_\theta + \frac{2}{r^2}\partial_\theta A_r - \frac{1}{r^2}A_\theta\Big)\boldsymbol{e}_\theta + \boldsymbol{e}_z\Delta A_z \tag{12}$$

$$\boldsymbol{B}\cdot\nabla\boldsymbol{A} = \boldsymbol{e}_r\Big(\boldsymbol{B}\cdot\nabla A_r - \frac{B_\theta A_\theta}{r}\Big) + \boldsymbol{e}_\theta\Big(\boldsymbol{B}\cdot\nabla A_\theta + \frac{B_\theta A_r}{r}\Big) + \boldsymbol{e}_z\big(\boldsymbol{B}\cdot\nabla A_z\big) \tag{13}$$

In spherical coordinates $(r,\theta,\phi)$:

$$\nabla f = \text{grad}\, f = \boldsymbol{e}_r\partial_r f + \frac{\boldsymbol{e}_\theta}{r}\partial_\theta f + \frac{\boldsymbol{e}_\phi}{r\sin\theta}\partial_\phi f \tag{14}$$

$$\nabla\cdot\boldsymbol{A} = \text{div}\,\boldsymbol{A} = \frac{1}{r^2}\partial_r\big(r^2A_r\big) + \frac{1}{r\sin\theta}\partial_\theta\big(A_\theta\sin\theta\big) + \frac{1}{r\sin\theta}\partial_\phi A_\phi \tag{15}$$

$$\begin{aligned}\nabla\times\boldsymbol{A} = \text{rot}\,\boldsymbol{A} = {} & \frac{\boldsymbol{e}_r}{r\sin\theta}\big(\partial_\theta(A_\phi\sin\theta) - \partial_\phi A_\theta\big) + \frac{\boldsymbol{e}_\theta}{r}\Big(\frac{1}{\sin\theta}\partial_\phi A_r - \partial_r(rA_\phi)\Big) \\ & + \frac{\boldsymbol{e}_\phi}{r}\big(\partial_r(rA_\theta) - \partial_\theta A_r\big)\end{aligned} \tag{16}$$

$$\Delta f = \frac{1}{r}\partial_{rr}(rf) + \frac{1}{r^2\sin\theta}\partial_\theta\big(\sin\theta\partial_\theta f\big) + \frac{1}{r^2\sin^2\theta}\partial_{\phi\phi}f \tag{17}$$

$$\begin{aligned}\Delta\boldsymbol{A} = {} & \Big(\Delta A_r - \frac{2}{r^2}A_r - \frac{2}{r^2\sin\theta}\big[\partial_\theta(\sin\theta A_\theta) + \partial_\phi A_\phi\big]\Big)\boldsymbol{e}_r \\ & + \Big(\Delta A_\theta + \frac{2}{r^2}\partial_\theta A_r - \frac{A_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\partial_\phi A_\phi\Big)\boldsymbol{e}_\theta \\ & + \Big(\Delta A_\phi + \frac{2}{r^2\sin\theta}\partial_\phi A_r + \frac{2\cos\theta}{r^2\sin^2\theta}\partial_\phi A_\theta - \frac{A_\phi}{r^2\sin^2\theta}\Big)\boldsymbol{e}_\phi\end{aligned} \tag{18}$$

$$\begin{aligned}\boldsymbol{B}\cdot\nabla\boldsymbol{A} = {} & \boldsymbol{e}_r\Big(\boldsymbol{B}\cdot\nabla A_r - \frac{B_\theta A_\theta}{r} - \frac{B_\phi A_\phi}{r}\Big) + \boldsymbol{e}_\theta\Big(\boldsymbol{B}\cdot\nabla A_\theta - \frac{B_\phi A_\phi}{r}\cot\theta + \frac{B_\theta A_r}{r}\Big) \\ & + \boldsymbol{e}_\phi\Big(\boldsymbol{B}\cdot\nabla A_\phi + \frac{B_\phi A_r}{r} + \frac{B_\phi A_\theta}{r}\cot\theta\Big)\end{aligned} \tag{19}$$

Bessel functions $u(r) = J_n(r)$ and $u(r) = Y_n(r)$ are solutions to the ODE

$$r^2u'' + ru' + \big(r^2 - n^2\big)u = 0. \tag{20}$$

Both $J_n(r)$ and $Y_n(r) \to 0$ as $r\to\infty$; $J_n(0) = \delta_{n0}$, and $|Y_n(r)|\to\infty$ as $r\to 0$.